DISCRETE DECENTRALIZED OBSERVATION SCHEMES OF LARGE SCALE INTERCONNECTED SYSTEMS

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Abstract: **-** A new approach for the design of discrete decentralized observation schemes for large-scale interconnected systems is considered. The design is based on stability result that employs the notion of block diagonal dominance in matrices and the reasonable bound estimates for the discrete Lyapunov matrix equation. The major contribution of this paper is the demonstration of how the observer gains can be tailored systematically to the existing interconnection pattern within the overall system. Although the present results are developed in the context of decentralization observation, they can be extended to the design of decentralized stabilization and to the design of decentralized model reference adaptive identification schemes. Simulation results on a numerical example are given to verify the proposed design.

Key-Words: - Large scale system, Observers, Diagonal dominance, Continuous Lyapunov equation, Discrete Lyapunov equation, Decentralized control.

1 Introduction

Large–scale interconnected systems can be found in such diverse fields as electrical power systems, space structures, manufacturing process, transportation, and communication. An important motivation for the design of decentralized schemes is that the information exchange between subsystems of a large –scale system is not needed ; thus, the individual subsystem controllers are simple and use only locally available information. Decentralized control of large-scale systems has received considerable interest in the systems and has benefit from numerous studies. Early work in this area can be found in [5.6.7]. More recently M. Sundareshan and R. M. Elbanna [1],[2] presented a systematic constructive procedure based on a stability result that employs the notion of block-diagonal dominances in matrices. But the implementation of these controllers is very complicated and the obtained gains are very high.

To reduce the controller's gains F.Elmarjany and N.Elalami [3] studied the decentralized stabilization via eigenvalues assignment and developed the sufficient condition under which exponential stabilization with a prescribed convergence rate is achieved. The obtained gains are smaller than those found in other designs. M.Zazi and N.Elalami [4] extended the previous works to discrete time systems and developed a new and simple approach for the design of discrete decentralized controllers of large scale interconnected linear systems.

In many practical situations, complete state measurements are not available at each individual subsystem for decentralized control; consequently, one has to consider decentralized feedback control based on measurements only or design decentralized observers to estimate the state of individual subsystems that can be used for estimated state feedback control. There has been a strong research effort in literature towards the problem of designing observers. Some applications of these designs have been made to the observation problems arising in such diverse areas as spacecraft control and control of industrial manipulators.

An approach to decentralized observation that has yielded useful results [2] is to first construct a set of local observers for the independent subsystems and then to incorporate appropriate compensatory signals in order to account for the presence of interconnections among the subsystems. The objective of this paper is to extend this work to discrete time systems and develop a new and simple algorithm for the design of discrete decentralized observers of large scale interconnected linear systems.

2 Problem Formulation

Consider a large-scale discrete system s described as an interconnection of N subsystems $s_1, s_2, \ldots s_N$, by:

$$x_i(k+1) = A_i x_i(k) + B_i u_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij} x_j(k)$$

$$y_i(k) = C_i\, x_i(k) \quad i = 1........N \qquad (1)$$

$x_i \in \Re^{ni}$ is the state of the subsystem s_i, $u_i \in \Re^{mi}$ is its input vector and $y_i \in \Re^{pi}$ is its output vector.

$\sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij}$ is the term due to interconnection of the other subsystems. Ai, Bi, and Ci are matrices of appropriate dimensions.

It assumed that all pair (A_i, B_i) are controllable and (A_i, C_i) are completely observable $\forall$ i, j=1,2,…..N.

