# An Improved Assignment Algorithm Based Rotational Angular Sorting Methods

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*Abstract:* - The data assignment problem occurs for multiple targets tracking application. It is crucial for the overall performance. In this paper, two observations of date assignment are considered, and then a new rotational sorting algorithm based on maximum likelihood principle was presented. The given algorithm of $O(logN)$ and $O(N^2)$ complexity developed is faster than the more popularly used Hungarian type $O(N^3)$ algorithm.

*Key-Words:* - Data assignment problem; Hungarian algorithm; maximum likelihood principle

## 1 Introduction

Multiple Targets Tracking problem is a well-know application of information processing commonly encountered in surveillance, computer vision and aerospace engineering, etc. The problem has been extensively studied[1-4]. At the heart of it lies a combinatorial data assignment (or data association), that is, determining which measurement in the measurement set corresponds to which target. Despite extended efforts in the past, the key problem has not been solved successfully as yet mostly because of the difficulty associated with the date assignment problem mainly because it remains NP hard when the number of the sensors exceed two even in the simplified cases with stationary targets[3]. In fact, for the case of N targets, standard state model based methods, like track splitting, probabilistic data association, Maximum Likelihood etc., involve searching over the N! possible combinations. So there is $O(N^{3)}$ Complexity of data assignment process embedded in the Maximum Likelihood(ML)-based solution of a Multiple Target Tracking problem.

We will consider the general situation of tracking multiple moving targets involving associations by passive sensor array which does not emphasize the density of the scenarios [5]. Based on the formulation of [1,2], we resort to a maximum likelihood function of bearings-only measurements are maximized with respect to both data associations and target initial states by minimizing the magnitude of the average square errors (ASE) of its exponent. A global minimum can be attained in principle by repeating the following two steps until the solution converges: 1) minimizing the ASE with respect to initial target state vector for fixed data assignment matrices; 2) minimizing the ASE with respect to data assignment matrices constructed for the given initial state vector. The first step of the approach usually exploits the Gauss-Newton downhill type algorithm, while the second step resorts to most famous Hungarian type algorithms [6-8], however, all of which have the complexity of $O(N^3)$

In this paper, two observations of date assignment are considered. As we proved one of the observation in this paper, the rotational angular sorting-based new algorithm is capable of reducing the $O(N^3)$ complexity of Hungarian type algorithms to $O(N^2)$. Further more, according to the result of experiment, we believe that the new algorithm has the $O(logN)$ complexity by utilizing dichotomous search method. It will be much faster than those of $O(N^3)$ Hungarian type algorithms most frequently used.

The rest of the paper will be organized as follows. In section 2, we review the formulation of the problem, presenting a modified conditional likelihood function there. The observations, theorems and experiments for establishing the optimality of the data association problem are given in section 3. Section 4 includes some concluding remarks.

# 2 Problem Formulation and ML Principle

## 2.1 Problem Formulation

The detailed formulation of the Multiple Target Tracking problem could be found in [1]. Consider a system of N moving targets to be tracked by a passive array of M sensors capable of measuring only the bearings of all the targets. It is required to estimate the target tracks by resolving the data association problem linking targets to sensor measurements. A typical multitarget-multisensor encounter is illustrated in Figure 1

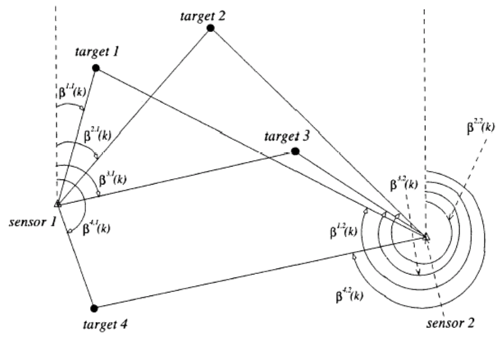


Figure 1. Typical Target-Sensor Encounter

The state of a target t at time index j is described by

$$X^t(j) = (r_x^t(j), r_y^t(j), v_x^t(j), v_y^t(j))' \quad (1)$$

Where $(r_x^t(j), r_y^t(j))$ denotes its Cartesian co-ordinates and $(v_x^t(j), v_y^t(j))$ its velocity components The sensor i is located at the positions $(r_{xs}^i, r_{ys}^i)$, i=1,2,…S. We assume that at time j, the measuring bearing data from target t for sensor i can be written using the state vector $X^t(j)$ as,

$$\beta^{t,i}(j) = \tan^{-1}\left[\frac{r_x^{t,i}(j)}{r_y^{t,i}(j)}\right] + w^{t,i}(j) \bmod 2\pi, 0 \le \beta^{t,i}(j) < 2\pi \quad (2)$$

Where $r_x^{t,i}(j) = r_x^t(j) - r_{xs}^i$, $r_y^{t,i}(j) = r_y^t(j) - r_{ys}^i$ and $w^{t,i}(j)$'s denote noise components of the ith sensor assumed to be white, Gaussian noise with zero mean and variance $\sigma_i^2$.

