Spacecraft Formation Flying Control

Sylvain Cadic1,2 Xu Bo1, Lu yuping1
1.Nanjing University of Astronautics and Aeronautics (NUAA), China
2. Supelec campus de Rennes, France

Abstract: Distributed spacecraft formation flying (DSFF) is a new technology in space mission design, aiming at replacing large satellites with multiple small satellites. It requires stringent control of the relative positioning of micro-satellites inside a formation flying. In this paper, the regulation of the relative distance between two satellites in a leader-follower formation is suggested, which implies a non natural motion of the follower. The system behaviour is described by the well known linear model for the relative motion between two satellites: the Hill-Clohessy-Wiltshire (HCW) equations. Then a linear quadratic regulator (LQR) is developed in order to guarantee closed-loop stability of the formation; an integral-action controller improves the regulation and eliminates the steady-state error. Then illustrative numerical examples are simulated to demonstrate the efficiency of the proposed approach.

Key-words: satellite formation flying, LQR control, linear state observer

1. Introduction

Since the first launch in the 1950's, satellites have proliferated in our sky for the purpose of Earth observing, deep space exploring, military surveillance, commercial and military communication, weather prediction etc. But conventional monolithic satellites still have some deficiencies[2,3,6]:

High cost, directly related to its size and weight: the larger the satellite is, the larger and more costly the required launch vehicle is.

Non-flexibility: one satellite on a fixed orbit matches with only one fixed mission with a limited observing baseline.

Bad redundancy: in case a failure occurs, the entire mission fails.

As a response to those deficiencies, an innovating technology has recently emerged.

Distributed spacecraft formation flying (DSFF), to distribute the functionality of conventional monolithic satellites among a formation flying of numerous micro-satellites working together. Thus a large amount of advantages are provided:

Size reduction naturally leads to cost reductions.

According to different missions or error conditions, the formation of the multiple satellites can be changed autonomously or manually, which grants more flexibility and a more efficient use of resources.

Extensive co-observing programs can be conducted without using extensive ground support: in the leader-follower architecture, only the leader satellite communicates with the ground station all the time, while the followers communicate only if necessary.

Increased precision and observational baseline.

Enhanced survivability and increased reliability: even if a certain number of satellites in the formation fail, the mission may still be accomplished.

On one hand, the DSFF technology grants flexibility, reliability and autonomy to the formation. On the other hand, it requires a fastidious control of the architecture of the formation. Indeed, a good communication between the satellites inside the flying formation is incontrovertible. It means that the exact position of each satellite must be known at any time. Moreover, the very high density of satellites in a small area also requires a stringent control of relative distances between satellites in order to avoid collisions.

In this paper, the regulation of the relative distance between a leader satellite and its follower using the Linear Quadratic Regulator (LQR) synthesis is performed.

Before proceeding to the development of that controller, we shall briefly establish the model of spacecraft relative position dynamics.

2. The Hill-Clohessy-Wiltshire (HCW) Equations

In this section, we begin with the classical Hill's equations that describe the motion of a follower spacecraft relative to a leader spacecraft. In order to present the Hill's equations we assume that the leader spacecraft is on a circular geostationary orbit around the Earth with constant angular velocity ω; and a rectangular moving coordinate frame is attached to the leader spacecraft with the x-axis directed radially outward along the local vertical, the y-axis pointing along the direction of motion, and the z-axis normal to the reference orbit plane.

The linearized dynamic equations governing the motion of the follower spacecraft relative to the leader spacecraft are then given by [1,7]:

$$\begin{aligned} \ddot{x} &= 3\omega^2 + 2\omega\dot{y} + \Gamma_x \\ \ddot{y} &= -2\omega\dot{x} + \Gamma_y \\ \ddot{z} &= -\omega^2 z + \Gamma_z \end{aligned} \quad (1)$$

$U = (\Gamma_x, \Gamma_y, \Gamma_z)$ is the thrust acceleration vector, that includes manual command and natural disturbances.

