A New Sliding Mode Control for Satellite Formation

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Abstract: A new sliding mode control based on developed power reaching law method is derived to control a satellite formation with initial deployment inaccuracy and perturbation in space. It is proved that the new method could guarantee system stability. C-W equations are used to model the follower satellite relative motion with respect to the leader. The numerical simulation results show that the new control exhibits not only strong robustness to the unknown perturbation bounded, but also effectiveness of eliminating chattering caused by high-frequency switch control.

Key-words: satellite formation flying, power reaching law, sliding mode control, chattering

1. Introduction

Satellite formation flying is described that several satellites are keeping a special relative position, communicating with each other, and working together in order to perform a unified mission when they reside near a natural orbit. Recently, Satellite formation flying is attracting more and more countries' attention since it can provide numerous advantages: cost reduction, better flexibility, increased precision and observational baselines, enhanced survivability and raised reliability. For the development of satellite formation, control is a crucial technique and many control methods have been used to design controllers. The LQ and PID control methods are first applied to the formation flying system[9 10 11]. However, both of above are based on the linear C-W model which has some limitations in representing the true motion. Thus, uncertain-model-based controllers have been researched in some literatures. For example, Bernstein[12] proposed an adaptive asymptotic tracking strategy for the spacecraft attitude of the formation system in the case that the spacecraft's inertia matrix is not known. Due to planned missions or unplanned changes, the formation may need to be reconfigured. Hybrid control architecture is used to address the reconfiguration issue. Additionally, sliding mode control (SMC) algorithm is explored to the formation control problems[5], and it provides several benefits. The drawback of this method is that there is a phenomenon called chattering which effects the applying of the SMC.

In this paper, a new sliding mode control based on developed power reaching law (DPRL) method is derived to improve the accuracy and the robustness of control, and the proposed method also can eliminate chattering. C-W equations are used to model the follower satellite relative motion with respect to the leader. In the simulation section, the control results of the general power reaching law and the DPRL are compared. The simulation results show that the new control exhibits not only strong robustness to the unknown perturbation bounded, but also effectively eliminating chattering caused by high-frequency switch control.

2. Sliding Mode Control

In this section, the theoretical basis for incorporating a multiple satellite formation control problem will be introduced[5], consider a nonlinear dynamical system of the form

$$\begin{aligned} \dot{x} &= f(x,t) + G(x,t)u(t) \\ y(t) &= h(x,t) \end{aligned} \tag{1}$$

where $x(t), u(t)$, and $y(t)$ are n, m , and m dimensional real function vectors,

$f(x,t), G(x,t)$ and $h(x,t)$ are analytic vector or matrix functions of the variables x and t.

We define the relative degree vector as α, $\alpha \in \mathbb{R}^m$. The reference trajectory is represented by $\bar{y}(t)$, and the tracking error is defined as $e(t) = \bar{y}(t) - y(t)$. As the definition of the relative degree, we can get

$$y^{(\alpha)}(t) = A^*(x,t) + B^*(x,t)u(t) \tag{2}$$

We shall design a controller with a sliding plane

$$\sigma(e) = e^{(\alpha-1)}(t) + \mathrm{c}_1 e^{(\alpha-2)}(t) + \cdots \mathrm{c}_\alpha \int e(t)dt \tag{3}$$

where $\mathrm{c}_1, \mathrm{c}_2, \cdots \mathrm{c}_\alpha$ are strictly positive constants.