Let us also consider the observation scheme:

$$\hat{x}_i(k+1) = (A_i - l_i C_i)\hat{x}_i(k) + B_i u_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} \Gamma_{ij}\hat{x}_j(k) \qquad (2)$$

$l_{ij} \in R^{n_i \times n_j}$ are the observer gains to be suitably selected. When $H_{ij} \neq 0$, the selection of $\Gamma_{ij} = H_{ij}$ $\forall$ i, j=1,2,…..N results in the dynamics of the estimation error being governed by:

$$e_i(k+1) = (A_i - l_i C_i)e_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij} e_j(k) \qquad (3)$$

The problem of interest then is the choice of l_i such that the overall error system

$$e(k+1) = (A + H - lC)e(k) \qquad (4)$$

Where $e=[e_1^T\ e_2^T\ \ldots e_N^T]$ A=diag $[A_1\ A_2 \ldots A_N]$, l=diag $[l_1\ l_2 \ldots l_N]$, and C=diag $[C_1\ C_2\ \ldots C_N]$,is asymptotically stable.

Recognition of this relation between the stability of the error system and the design of the observation scheme enables one to employ the available results from the stability and the stabilization of large-scale systems to the observer design problem.

We first show how to find a local controller in the continuous and discrete-time case.

Consider a large scale continuous time system

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij} x_j(t)$$

$$y_i = C_i\, x_i(t) \quad i = 1........N \qquad (5)$$

The objective is to find a decentralized linear constant feedback control $u_i(t) = -K_i x_i(t)$ to exponentially stabilize the system(5).

Applying the i controller to the plant (5) give:

$$\dot{x}_i(t) = F_i x_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} H_{ij} x_j(t) \qquad i = 1...N \qquad (6)$$

Where $F_i = A_i - B_i K_i$

Let $\lambda_{min}(X) et\, \lambda_{max}(X)$ respectively denote the minimum and the maximum of reel matrix X, the notation λ(X) and σ(X) denote the eigenvalue and singular value of the matrix X, also for any $P \in \Re^{n*m}$ $\|P\| = \lambda_{max}^{\frac{1}{2}}(P^T P) = \sigma_{max}(P)$ Let $\mu(.)$ Let

Let $\mu(.)$ denote the matrix measure induced by some vector or matrix norm and defined by the formula

$$\mu(A) = \lim_{\theta \to 0^+} \frac{\|I + \theta A\| - 1}{\theta}$$

The matrix measure induced by the 2-normis denoted by μ_2(A) and $\mu_2(A) = \frac{1}{2}\lambda_1(A + A^T)$. Moreover, the matrix measure μ_M (A) is given by $\mu_M(A) = \frac{1}{2}\lambda_1(MAM^{-1} + A^T) = \mu_2(M^{1/2} A M^{-1/2})$

Definition 2.1 Let $A \in \Re^{n\times n}$ be partitioned in the form:

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & & & \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix} \qquad (7)$$

Where $A_{ii} \in \Re^{ni\times ni}$ and $A_{ij} \in \Re^{ni\times nj}$ i, j=1, 2…n.

If A_{ii} are non singular and

$$\left\|A_{ii}^{-1}\right\|^{-1} \geq \sum_{\substack{j=1\\ j\neq i}}^{n} \left\|A_{ij}\right\|, \forall \text{ i=1,2…n} \qquad (8)$$

Then A is said to be block diagonal dominant relative to the partitioning in (7). If strict inequality holds in (8), then A is strictly block diagonal dominant.

Lemma 2.2

Let the matrix A partitioned as in (7) satisfy the conditions:

(i) $A=A^T$

(ii) A_{ii}=1,2….s. are positive definite

(iii) A is strictly block diagonal dominant

Then, all eigenvalues of A are real and positive.

Lemma 2.3 Let M be a positive definite matrix satisfying μ_M $(F_i)< 0$. Let spec $(F_i) \subset$ LHP $\forall$ i=1, 2….N and let P_i the symmetric matrix solution of the Lyapunov equation

$$F_i^T P_i - P_i F_i + Q_i = 0 \qquad (9)$$

For an arbitrarily selected symmetric matrix $Q_i \in \Re^{ni \times ni}$

$$\lambda_1(P_i) \le \frac{\lambda_1(M)\lambda_1(M^{-1}Q_i)}{-2\mu_M(F_i)} \quad (10)$$

