Design and performance analysis of a linear quadratic Gaussian controller in a manufacturing process

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Abstract: - This study presents the design process and performance analysis of Linear Quadratic Gaussian (LQG) controller, which is intended to operate within a *hybrid* production planning and control structure in a manufacturing process. The hybrid structure proposed provides the integration between the production planning and control activities performed at different levels in the plant. A brief description of this structure is given, but the article is primarily dedicated to the design and performance aspects of the LQG controller. The controller is based on a dynamic, discrete-time and a stochastic model, aggregating products and production processes, hence making the use of LQG control technique suitable. Implementation of the resulting control law is performed through the use of the Separation Theorem, which involves the design of a Kalman estimator and a linear quadratic controller, separately. Preliminary simulation results show that the resulting LQG controller is well-suited for production planning and control studies at systems engineering level, within the proposed hybrid structure, and is capable of regulating production under considerable noise or uncertainty reasonably successfully.

Key-Words: Systems engineering, Manufacturing processes, Hybrid control, Linear quadratic Guassian control, Control system design, Kalman filtering.

1 Introduction

The concept of production planning and control, and its dependence on the technology and the organizational forms were studied by many researchers in the past [1], [2], [3], [4], and [5]. The most common finding of these studies is that the new technologies yield higher productivity, provided that they are implemented through appropriate organizational forms [1] and [2]. This aspect is particularly emphasized in [2] where the contribution of new technologies without sound management systems was found to be limited; the conclusions reported were based on an empirical study where four major factors of automation technology were evaluated for fifteen machinery firms. Hence, a system engineering point of view of production planning and control appears to be more beneficial, compared to purely technological or purely managerial approaches. The present study is a typical example for the use of systems engineering techniques in handling production planning and control problems in a manufacturing process.

The application of control systems engineering techniques to manufacturing systems can be traced to [6], [7], and [8], all of which fall within the body of classical control theory. Classical control theory has its limitations when it comes to dealing with multi-input multi-output systems, time-varying systems, or non-linear systems. These were partially overcome with the application of modern control theory to the analysis and design of production-inventory systems, broadening the scope of applications [9], [10], and [11]. The model developed in this study has been borrowed from control systems engineering area, and was adopted earlier for production planning and control purposes [12], and [13].

Later, Yurtseven modified and extended the model to design a hybrid production planning and control system for a manufacturing process [14], and Yurtseven and Buchanan then proposed a similar model that could be used for assessing the effect of new technologies on production [15].

There has been a growing interest in the application of control system engineering techniques to the modelling and control of supply chain systems. The work reported by Perea, Grossmann, et al., employs some ideas from process control to modeling and control of supply chains [16]. Lin, Wong, et. al., report a controller design study and its use on the reduction of bullwhip for a model supply chain [17]. The modeling approach is based on the Z-transform and the controller design is achieved in the frequency domain. Hoberg, Bradley, et. al., applies linear control theory to study the effect of various policies on order and inventory variability which is considered to be the key drivers of supply chain performance [18]. Agaran, Buchanan, Yurtseven believe that the dominant dynamic characteristics of a complex system, such as a supply chain or a complex production-inventory system can be modeled and controlled effectively with the powerful analytical tools of *Modern Control Theory*, as opposed to the classical control theory [19]. They state that the advantages of Modern Control Theory over its classical counterpart; the latter is limited to the analysis of relatively simple systems that are linear, time-invariant, and small dimensioned (i.e., with small number of inputs and outputs). In Modern Control Theory one can handle large-scale systems with several inputs and outputs without too much difficulty. In addition, the powerful techniques developed for linear and time-invariant systems can be extended to non-linear, time-varying, and stochastic systems effectively. In addition, it is possible to filter stationary or non-stationary noise present in signals through high-performance filters such as Kalman filters, design optimal control policies, and make use of adaptive techniques to update model parameters and control policies for more effective control.

The control-theoretic approach, like the other analytical tools, suffer from a major disadvantage; it is almost impossible to formulate complex issues such as organizational resistance to change, inter-functional or inter-organizational conflicts, team-oriented performance measures, customer relationship management, etc., adequately. Min and Zhou suggest that the analytical tools alone are not sufficient to represent the realities of complex systems [20]. According to them, the traditional mathematical programming techniques can be used to model inter-functional integration, but realistic representations of such systems can be found through IT-based models that make use of model-based decision support systems (DSS). Such DSS's have the potential of representing all the analytical and non-analytical aspects of complex systems in a more realistic manner. Hence, the work reported here is seen as a part of an on-going research where the overall objective is to develop a DSS for managing the manufacturing system under study. In other words, the model and the associated controller developed in this study will be a part of a DSS; it will be integrated with some other analytical/non-analytical tools within the DSS to cope with the ill-structured, strategic, and behavioral issues involved in the system.