The objective of sliding mode control is driving the states of the system to the sliding plane and keeping them on it. In order to get the control vector which can stabilize the system, we select a candidate Lyapunov function as: $V = \sigma^T(e)\sigma(e)/2$. The derivative of V is

$$\dot{V} = \sigma^T(e)\dot{\sigma}(e) \tag{4}$$

Substituting (2) into $\dot{\sigma}(e)$ gives

$$\begin{aligned}\dot{\sigma}(e) &= \bar{y}^{(\alpha)}(t) + \mathrm{c}_1 e^{(\alpha-1)}(t) + \cdots \mathrm{c}_\alpha e(t) \\ &\quad - A^*(x,t) - B^*(x,t)u(t)\end{aligned} \tag{5}$$

If $\tilde{u}(t)$ is defined as

$$\begin{aligned}\tilde{u}(t) &= \bar{y}^{(\alpha)}(t) + \mathrm{c}_1 e^{(\alpha-1)}(t) + \cdots \mathrm{c}_\alpha e(t) \\ &\quad - A^*(x,t)\end{aligned} \tag{6}$$

We can get

$$\dot{\sigma}(e) = \tilde{u}(t) - B^*(x,t)u(t) \tag{7}$$

Substituting (7) into (4) yields

$$\begin{aligned}\dot{V} &= \sigma^T(e)(\tilde{u}(t) - B^*(x,t)u(t)) \\ &= \sigma^T(e)B^*(x,t)(B^{*-1}(x,t)\tilde{u}(t) - u(t))\end{aligned} \tag{8}$$

For the purpose of enabling the closed loop system to be globally asymptotically stable, $\sigma^T(e)\dot{\sigma}(e)$ must be always negative. There are many ways to achieve this goal. One solution can take the following form

$$u(t) = B^{*-1}(x,t)\tilde{u}(t) + u_b(t) \tag{9}$$

where

$$u_b(t) = \begin{cases} \rho\mathrm{sgn}(B^*(x,t)\sigma(e)), \rho > 0, \sigma(e) \neq 0 \\ 0, \quad \sigma(e) = 0 \end{cases} \tag{10}$$

ρ is defined as a diagonal matrix,

$$\rho = \mathrm{diag}(\rho_1, \rho_2, \cdots, \rho_m) \tag{11}$$

and the vector sign function is a column of sign functions

$$\mathrm{sgn}\sigma(e) \triangleq [\mathrm{sgn}\sigma_1(e)\ \mathrm{sgn}\sigma_2(e)\ \cdots\ \mathrm{sgn}\sigma_m(e)]^T \tag{12}$$

The main drawback of sliding mode control is that when an unstable high frequency plant mode is excited, the discontinuous control may exhibit a chattering phenomenon. The most common way to avoid chattering is to introduce a boundary layer on the sliding plane. In this paper, an improved SMC method based on power reaching law (PRL) algorithm with a saturation function is proposed to eliminate chattering. Within the boundary layer, we implement a linearized smooth transition between positive and negative, as the system state crosses the sliding plane.

3. Satellite Formation Flying Control Design

In this part, we will discuss the dynamic model of the formation flying and the DPRL algorithm.

3.1 The dynamic model of the satellite formation flying

We start by using the linearized C-W Equations to describe the legal relative motion between a leader and follower satellite

$$\begin{aligned}&\ddot{x} - 2\omega\dot{y} - 3\omega^2 x = u_x + d_x \\ &\ddot{y} + 2\omega\dot{x} = u_y + d_y \\ &\ddot{z} + \omega^2 z = u_z + d_z\end{aligned} \tag{13}$$

In equation (13), x, y and z are the follower satellite's position relative to a leader satellite in a circular orbit: x is in the radial direction from the Earth, y is in leader satellite's tangential velocity direction, and z completes a right-hand coordinate system. The leader satellite mean angular velocity, around the Earth, is $\omega = \sqrt{\dfrac{\mu}{a^3}}$, where μ is the Earth's gravitational constant and a is the semi-major axis (if the reference trajectory is a circular , a is the radius) of the leader satellite's elliptic orbit.

$u_x, u_y, u_z, d_x, d_y, d_z$ are control variables and disturbances respectively.

3.2 The power reaching law algorithm

The sliding mode motion includes the reaching portion which describes the motion from any initial condition to the sliding plane and the portion sustaining the states staying on the sliding plane. According to the theory of sliding mode control, the reaching condition only can guarantee the states arriving at the sliding plane in a finite time without any information about the exact trajectory. However, the PRL can improve the dynamic quality of the reaching motion[4].