In particular, if μ_2(A)<0, then we have

$$\lambda_1(P_i) \le \frac{\lambda_1(Q_i)}{-2\mu_2(F_i)} \quad (11)$$

Theorem 2.4

Let spec (F_i) $\subset$ LHP $\forall$ i=1, 2,….N and let P_i the symmetric matrix solution of the Lyapunov equation

$$F_i^T P_i - P_i F_i + Q_i = 0$$

For an arbitrarily selected symmetric matrix $Q_i \in \Re^{ni \times ni}$, then (7) is asymptotically stable if

$$\lambda_{\min}(Q_i) \succ \alpha_i \sum_{\substack{j=1 \\ j \ne i}}^{N} \|H_{ij}\| + \sum_{\substack{j=1 \\ j \ne i}}^{N} \alpha_j \|A_{ji}\| \quad (12)$$

Where $\alpha_i = \dfrac{\lambda_1(M)\lambda_1(M^{-1}Q_i)}{-2\mu_M(F_i)}$ $\forall$ i=1,2…..N

Proof: see [3]

The objective of this paper is to extend this work to discrete systems, for that we must initially seek the sufficient condition for the existence of discrete decentralized controllers.

Let us consider a discrete large-scale system described as interconnections of N subsystems by:

$$x_i(k+1) = F_i x_i(k) + \sum_{1}^{N} H_{ij} x_j(k) \quad (13)$$

$\forall$ i=1, 2…..N

F_i is asymptotically stable matrix.

Theorem 2.5

Let F_i an asymptotically stable matrix $\forall$ i=1, 2… N and let P_i the symmetric matrix solution of the discrete algebraic Lyapunov matrix equation

$$F_i^T P_i F_i - P_i + Q_i = 0$$

For an arbitrarily selected symmetric matrix $Q_i \in \Re^{ni \times ni}$, then (13) is asymptotically stable if

$$\lambda_{\min}(Q_i) \succ \lambda_{\max}(P_i)\|F_i\| \sum_{\substack{j=1 \\ j \ne i}}^{N} \|H_{ij}\| + \sum_{\substack{j=1 \\ j \ne i}}^{N} \lambda_{\max}(P_j)\|H_{ij}\|\|F_j\| + \sum_{\substack{k=1 \\ k \ne i}}^{N} (\lambda_{\max}(P_k)\|H_{ki}\| (\sum_{\substack{j=1 \\ j \ne i}}^{N} \|H_{kj}\|)) + \sum_{\substack{j=1 \\ j \ne i}}^{N} \lambda_{\max}(P_j)\|H_{ji}\|^2 \quad (14)$$

Proof: selecting

$V(x(k)) = x^T(k)\, P\, x(k)$,

$P = diag(P_1, P_2, \cdots P_N)$ as a discrete Lyapunov function and evaluating its variation along the trajectories of (13)

Δ V(x)=V(x(k+1))-V(x(k))

$= x^T(k)\,(F^T P H + H^T P F + H^T P H - Q)\,x(k)$

$= -x^T(k)\, W\, x(k)$

Where

$Q = diag(Q_1, Q_2, \cdots\cdots Q_s)$

$F = diag[F_1 \quad F_2 \quad \cdots \quad F_N],\quad H = [H_{ij}]\, i, j = 1..N$

$$W = Q - F^T P H - H^T P F - H^T P H \quad (15)$$

Satisfies $W = W^T$

The diagonal elements:

$$W_{ii} = Q_i - \sum_{j=1(j \ne i)}^{N} H_{ji}^T P_j H_{ji}, \text{ i=1,2…..,N.}$$

$W_{ii}^T = W_{ii}$,

$$W_{ij} = -F_i^T P_i H_{ij} - H_{ji}^T P_j F_j - \sum_{\substack{k=1 \\ k \ne i}}^{N} H_{ki}^T P_k H_{kj}$$

It is simple to observer that for i $\ne$ j:

$$\|W_{ij}\| \le \lambda_{\max}(P_i)\|F_i\|\|H_{ij}\| + \lambda_{\max}(P_j)\|H_{ji}\|\|F_j\| + \sum_{\substack{k=1 \\ k \ne i}}^{N} \lambda_{\max}(P_k)\|H_{ki}\|\|H_{kj}\|$$

If condition (14) is checked then $W_{ii} \succ 0$ and $\|W_{ii}^{-1}\|^{-1} \succ \sum_{\substack{j=1 \\ j \ne i}}^{N} \|W_{ij}\|$ and From lemma 2.2 , W is positive definite. For more details see [4].