The work reported in this article will be presented in the following order: the principles adopted in the design of the overall hybrid production planning and control system will be summarized in the next section. Descriptions of the plant model, the LQG controller structure, the controller design procedure will follow this.

2 Principals of Design

The reader will find here only a summary of the ideas considered during the design of the hybrid production and planning control system; details can be found in [14]. The hybrid control structure proposed is shown in Fig. 1. This structure is based on a concept developed by Kohn, James, et.al. [21]. The plant or process under consideration is a workshop. The production strategies developed by the top management is translated into a set of production targets by Translator I, and then fed into the Production Planning unit. Typically, weekly production plans are prepared within this unit. Note that the function of Translator I is to formulate production strategies set by the top management, which may be a mixture of quantitative and qualitative statements, into quantitative production targets in a specific format. Translator II translates these production plans into a specific form acceptable by the Scheduler. In turn, the Scheduler has the task of producing specific, typically daily schedules. Translator III transforms these schedules into specific control settings, or production trajectories that are used by the controller. The hybrid nature of the control structure provides the "glue" between the event-based systems and continuum systems in the control hierarchy. The design of the control hierarchy, with its coherent control objectives and coordination schemes, requires a formal design procedure. The models that are used for production planning, scheduling, and controlling activities will have to be different; they normally have an increasing size, complexity, and level of detail as one goes down the hierarchy. Similarly, the time horizon considered by these models will need to decrease with decreasing level of hierarchy. Some discussion related to this topic can be found in [21] and [22].

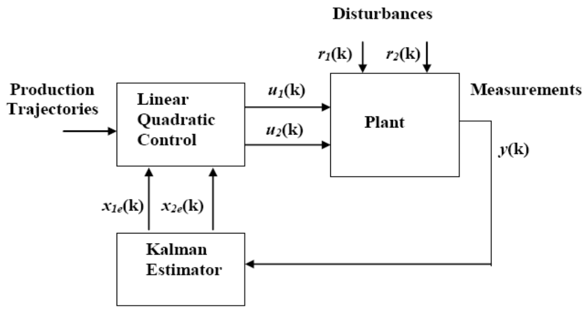

Fig. 1. The proposed hybrid production planning and control structure

3 The Controller Structure

The objective of the controller is to ensure that the production schedules prepared by the Scheduler are implemented properly. The controller may also be used as a pre-planning tool, providing the opportunity to systems engineers to test the possible contributions of the newer technologies and/or organizational forms into production [14]. The controller design is based on a dynamic model, providing the opportunity to investigate the variations in production under different control policies, at different production stages, as time progresses. The model is a discrete-time type, hence well suited to the discrete nature of the manufacturing process. Furthermore, its stochastic nature allows the systems engineer to incorporate the uncertainties involved in the process, providing some flexibility in the modeling of such complex phenomena. In order to keep the model at a reasonable size, products and production processes are aggregated. The aggregate aspect of the model allows the systems engineer to suppress the details and bring out the dominant characteristics of the production process, providing a systems perspective.

A block diagram of the LQG controller is shown in Fig. 2. The input to the controller is the vector of production trajectories. The controller generates the optimum control vector with components of u_{11}(k), u_{12}(k), u_{21}(k), and u_{22}(k) , in period k. The former two represent increased or decreased number of machines at stages 1 and 2, respectively, and the latter two are the amount of overtime or under-time work exercised, at stages 1 and 2, respectively. The plant output $\boldsymbol{y}$(k) is the available measurements. Due to the difficulties and cost involved in the measurement process, it is assumed that only x_{21}(k) and x_{22}(k) can be measured, which are the inventory levels at stages 1 and 2, respectively, in period k. A Kalman estimator is employed to estimate $\boldsymbol{x_{1e}}$ and $\boldsymbol{x_{2e}}$, which are the best estimates of $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$, respectively. The vector $\boldsymbol{r_1}$(k) is a stochastic variable representing the unpredictable variations in the number of disabled or repaired machines during the period k. Similarly, The vector $\boldsymbol{r_2}$(k) is another stochastic variable, representing the unpredictable variations in demand to the products in period k.