The PRL is defined as

$$\dot{\sigma} = -\lambda|\sigma|^{\beta}\operatorname{sgn}(\sigma),\ \lambda > 0, 1 > \beta > 0 \tag{14}$$

Integrating (14) yields

$$|\sigma|^{1-\beta} = -\lambda(1-\beta)t + |\sigma_0|^{1-\beta} \tag{15}$$

The time that σ takes to arrive at zero from σ_0 is

$$t = |\sigma_0|^{1-\beta} / \lambda(1-\beta) \tag{16}$$

Hence, the state's arriving at the sliding plane in a finite time is guaranteed.

Next, we develop a more effective power reaching law method based on saturation function.

$$\dot{\sigma} = -\lambda|\sigma|^{\beta}\operatorname{sat}(\sigma) - \int \mathrm{M}\operatorname{sat}(\sigma)dt \qquad \lambda > 0, 1 > \beta > 0, \mathrm{M} > 0 \tag{17}$$

where

$$\lambda \triangleq \operatorname{diag}[\lambda_1, \lambda_2, \cdots, \lambda_{\mathrm{m}}],$$

$$|\sigma|^{\beta} \triangleq \operatorname{diag}[|\sigma_1|^{\beta_1}, |\sigma_2|^{\beta_2}, \cdots, |\sigma_{\mathrm{m}}|^{\beta_{\mathrm{m}}}]$$

$$\operatorname{sat}\sigma(e) \triangleq [\operatorname{sat}\sigma_1(e)\ \operatorname{sat}\sigma_2(e) \cdots \operatorname{sat}\sigma_m(e)]^T$$

The gains λ_i, β_i and W_i are chosen based on the bounds of the disturbance with little concern for the shape of the general disturbance function. The term of integration can reduce the shaking caused by the velocity inertia.

Substituting (5) into (17), we get the DPRL control function:

$$u(t) = B^{*-1}(x,t)(\tilde{u}(t) + \lambda|\sigma|^{\beta}\operatorname{sat}(\sigma) + \int \mathrm{M}\operatorname{sat}(\sigma)dt) \tag{18}$$

where $B^*(x,t) = I$, I is identity matrix. We choose C-W equation as the model. Therefore, (18) can be represented as

$$u(t) = \tilde{u}(t) + \lambda|\sigma|^{\beta}\operatorname{sat}(\sigma) + \int \mathrm{M}\operatorname{sat}(\sigma)dt \tag{19}$$

The saturation function is

$$\operatorname{sat}(\sigma) \triangleq \begin{cases} 1 & \sigma > \Delta \\ \kappa\sigma & |\sigma| \le \Delta \quad \kappa = \dfrac{1}{\Delta} \\ -1 & \sigma < -\Delta \end{cases} \tag{20}$$

Δ represents the boundary of the saturation. Because of the linearized feedback control in the boundary layer, the chattering phenomenon caused by the high-frequency switch control is eliminated and the system states can reach the sliding plane in a finite time.

3.3 The analysis of the stability

Assuming $|\tilde{u}(t)| \le \mathrm{L}$, and considering (19), we can get the desired control input by selecting proper λ and M. The control function can be written as

$$u(t) = \tilde{u}(t) + \lambda|\sigma|^{\beta}\operatorname{sat}(\sigma) + \int \mathrm{M}\operatorname{sat}(\sigma)dt \tag{21}$$

where

$$\begin{cases} \tilde{\lambda}_i > \dfrac{\mathrm{L}}{|\Delta|^{\beta}} & i = 1, 2, \cdots, m \\ \tilde{\mathrm{M}}_i > \mathrm{L} \end{cases} \tag{22}$$