Many researchers have developed results of upper bounds for discrete Lyapunov matrix $F^T P F - P + Q = 0$.All the existing results are based on the assumptions of $\lambda_1(F\,F^T) \prec 1$. This is obviously restrictive because the stability of F does not guarantee this assumption.

To cover the case that $\lambda_1(F\,F^T)$ is not inside the unit circle, Dong-Gi Lee,Gwang-Hee Heo, and Jong-Myung Woo [8] uses the similarity transformation and sets:

$\hat{P} = T^T P\, T,\quad \hat{Q} = T^T Q\, T,\quad \hat{F} = T^{-1} F\, T$.

Then the modified Lyapunov equation is obtained:

$(T^T F^T T^{-T})(T^T P\, T)\,(T^{-1} F T) - T^T P\, T + T^T Q\, T = 0$

Theorem 2.6
Let the positive defined matrix P be the solution to (16) If $\sigma_1(\hat{F}) < 1$

$$\lambda_i(P) \le \lambda_1(E)\left[\lambda_i\left[\frac{\lambda_1(E^{-1}Q)\hat{F}^T\hat{F}}{[1-\sigma_1^2(\hat{F})]} + E^{-1}Q\right]\right]$$
$$1 \le i \le n \qquad (16)$$

Where $E = T^{-T}T^{-1}$

3 An algorithm for discrete decentralized observation

As discussed earlier, the error system is described by

$$e_i(k+1) = (A_i - l_iC_i)e_i(k) + \sum_{\substack{j=1 \\ j\neq i}}^{N} H_{ij}e_j(k)$$

$$e_i(k) = x_i(k) - \hat{x}_i(k),\; F_i = A_i - l_iC_i \qquad (17)$$

Theorem 3.1
Let F_i an asymptotically stable matrix and let P_i be the solution of the discrete Lyapunov equation. For an arbitrarily selected Q_i, where Q_i is a positive definite matrix, then (17) is asymptotically stable if:

$$\lambda_{\min}(Q_i) \succ \alpha_i\|F_i\|\sum_{j=1(j\neq i)}^{N}\|H_{ij}\| + \sum_{j=1(j\neq i)}^{N}\alpha_j\|H_{ij}\|\|F_j\|$$
$$+\sum_{\substack{k=1\\k\neq i}}^{N}(\alpha_k\|H_{ki}\|(\sum_{\substack{j=1\\j\neq i)}}^{N}\|H_{kj}\|)) + \sum_{\substack{j=1\\j\neq i}}^{N}\alpha_j\|H_{ji}\|^2 \qquad (18)$$

Where

$$\alpha_i = \lambda_1(E)\left[\lambda_1\left[\frac{\lambda_1(E^{-1}Q_i)\hat{F}_i^T\hat{F}_i}{[1-\sigma_1^2(\hat{F}_i)]} + E^{-1}Q_i\right]\right]$$