One interesting property is that, although the equations describing the in-plane (x, y) motion are coupled, the out-of-plane (z) motion is uncoupled. The velocity dependant terms $2\omega\dot{x}$ and $2\omega\dot{y}$ represent damping in the system. It is a non-dissipative and is present only because the motion is described in a rotating coordinate frame.

The general solutions of the HCW equations can be easily obtained (considering $\Gamma_x = \Gamma_y = \Gamma_z = 0$)[4]:

$$\begin{aligned} x(t) &= (-3x_0 - \frac{2\dot{y}_0}{\omega})\cos(\omega t) + \frac{\dot{x}_0}{\omega}\sin(\omega t) + 4x_0 + \frac{2\dot{y}_0}{\omega} \\ y(t) &= \frac{2\dot{x}_0}{\omega}\cos(\omega t) + (6x_0 + \frac{4\dot{y}_0}{\omega})\sin(\omega t) + (6\omega x_0 + 3\dot{y}_0)t + y_0 - \frac{2\dot{x}_0}{\omega} \\ z(t) &= z_0\cos(\omega t) + \frac{\dot{z}_0}{\omega}\sin(\omega t) \end{aligned} \quad (2)$$

An interpretation for these solutions shows that the combined effects of relative motion in all components of the HCW frame represents the general case of a neighbouring orbit which is:

elliptic (due to oscillations in x)

inclined (due to oscillations in z)

of a different period than the target orbit (due to the steady drift along the y-axis).

For this project we are interested in a follower satellite with a periodic motion centered on the leader satellite. Thus two constraints appear:

$$\begin{aligned} \dot{y}_0 &= -2\omega x_0 \\ y_0 &= \frac{2}{\omega}\dot{x}_0 \end{aligned} \quad (3)$$

The new expression can be written as follows:

$$\begin{aligned} x(t) &= x_0\cos(\omega t) + \frac{\dot{x}_0}{\omega}\sin(\omega t) \\ y(t) &= \frac{2\dot{x}_0}{\omega}\cos(\omega t) - 2x_0\sin(\omega t) \\ z(t) &= z_0\cos(\omega t) - z_0 w\sin(\omega t) \end{aligned} \quad (4)$$

If we consider a constant relative distance ρ between the leader and the follower, we have to assume 4 cases:

parallel orbit:

$$x(t) = \rho; y(t) = 0; z(t) = 0$$

follower tracking leader on the same orbit:

$$x(t) = 0; y(t) = \rho; z(t) = 0$$

in-plane (x,y) circle around the leader:

$$x(t)^2 + y(t)^2 = \rho^2; z(t) = 0$$

sphere around the leader:

$$\begin{cases} x(t)^2 + y(t)^2 + z(t)^2 = \rho^2 \\ z(t) = \pm\sqrt{3}x \end{cases} \quad (5)$$

Whatever the case, quick calculi show there is no solution unless ρ =0. For any other value of ρ, the motions are not natural. In (1), U the commanded thrust but also the natural disturbance such as solar pressure, Earth oblateness, eventually shocks etc. The following developed controller is supposed to force the behaviours described in (5) and reject the natural disturbances.

3. LQR Synthesis

Given the HCW equations in (1) the state-space representation of our system is:

$$\begin{aligned} \dot{X} &= AX + BU \\ Y &= CX \end{aligned} \quad (6)$$

With $X = [x, y, z, \dot{x}, \dot{y}, \dot{z}]^T$, $U = [\Gamma_x, \Gamma_y, \Gamma_z]^T$,

$$A=\begin{pmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\3\omega^2&0&0&0&2\omega&0\\0&0&0&-2\omega&0&0\\0&0&-\omega^2&0&0&0\end{pmatrix},$$

$$B=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\\1&0&0\\0&1&0\\0&0&1\end{pmatrix},\quad C=\begin{pmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\end{pmatrix}$$

The purpose of LQR synthesis is to compute a state feedback control $U=-KX$, where K is a gain matrix, so as to minimize the performance index:

$$J=\int_0^\infty (X^*QX+U^*RU)dt\,. \qquad (7)$$

This integral is the global energy of the system: the first term is the satellite energy and the second term represents the energy of the control signals. The energy must be minimized. Q is a positive-definite Hermitian matrix; R is a positive-definite Hermitian matrix. They are both weight matrixes that represent the expenditure of the energy. They are arbitrary parameters that are adjusted relating to the system[5].