When σ is outside the boundary layer

$$\operatorname{sat}(\sigma) = \operatorname{sgn}(\sigma)$$

Substituting (21) into (4) yields:

$$\begin{aligned} \sigma^T(e)\dot{\sigma}(e) &= \sigma^T(e)[\tilde{u}(t) - \tilde{\lambda}|\sigma|^{\beta}\operatorname{sgn}(\sigma) - \int \tilde{\mathrm{M}}\operatorname{sgn}(\sigma)dt\ ] \\ &\le \sigma^T(e)[\mathrm{L}\operatorname{sgn}(\sigma) - \tilde{\lambda}|\sigma|^{\beta}\operatorname{sgn}(\sigma) - \int \tilde{\mathrm{M}}\operatorname{sgn}(\sigma)dt\ ] \end{aligned} \tag{23}$$

As $\tilde{\lambda} > 0, \tilde{W} > 0$, $|\sigma| > 0$, And $\tilde{\lambda}$, $\tilde{\mathrm{M}}$ is derived from (22), we get

$$\sigma^T(e)[\mathrm{L}\operatorname{sgn}(\sigma) - \tilde{\lambda}|\sigma|^{\beta}\operatorname{sgn}(\sigma) - \int \tilde{\mathrm{M}}\operatorname{sgn}(\sigma)dt\ ] < 0 \tag{24}$$

then, $\sigma^T(e)\dot{\sigma}(e) < 0$.

Within the boundary layer

$$\operatorname{sat}(\sigma) = \kappa\sigma$$

From (22), we have

$$\sigma^T(e)\dot{\sigma}(e) = \sigma^T(e)[\tilde{u}(t) - \tilde{\lambda}|\sigma|^\beta \kappa\sigma - \int \tilde{\mathrm{M}}\kappa\sigma dt\ ] \tag{25}$$
$$< 0$$

Thus, by using the developed power reaching law, the closed loop system is globally asymptotically stable.

3.4 Numerical Simulation

A simulation of a follower satellite motion around a leader was developed to test the control methods' effectiveness against the disturbances and the chattering. Since the leader satellite is assumed to remain in a circular orbit, the model consists of a reference trajectory which is defined by the C-W equations and an actual satellite plant based on (13). In this case, the representative disturbance is a sine wave with amplitude of 0.4 and a frequency of 0.6 rad/s. This disturbance acts in the x, y and z axis of the follower satellite simultaneously, and was chosen to approximate the J_2 perturbation which a satellite in the given orbit would experience. The orbit parameters of the leader are

a=7171 km, $i = 30^\circ, e = 0, \Omega = i = M = 0$

The initial condition of the relative position and the relative velocity are given as follows:

$x(t_o)$=0 km, $y(t_o)$=0.5 km,

$z(t_o)$=0.500$\times\sqrt{3}$km, $\dot{x}(t_o)$=-1ω,

$\dot{y}(t_o)$=0, $\dot{z}(t_o)$=0

We choose a sliding plane in the form

$$\sigma(e) = \dot{e}(t) + c_1 e(t) + c_2 \int_{t_0}^{t} e(t)dt$$

where $c_1 = 0.1, c_2 = 0.01$. According to (22) $\beta_i = 0.5$, $\mathrm{M} = 1.0, \lambda = 4.0$.

Since the controls are similar for all three channels, only the "x" channel will be shown in detail here.

Fig.1 is a plot of σ_x vs. time for the sliding mode controller with general PRL and DPRL. As described earlier in this work, the σ_x using DPRL is much more smooth and without high frequency chattering. Fig. 2 depicts the error comparison between the general PRL and the developed one. The error of DPRL is much smaller and has a low oscillation frequency as the figure showing. Fig. 3 shows the comparison of the control input vs. time for two algorithms. As we can see, the DPRL provides an exact, continuous signal to the plant and eliminates the chattering effectively.