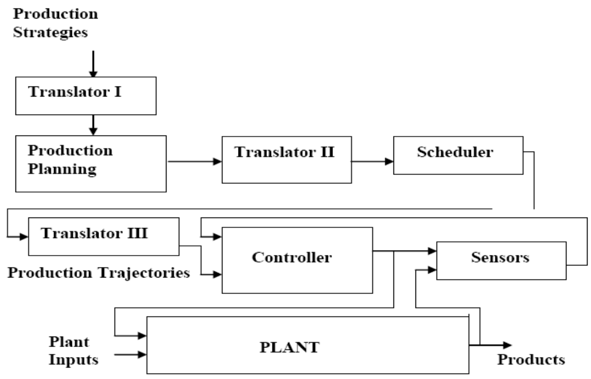

Fig. 2. A block diagram of the LQG controller.

The objective of the controller is to regulate the plant around the nominal operating conditions. The linear model equations that represent small deviations from the nominal operating conditions are given by [13]:
The transition of the number of machines in two successive periods is given by:

$$\boldsymbol{x_1}(k+1) = \boldsymbol{x_1}(k) + \boldsymbol{u_1}(k) + \boldsymbol{r_1}(k)\,,\quad k = 0, 1, 2,\ldots\ldots, N-1 \qquad (1)$$

where the variables are as defined above.
The corresponding vectors are defined in the forms of:

$$\boldsymbol{x_1}(k) = (\boldsymbol{x_{11}}(k),\ldots\ldots\ldots\ldots, \boldsymbol{x_{1j}}(k))'$$
$$\boldsymbol{u_1}(k) = (\boldsymbol{u_{11}}(k),\ldots\ldots\ldots\ldots, \boldsymbol{u_{1j}}(k))'$$
$$\boldsymbol{r_1}(k) = (\boldsymbol{r_{11}}(k),\ldots\ldots\ldots\ldots, \boldsymbol{r_{1j}}(k))'$$

where $\boldsymbol{x_{1j}}$(k), $\boldsymbol{u_{1j}}$(k) and $\boldsymbol{r_{1j}}$(k) represent these quantities at the jth production stage in period k = 0, 1, 2,…., N-1. Note that the symbol ´ indicates a matrix transposition operation.
The inventory level at the beginning of period k is given by:

$$\boldsymbol{x_2}(k+1) = \boldsymbol{x_2}(k) + \boldsymbol{p}(k) - \boldsymbol{r_2}(k),\quad k = 0, 1, 2,\ldots.., N-1 \qquad (2)$$

where the variables are as defined earlier. The vectors have the forms of:

$$\boldsymbol{x_2}(k) = (\boldsymbol{x_{21}}(k),\ldots\ldots\ldots\ldots, \boldsymbol{x_{2i}}(k))'$$
$$\boldsymbol{p}(k) = (\boldsymbol{p_1}(k),\ldots\ldots\ldots\ldots, \boldsymbol{p_i}(k))'$$
$$\boldsymbol{r_2}(k) = (\boldsymbol{r_{21}}(k),\ldots\ldots\ldots\ldots, \boldsymbol{r_{2i}}(k))'$$

$\boldsymbol{x}_{2i}$(k), $\boldsymbol{p}_i$(k) and $\boldsymbol{r}_{2i}$(k) represent these quantities at the ith production stage in period k = 0, 1, 2,…., N-1.

The relation between the production time and the amount of products can be described by a linear approximation as

$$\mathbf{t}\ \boldsymbol{p}(k) = \boldsymbol{b}_p(k) \tag{3}$$

where $\boldsymbol{b}_p(k)$ is the production time in period k with $\boldsymbol{b}_p(k) = (\boldsymbol{b}_{p1}(k),\ldots, \boldsymbol{b}_{pj}(k))'$, $\boldsymbol{b}_{pj}(k)$ representing this quantity at production stage j; **t** is the machining matrix with **t** = (t_{ij}) with a dimension of jxi; t_{ji} represents the time required to produce one unit of product

i (= 1,2,… .,I) at stage j (= 1, 2,….., J).

The amount of products can then be written as:

$$\boldsymbol{p}(k) = (\mathbf{t}^+)\, \boldsymbol{b}_p(k) \tag{4}$$

where ($\mathbf{t}^+$) is the pseudo inverse of **t**. **t** is a square matrix when the number of products is equal to the number of production stages, and its inversion is easy. However, in some cases this is not a square matrix and its inverse has to be calculated through a special algorithm [13].