Let $$Q=\begin{pmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&0.5&0&0&0\\0&0&0&0.05&0&0\\0&0&0&0&0.05&0\\0&0&0&0&0&0.05\end{pmatrix}$$

and $$R=\frac{1}{\omega^2}\begin{pmatrix}1&0&0\\0&1&0\\0&0&0.5\end{pmatrix} \qquad (8)$$

For that choice, we have considered that commanded thrust predominates and the satellite energy is mainly spent on in-plane motion[5,9].

(A,B) is controllable and (A,C) is observable, so there is one and only one optimal controller:

$$K=R^{-1}B^*P \qquad (9)$$

where P is the solution of the Riccati equation:

$$PA+A^*P-PBR^{-1}B^*P+Q=0 \qquad (10)$$

4. State Observer Synthesis

Previously we have computed an optimal state feedback control. However the state vector X is unknown and must be rebuilt. We use a linear rebuilder in order to create a good steady-state estimator $\hat{X}$ with the following structure:

$$\dot{\hat{X}}=A\hat{X}+BU+L(Y-C\hat{X}) \qquad (11)$$

The matrixes A, B and C are well identified. We can show that if $A-LC$ is stable, then estimator error decreases to zero. L is chosen by pole placement: eigenvalues of $A-LC$ shall all have a negative real part.

The state-feedback command $U=-K\hat{X}$ can finally be applied (Fig.1).

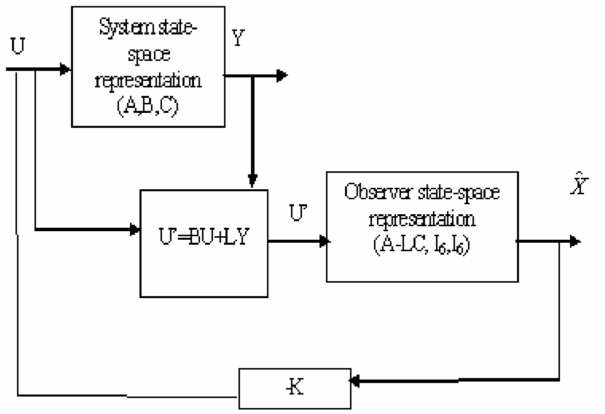

Fig.1. Optimal state feedback control with linear state observer

At this point, the controller can force the behavior of the system as expected in (e). The system is stable and follows standards various commands in an acceptable transitional period. However we can notice a non negligible overshoot, and a little steady state error. Plus, the controlled system can not reject the natural disturbances.

In order to increase robustness and accuracy, a command by integral action is added to the previous controller.

5. Command By Integral Action

In automation, recourse to integral controller guarantees a null steady-state error when constant commands and disturbances are applied. Concerning our system those signals are not perfectly constant, but they both are oscillating signals at pulsation ω. Motion period is about 24 hours, while the time response of our controlled system is about 7 minutes. During this short time we can approximate input signals as constants, and see the efficiency of the suggested method.

We consider the new system:

$$\begin{aligned} \dot{X} &= AX + B(U + D) \\ Y &= CX \end{aligned} \quad (12)$$

Where D represents the disturbing accelerating forces. We also add a new variable $q(t)$, as the integral of the error between the position required Y_c and the real position Y:

$$q(t) = \int_0^t (Y(\tau) - Y_c(\tau))d\tau \quad (13)$$

Concatenating X and q into a larger state-space vector, we deal with a new system:

$$\begin{aligned} \begin{pmatrix} \dot{X} \\ \dot{q} \end{pmatrix} &= \begin{pmatrix} A & 0 \\ C & 0 \end{pmatrix} \begin{pmatrix} X \\ q \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} U + \begin{pmatrix} B & 0 \\ 0 & -I \end{pmatrix} \begin{pmatrix} D \\ Y_c \end{pmatrix} \\ Y &= \begin{pmatrix} C & 0 \end{pmatrix} \begin{pmatrix} X \\ q \end{pmatrix} \end{aligned} \quad (14)$$