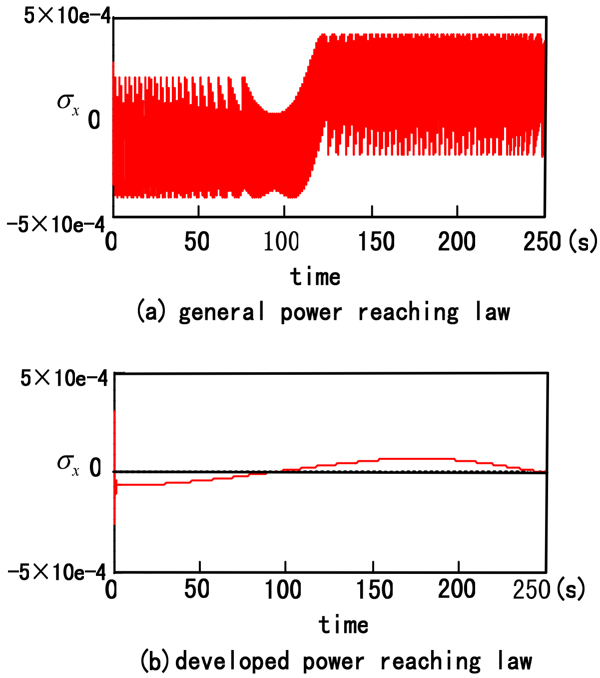

Fig.1 sliding quantity σ_x

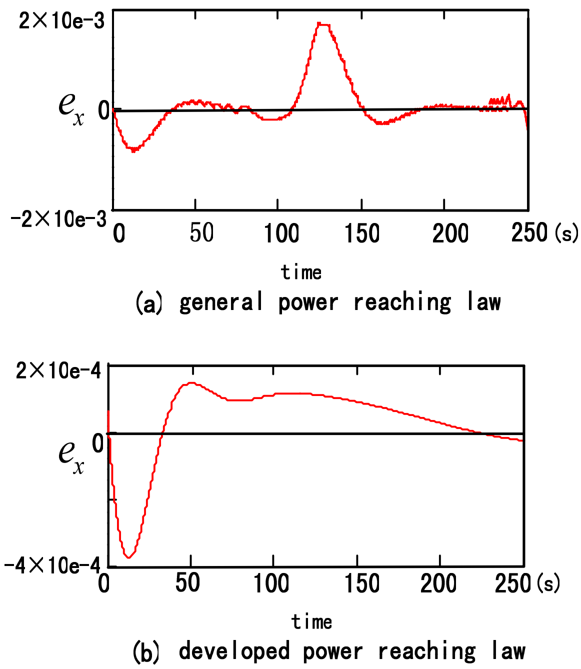

Fig.2 tracking error

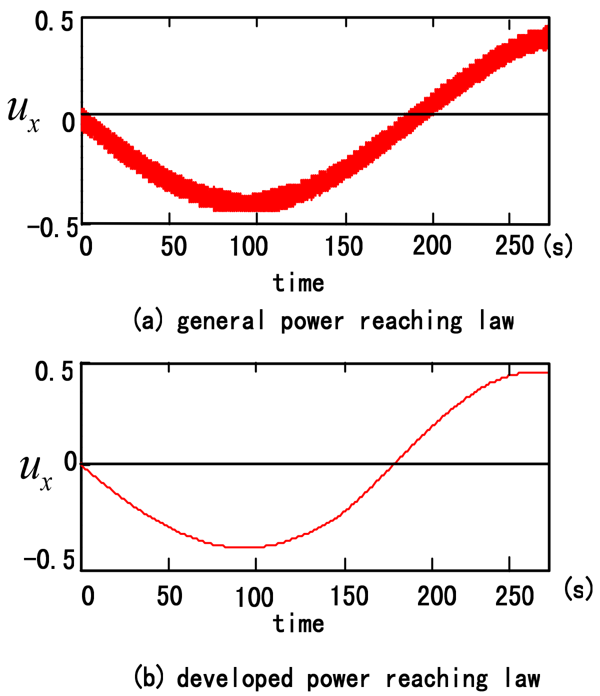

Fig.3 the control input signal

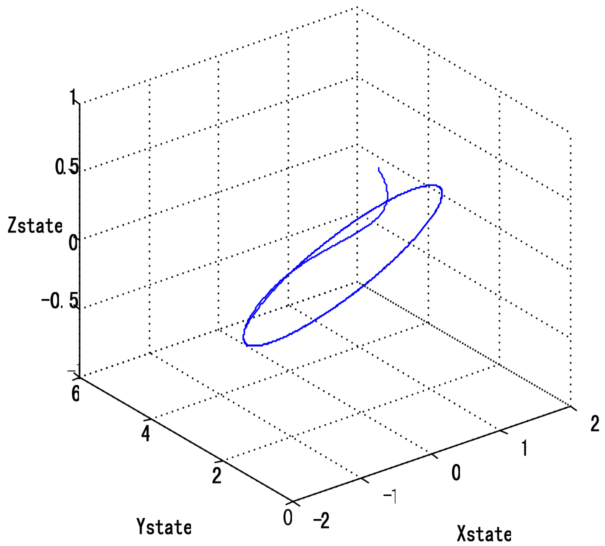

Fig.4 the practical trajectory

Fig. 4 is a 3-D plot of the exact trajectory of the relative motion about the formation. It is apparent from the figure that the relative elliptic motion is kept very well after correcting the initial errors.

Overall, it appears that the SMC with DPRL algorithm is an effective method of providing a continuous, smooth signal in the presence of bounded, unknown disturbances.

4. Conclusion

In this paper, the DPRL algorithm used in sliding mode control is derived and analyzed. The linearized C-W equations are introduced and used as the model of the satellite formation flying. In the simulation, the results of the proposed controllers applying the DPRL algorithm are compared with the controllers using the general PRL method. As shown in part 3.4, the DPRL is very effective at reducing the sliding quantity to zero and keeping it there. And it also successfully eliminates the chattering caused by the high frequency switch control and compensates for bounded disturbances and uncertainties without their estimation.

References

[1].Gao weibing. *Design approaches and theory of variable structure control systems*, Science Press，1996

[2].Liulin. *An Introduction of Astrodynamics*. Nanjing University Press，2006