Proof:. From theorem 2.5, the system (17) is asymptotically stable if the condition (14) is satisfied. From theorem 2.6 the solution has an upper bound (16), through which we obtain the following inequality:

$$\lambda_{\max}(P_i)\|F_i\|\sum_{\substack{j=1\\j\neq i}}^{N}\|H_{ij}\| + \sum_{\substack{j=1\\j\neq i}}^{N}\lambda_{\max}(P_j)\|H_{ij}\|\|F_j\| +$$
$$\sum_{\substack{k=1\\k\neq i}}^{N}(\lambda_{\max}(P_k)\|H_{ki}\|(\sum_{\substack{j=1\\j\neq i}}^{N}\|H_{kj}\|)) + \sum_{\substack{j=1\\j\neq i}}^{N}\lambda_{\max}(P_j)\|H_{ji}\|$$

$$\prec \alpha_i\|F_i\|\sum_{\substack{j=1\\(j\neq i}}^{N}\|H_{ij}\| + \sum_{\substack{j=1\\j\neq i}}^{N}\alpha_j\|H_{ij}\|\|F_j\| +$$
$$\sum_{\substack{k=1\\j\neq i}}^{N}(\alpha_k\|H_{ki}\|(\sum_{\substack{j=1\\j\neq i}}^{N}\|H_{kj}\|)) + \sum_{\substack{j=1\\j\neq i}}^{N}\alpha_j\|H_{ji}\|^2$$

i=1,2…… N
The system (17) is asymptotically stable if the condition

$$\lambda_{\min}(Q_i) \succ \alpha_i\sum_{j=1(j\neq i)}^{N}\|F_i\|\|H_{ij}\| + \sum_{j=1(j\neq i)}^{N}\alpha_j\|H_{ij}\|\|F_j\|$$
$$+\sum_{\substack{k=1\\k\neq i}}^{N}(\alpha_k\|H_{ki}\|(\sum_{\substack{j=1\\j\neq i)}}^{N}\|H_{kj}\|)) + \sum_{\substack{j=1\\j\neq i}}^{N}\alpha_j\|H_{ji}\|^2$$

is satisfied. This condition is most robust that the condition (14).

The implementation of this observer by using theorem 3.1 is very complicated because the resolution of equation of algorithm imposes constraining conditions on the interconnections matrices and leads to restricted classes of the interconnected system. Moreover the obtained gains are very high. In this paper we propose a new and simple algorithm for decentralised observation.

Let us consider the following transformation:

$$X_i(k) = \gamma^{-k}x_i(k), \qquad U_i(k) = \gamma^{-k}u_i(k) \qquad (19)$$

γ is a positive scalar ($\gamma \ge 1$).

$$\hat{X}_i(k+1) = \gamma^{-(k+1)}\hat{x}_i(k+1) = \gamma^{-1}(A_i - L_iC_i)\hat{x}_i(k) + \gamma^{-1}B_iU_i(k) + \sum_{\substack{j=1\\j\neq i}}^{N}\gamma^{-1}H_{ij}\hat{X}_j(k) \qquad (20)$$

Then the estimation error of the modified system is governed by:

$$\varepsilon_i(k+1) = \gamma^{-1}(A_i - L_iC_i)\varepsilon_i(k) + \sum_{\substack{j=1\\j\neq i}}^{N}\gamma^{-1}H_{ij}\varepsilon_j(k) \qquad (21)$$

The goal is to select the observer gain L_i and the required scalar α, such that the overall error system (21) is asymptotically stable.

From theorem 3.1, the system (21) is asymptotically stable if :

$$\lambda_{\min}(Q_i) \succ \gamma^{-1}\alpha_i\|F_i\|\sum_{j=1(j\neq i)}^{N}\|H_{ij}\| + \gamma^{-1}\sum_{j=1(j\neq i)}^{N}\alpha_j\|H_{ij}\|\|F_j\|$$
$$+\gamma^{-2}\sum_{\substack{k=1\\k\neq i}}^{N}(\alpha_k\|H_{ki}\|(\sum_{\substack{j=1\\j\neq i)}}^{N}\|H_{kj}\|)) + \gamma^{-2}\sum_{\substack{j=1\\j\neq i}}^{N}\alpha_j\|H_{ji}\|^2 \qquad (22)$$

We shall now give the procedure for the construction of the observer gains l_i:

Step1: γ =1, select L_i such that spec (A_i-L_i C_i) is inside the unit circle. This selection can be made by a standard pole placement design.