Then we suppose a command U_c that makes $Y=Y_c$ does exist. Let X_c and q_c the corresponding values, and let the following new variables:

$$X' = X - X_c \qquad Y' = Y - Y_c$$

$$q' = q - q_c \qquad U' = U - U_c$$

Because the system in linear, those new variables are linked by the state equations:

$$\begin{aligned} \begin{pmatrix} \dot{X}' \\ \dot{q}' \end{pmatrix} &= \begin{pmatrix} A & 0 \\ C & 0 \end{pmatrix} \begin{pmatrix} X' \\ q' \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} U' \\ Y &= \begin{pmatrix} C & 0 \end{pmatrix} \begin{pmatrix} X' \\ q' \end{pmatrix} \end{aligned} \quad (15)$$

The problem is now to regulate Y' to 0, thanks to a sate feedback control:

$$U'(t) = -K \begin{pmatrix} X'(t) \\ q'(t) \end{pmatrix} = -K_1 X'(t) - K_2 q'(t) \quad (16)$$

We solve it with LQR synthesis described previously, but with a larger state vector $\begin{pmatrix} X' \\ q' \end{pmatrix}$.

The sizes of weight matrixes Q and R must also be readapted.

The principle is illustrated *Fig.2* with a block representation.

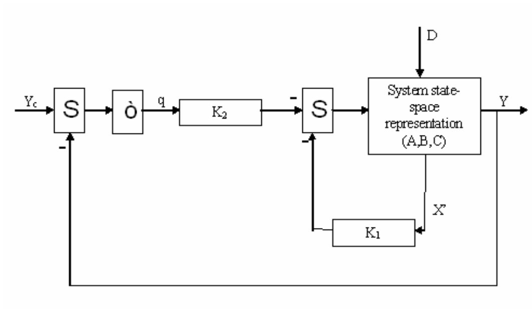

Fig.2 . Command by integral action state representation

6. Simulation

The followings illustrate the previous theoretical results. First of all we consider the fourth case in (e), sphere around the leader, with a simple LQR controller. Originally, the follower will be closed to his leader and quickly move to a 3D-motion on an inclined circle as shown on *Fig.3a*. The imposed motion equation is given by

$$x_c(t) = 200\cos(\omega t)$$

$$y_c(t) = -400\sin(\omega t)$$

$$z_c(t) = 200\sqrt{3}\cos(\omega t)$$

On *Fig.3b* we check that the relative distance is constant during 86164s (time period for a geostationary spacecraft) after the settling period. However we can notice an important overshoot before the correct value is reached.

Then we consider the first case of (e), parallel orbit, to focus on command by integral action comparing to simple LQR synthesis.

$$x_c(t) = 200km; \; y_c(t) = 0km; \quad z_c(t) = 0km$$

We also add disturbances modeled by oscillating signals in the three directions, with a angular velocity ω, and an amplitude 10^{-6} m/s² (value similar to manual thrust).

The comparison of the two methods during the settling period illustrates the expected results (*Fig.4a*): the command by integral action is really faster and has no overshoot.

During the permanent motion, we zoom in around the expected value, 200 km. While a simple LQR synthesis reveals a small steady-state error and is affected by disturbances, the method by integral action rejects both of them. Robustness and accuracy have been improved.