[3].Xi xiaoning. *Fundamentals of Near-earth Spacecraft Orbit*, National University of Defense Technology, 2003

[4].Liu jinkun. *Matlab simulation for sliding mode control*. Tsinghua University Press，2005

[5].His-Han Yeh. Nelson, E. Sparks, A. Nonlinear tracking control for satellite formations，*Journal of Guidance, Control, and Dynamics*, Vol 25, No. 2, 2002, pp376-86

[6].Levant,A. Universal single input single output (SISO) sliding-mode controllers with finite time convergence， *IEEE Transactions on Automatic Control*, Vol 46, No 9, 2001, pp 1447-1451

[7].Timothy E.Massey. *Satellite formations control using continuous sliding mode*，2003

[8].Fridman, L. Levant,A., *Sliding mode control in engineering*, Marcel Dekker Inc.,2002, pp53-101

[9].Richard H. Vassar and Richard B.Sherwood, Formation keeping for a pair of satellites in a circular orbit, *Journal of Guidance, Control, and Dynamics*, Vol. 8, No. 2, 1985, pp. 235-242

[10].David C. Redding and Neil J. Adams, Linear-quadratic station keeping for the STS orbiter, *Journal of Guidance, Control, and Dynamics*, Vol. 12, No. 2, 1989, pp. 248-255

[11].Qiguo Yan, Vikram Kapila and Andrew G. Sparks, Pulse-based periodic control for spacecraft formation flying, *American Control Conference*, Vol. 1, Chicago, IL, June 28-30, 2000, pp. 374-378

[12].Jasim Ahmed, Vincent T. Coppola, and Dennis S. Bernstein, Adaptive asymptotic tracking of spacecraft attitude motion with inertia matrix identification, *Journal of Guidance, Control, and Dynamics*, Vol. 21 No. 5, 1998, pp. 684-692