The production time in period k is calculated as follows:

$$\boldsymbol{b}_p(k) = \mathbf{r_p}\ \boldsymbol{x}_1(k) + \boldsymbol{u}_2(k),$$
$$k = 0, 1, 2,\ldots, N\text{-}1 \tag{5}$$

where $\boldsymbol{u}_2(k)$ is the vector of overtime or under-time with $\boldsymbol{u}_2(k) = (\boldsymbol{u}_{21}(k),\ldots, \boldsymbol{u}_{2j}(k))'$, $\boldsymbol{u}_{2j}(k)$ representing this quantity at production stage j. $\mathbf{r_p}$ represents the regular working time matrix with $\mathbf{r_p}$ = Diagonal ($\mathbf{r_{pj}}$), with a dimension of jxj, $\mathbf{r_{pj}}$ being the regular working time at stage j = 1, 2,…., J.

Substituting equations (4) and (5) into (2) yields

$$\boldsymbol{x}_2(k+1) = \boldsymbol{x}_2(k) + (\mathbf{t}^+)\, \mathbf{r_p}\, \boldsymbol{x}_1(k) + (\mathbf{t}^+)\, \boldsymbol{u}_2(k) - \boldsymbol{r}_2(k),$$
$$k = 0, 1, 2,\ldots, N\text{-}1 \tag{6}$$

The vector-matrix form of equations (1) and (6) can be written as:

$$\boldsymbol{x}(k+1) = \mathbf{a}\, \boldsymbol{x}(k) + \mathbf{b}\, \boldsymbol{u}(k) + \mathbf{c}\, \boldsymbol{r}(k),$$
$$k = 0, 1, 2,\ldots, N\text{-}1. \tag{7}$$

where

$$x(k) = \begin{bmatrix} x(k)_1 \\ x(k)_2 \end{bmatrix} \quad u(k) = \begin{bmatrix} u(k)_1 \\ u(k)_2 \end{bmatrix} \quad r(k) = \begin{bmatrix} r(k)_1 \\ r(k)_2 \end{bmatrix}$$

$$a = \begin{bmatrix} I & 0 \\ (t^+)r_p & I \end{bmatrix} \quad b = \begin{bmatrix} I & 0 \\ 0 & (t^+) \end{bmatrix} \quad c = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$$

4 The design procedure

The mathematics of the LQG control is well known, hence they will not be repeated here. Instead, the design approach adopted, and the criteria used in the selection of the critical design parameters will be explained, followed by a discussion on how simulation experiments were performed, and the results obtained. The reader will find the full information related to LQG control design in [23], [24], and [25]. The system vector-matrix equations and data that are used during the simulation studies for various purposes are given in the Appendix. All design and simulation studies were performed using Matlab.

The solution to the stochastic optimal control problem at hand is found through the well-known Separation Theorem or Certainty Equivalence Principle. According to this theorem, first an optimum estimator estimates the states of the model, ignoring the optimum control problem, and optimum control is then computed treating the estimated states as deterministic quantities. A *two product-two stage* case is considered in this study, with the following data:

$$t = \begin{bmatrix} 1 & 0.5 \\ 3 & 2 \end{bmatrix} \qquad \mathbf{r_p} = \begin{bmatrix} 4 & 0 \\ 0 & 10 \end{bmatrix}$$

Here, **t** represents the machining time matrix, and $\mathbf{r_p}$ represents the regular working time matrix. The reader should note that $\mathbf{r_p}$ is a diagonal matrix, whereas **t** is an off-diagonal matrix, as expected. The plant state-space and measurement equations are put into the following standard form to be able to perform the design:

$$\boldsymbol{x}(k+1) = \mathbf{A}\, \boldsymbol{x}(k) + \mathbf{B}\, \boldsymbol{u}(k) + \mathbf{G}\, \boldsymbol{w}(k) \tag{8}$$
$$\boldsymbol{y}(k) = \mathbf{C}\, \boldsymbol{x}(k) + \mathbf{D}\, u(k) + \mathbf{H}\, w(k) + \boldsymbol{v}(k) \tag{9}$$

where $\boldsymbol{w}(k)$ and $\boldsymbol{v}(k)$ are the random processes associated with process noise and measurement noise, respectively. **A**, **B**, **G**, **C**, and **D** are the corresponding system matrices, as given in the Appendix. **C** was chosen so that only the 3rd and 4th state variables are available for measurement. The reader should note that the uncontrolled plant is unstable.