Step2: choose an arbitrary matrix positive Q_i,

Step 3 :If condition (22) is checked then Calculate l_i such as spec (A_i-li Ci)= spec (γ^{-1}(A_i-L_i C_i))

If not, $\gamma = \gamma + 1$ and go to step 3.

4 Illustrative example

We consider the following example which was treated by Sundershan and Elbanna[2].

The discrete-time model is obtained from its continuous –time model by discretizing it using MATLAB c2d with the sampling period T=0.1

$$x_1(k+1) = \begin{bmatrix} 1.0637 & 0.2013 & 0.4264 & 0.8956 \\ 0.3275 & 1.0293 & 0.0616 & 0.1340 \\ 0.2587 & 0.3050 & 1.4371 & 0.7653 \\ 0.0142 & 0.0174 & 0.1502 & 1.1474 \end{bmatrix} x_1 +$$

$$+ B_1 u_1(k) + \begin{bmatrix} 0.0310 & 0.0426 & -0.0327 \\ 0.1431 & 0.0286 & -0.0322 \\ 0.0072 & -0.0200 & 0.0165 \\ -0.0049 & -0.0049 & 0.0088 \end{bmatrix} x_2(k)$$

$$y_1(k) = \begin{bmatrix} 1 & 0 & 2 & 0 \end{bmatrix} x_1(k)$$

$$x_2(k+1) = \begin{bmatrix} 1.0074 & 0.1082 & 0.0264 \\ 0.1083 & 1.1697 & 0.5633 \\ -0.5681 & -0.0286 & 1.2731 \end{bmatrix} x_2(k)$$

$$+ B_2 u_2(k) + \begin{bmatrix} 0.0053 & 0.0038 & 0.0249 & -0.2327 \\ 0.2557 & 0.0086 & 0.6204 & -0.0337 \\ -0.0547 & 0.0109 & 0.3322 & -0.0769 \end{bmatrix} x_2(k)$$

$$y_2(k) = \begin{bmatrix} 0 & 1.2 & 0 \end{bmatrix} x_2(k)$$

The resulting local observer gain matrices defined in (17) are :

$$l_1 = \begin{bmatrix} 1.7089 & -19.2365 & 1.0409 & 8.1873 \end{bmatrix}^T$$

$$l_2 = \begin{bmatrix} -1.0386 & 2.4032 & 4.1746 \end{bmatrix}^T$$

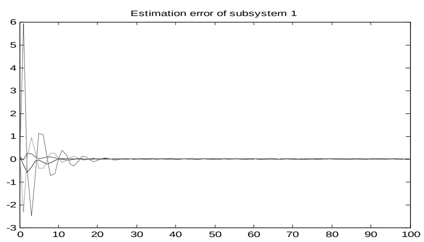
For comparison observed gains determined in [2] are

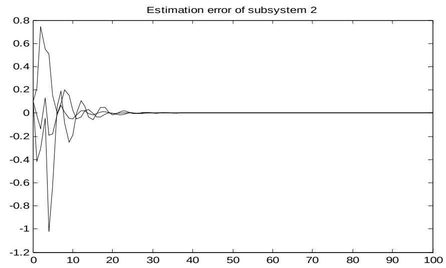
$$l_1 = \begin{bmatrix} -674 & -4424.2 & 379 & 1594.4 \end{bmatrix}^T$$

$$l_2 = \begin{bmatrix} -252.4 & 69.208 & 435.05 \end{bmatrix}^T$$

Which have considerably larger values.

From the simulation results we can notice that each estimation errors are asymptotically stable.





5 Conclusion

This paper has presented a procedure for designing linear decentralized observer for discrete large scale systems. Compared with existing results, our approach is more simple and easier to use and the gains obtained are smaller. The present results are developed in the context of decentralized observation, they have a wider application in that they can be extended to the design of decentralized model reference adaptive identification schemes. Future research is directed to the application of this approach to linear large scale uncertain systems.

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