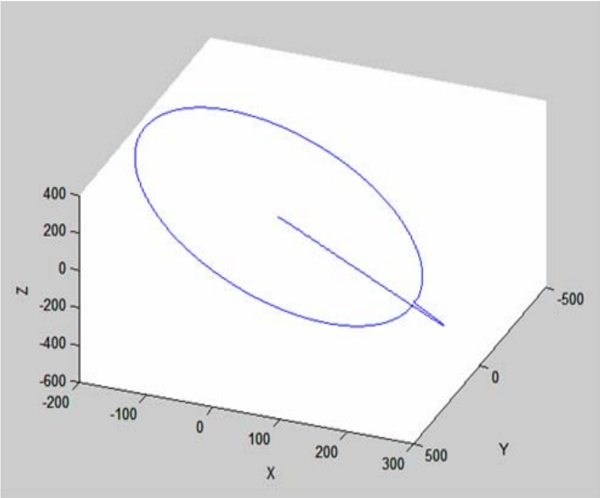

Fig.3a: r with relative 3D-motion leader on the origin of the frame

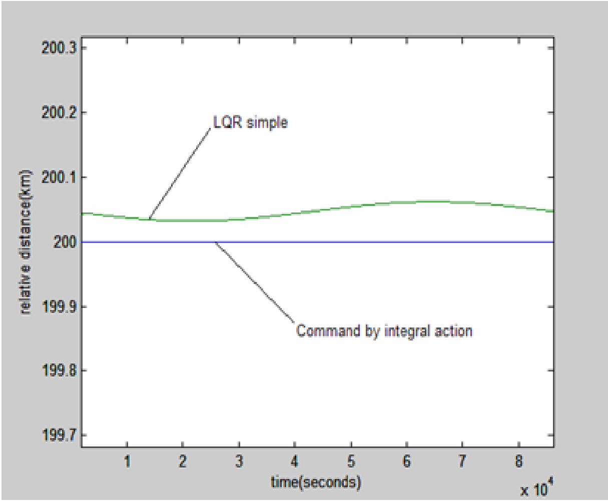

Fig.4b: Disturbance are rejected

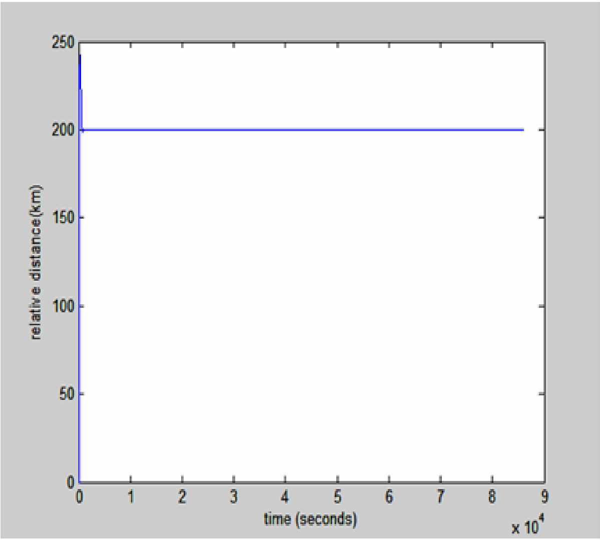

Fig.3b: relative distance

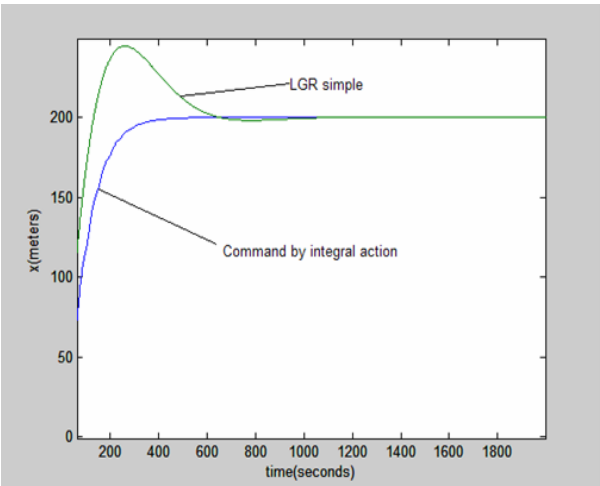

Fig.4a: Overshoot is rejected

7. Conclusion

In this paper we developed a rigorous linear control for relative distance of spacecraft in formation that guarantees closed-loop stability, accuracy, and certain robustness as well. In particular an illustrative numerical simulation demonstrated the efficiency of command by integral action, associated to linear quadratic regulation.

However, regulation on relative distance requires continuous adjustment and so continuous consumption of energy is necessary to force those behaviours, even without natural disturbances. A concrete evaluation of the needs in energy and a comparison with existing controllers for spacecraft information may show up the non sustainability of this system for the time being.

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