Firstly, the (deterministic) LQ controller was designed, ignoring the noise processes. This was achieved through the use of Matlab's *dlqr* command. The optimal control law is then computed to minimize the loss function J_c, where J_c = ($\boldsymbol{x}'\mathbf{Q_c}\, \boldsymbol{x} + \boldsymbol{u}'\, \mathbf{R_c}\, \boldsymbol{u}$). $\mathbf{Q_c}$ and $\mathbf{R_c}$ represent the weighting matrices for the state and control vectors, respectively. Several combinations of $\mathbf{Q_c}$ and $\mathbf{R_c}$ were simulated in order to tune the controller's performance. First, the *steady-state* optimal control law was calculated through the command *dlqr*. The Kalman estimator was designed through Matlab's KALMAN command. The execution of this command requires the formulation of a quadratic loss function, similar to the one given above. This loss function contains $\mathbf{Q_n}$ and $\mathbf{R_n}$, which are the process noise and measurement noise covariance matrices, respectively. Once again, their values were chosen after some tedious tuning studies. A *steady-state* Kalman estimator was designed, as opposed to a time-varying one, since it satisfies the requirements of the regulator under consideration. The reader should also note that the stochastic variables $\boldsymbol{r_1}$(k), given in equation (1), and $\boldsymbol{r_2}$(k), given in equation (2), are included into the expression $\mathbf{G}\boldsymbol{w}$(k).

4 Conclusion

The design and performance analysis of a LQG controller for a complex manufacturing system was presented. The controller is intended to operate in a *hybrid* production planning and control structure where three translators serve as the 'glue' between various subsystems of the production planning, scheduling and control activities in the structure. In this article, a brief description of the proposed hybrid control structure was given, and the design procedure and performance analysis of the LQG controller was presented, fully. It was shown how products and production stages can be aggregated to construct a dynamic, discrete-time, and a stochastic model, and how a LQG controller can be designed. Preliminary simulation studies conducted show that the resulting controller is able to regulate the plant under considerable noise or uncertainty reasonably successfully. Furthermore, more research needs to be conducted in the direction of designing the remaining components of the hybrid production planning and control system, and test system's performance under realistic operating conditions.

APPENDIX

The plant state-space and measurement equations that are used for the design of LQG controller are:

$$\boldsymbol{x}(k+1) = \mathbf{A}\,\boldsymbol{x}(k) + \mathbf{B}\,\boldsymbol{u}(k) + \mathbf{G}\,\boldsymbol{w}(k) \qquad \text{(A1)}$$

$$\boldsymbol{y}(k) = \mathbf{C}\,\boldsymbol{x}(k) + \mathbf{D}\,\boldsymbol{u}(k) + \mathbf{H}\,\boldsymbol{w}(k) + \boldsymbol{v}(k) \qquad \text{(A2)}$$

where $\boldsymbol{x}$(k) and $\boldsymbol{u}$(k) are as defined in the main text. $\boldsymbol{w}$(k) and $\boldsymbol{v}$(k) are the process noise and measurement noise, respectively, with the following properties:
$E\{\boldsymbol{w}\} = E\{\boldsymbol{v}\} = 0$; $E\{\boldsymbol{w}\boldsymbol{w}'\} = \mathbf{Q_n}$; $E\{\boldsymbol{v}\boldsymbol{v}'\} = \mathbf{R_n}$; $E\{\boldsymbol{w}\boldsymbol{v}'\} = 0$; E is the expected value operator; $\mathbf{Q_n}$ is the process noise covariance matrix; $\mathbf{R_n}$ is the measurement noise covariance matrix.

The system matrices are:

$$A = \begin{bmatrix} I & 0 \\ (t^{+})r_p & I \end{bmatrix} \quad B = \begin{bmatrix} I & 0 \\ 0 & (t^{+}) \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\mathbf{D}$ is a 2x4 zero matrix, $\mathbf{G}$ is a 4x4 identity matrix, $\mathbf{H}$ is a 2x4 zero matrix, with

$$t = \begin{bmatrix} 1 & 0.5 \\ 3 & 2 \end{bmatrix} \qquad \mathbf{r_p} = \begin{bmatrix} 4 & 0 \\ 0 & 10 \end{bmatrix}$$

While designing the Kalman estimator, several values of $\mathbf{Q_n}$ and $\mathbf{R_n}$ were simulated. Their numerical values were tuned to:

$$Q_n = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix} \quad ,R_n \text{ is an 2x2 identity matrix}$$

Similarly, while designing the linear quadratic control, the weighting matrices of the objective function were fixed to the following values after some tuning studies:
$\mathbf{Q_c}$ is a 4x4 identity matrix, where its diagonal elements represent the weighting of the state variables. **Rc** is a 4x4 diagonal matrix with diagonal elements equal to 0.1 which represent the weighting of the control variables.

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