The measuring bearing data vector of sensor i at time j forms a N-tuple vector as,

$$\beta^i(j) = (\beta^{1,i}(j), \beta^{2,i}(j), \cdots, \beta^{N,i}(j))' \quad (3)$$

Using the measuring data from multiple sensors, we want to estimate the positions $(r_x^t(j), r_y^t(j))$ for all targets *t*, *t=1,2,…,N* and all the time indices j, j=0,1,…,K. Since we have no a prior knowledge on the origin of each measurement, we have to associate each measurement vector with an N × N data assignment matrix $C^i(j)$ for all i and j, whose components consist of 0-1 elements with just one if its element to take on the value of 1 in each of the rows and columns: Here the entry $[C^i(j)]_{mt}$ =1 denotes that the *m*th element of measurement vector $\beta^i(j)$ is associated with the tth target. Then denote $\hat{\beta}^i(j) = (\hat{\beta}^{1,i}(j), \hat{\beta}^{2,i}(j), \cdots, \hat{\beta}^{N,i}(j))'$ as the cumulative N-tuple bearing estimate measurement vector of sensor i at time j, where $\hat{\beta}^{t,i}(j)$ $(0 \le \hat{\beta}^{t,i}(j) < 2\pi)$ is the estimated measurement for the target t from the sensor i at time j. This can be obtained from the initial target state vector $\hat{X}$ which describes the position and velocity of all the targets, by an arctangent function like (2). Then the problem can best be formulated by a ML-based approach as follows

## 2.2 ML Principle for Multiple Target Tracking

A solution to the ML-based multiple targets tracking algorithm is obtained by maximizing the conditional likelihood of the measurements of the sensors $((\beta^1(0), \beta^1(1), \cdots, \beta^1(k), \beta^2(0), \cdots, \beta^2(k), \cdots, \beta^s(0), \cdots, \beta^s(k))$ given $C^k = \{C^1(0), C^1(1), \cdots; C^1(k), C^2(0), \cdots; C^2(k), \cdots C^s(0), \cdots; C^s(k)\}$ and the initial target state vector $\hat{X}$

$$J = \frac{1}{c}\exp\left\{-\frac{1}{2}\sum_{i=1}^{s}\sum_{j=0}^{k}[C^i(j)\beta^i(j) \dot{-} \hat{\beta}^i(j)]' R_i^{-1}[C^i(j)\beta^i(j) \dot{-} \hat{\beta}^i(j)]\right\} \quad (4)$$

This is equivalent to minimizing the corresponding ASE,

$$ASE = \frac{1}{skN}\sum_{i=1}^{s}\sum_{j=0}^{k}[C^i(j)\beta^i(j) \dot{-} \hat{\beta}^i(j)]' R_i^{-1}[C^i(j)\beta^i(j) \dot{-} \hat{\beta}^i(j)]$$

$$= \frac{1}{skN}\sum_{t=1}^{N} E_t = \frac{1}{skN}\sum_{t=1}^{N}\sum_{i=1}^{s}\sum_{j=0}^{k}\left(\frac{\beta^{c(i,j,t),i} \dot{-} \hat{\beta}^{t,i}}{\sigma_i}\right)^2 \quad (5)$$

$R_i = \sigma_i^2 I$ of Eqs. (4) and (5) denotes the N × N diagonal noise covariance matrix at the ith sensor and c is a constant. Where $c(i, j, t) = m$ if $[C^i(j)]_{mt} = 1$.

With $C^k$ fixed, these N independent $E_t$'s can be minimized with respect to $\hat{X}_0^t$ by the Gauss-Newton iteration [1]. Consider the form of the average square error in (5). Each summand is nonnegative, and thus, for a given $\hat{X}_0$, minimizing E with respect to $C^k$ is equivalent to minimizing each individual term independently with respect to $C^i(j)$. Thus, we perform the following operations for minimizing (5) with respect to $C^k$, This requires a minimization of the following cost function:

$$T(i,j) = \sum_{n=1}^{N}\sum_{m=1}^{N} \rho_{nm}[C^i(j)]_{nm} \qquad (6)$$

The N2 square errors $\rho_{n,m} \equiv (\hat{\beta}_n \dot{-} \beta_m)^2$

Note that Eqs. (4) and (5) differ distinctly from those of the existing references on this topic[1], it is defined as,

$$a \dot{-} b = \begin{cases} a-b, & if \quad -\pi < a-b \le \pi \\ a-b-2\pi, & if \quad a-b > \pi \\ a-b+2\pi, & if \quad a-b \le -\pi \end{cases}$$

We see immediately that this new operation is needed to ensure the angular differences formed to be in $(-\pi, \pi]$ such that the mismatching of angles across the so-called Riemann sheet is avoided, including our previous work [10].

Most of the existing publications solve the data assignment problem in minimization equation (6) by general Hungarian type algorithms. But we now show that the following 2 observations of the coefficient matrix, and prove the observation 1 strictly, and verify observation 2 by experiment, then we draw a conclusion that the rotational sorting algorithm can be used for the data assignment process of Multiple Target Tracking problem, reducing the computational complexity from O($N^3$) of the Hungarian type algorithms to O($N^2$), even to O(log$N$)

## 3 Main Results

Before we introduce our observation, there are some definition should be purposed firstly

**Definition 1**: For arbitrary two angles $\alpha$ and $\beta$, $0 \le \alpha, \beta < 2\pi$, then defined

$$\min\{\alpha,\beta\} = \begin{cases} \beta, & if \quad \alpha \dot{-} \beta \ge 0 \\ \alpha, & otherwise \end{cases}$$

$$\max\{\alpha,\beta\} = \begin{cases} \alpha, & if \quad \alpha \dot{-} \beta \ge 0 \\ \beta, & otherwise \end{cases}$$

**Definition 2** (Rotational Sort): For a list of angles $\alpha_1, \alpha_2, \cdots, \alpha_n$, $0 \le \alpha_i < 2\pi$, i=1,2,…,n, the rotational sort of the angles is defined as a permutation $\alpha_{i1}, \alpha_{i2}, \cdots, \alpha_{in}$, such that $\alpha_{iu} - \alpha_{i1} \bmod 2\pi \ge \alpha_{iv} - \alpha_{i1} \bmod 2\pi$ , for any $1 \le u < v \le n$.

The notation $\alpha_1 \prec \alpha_2 \prec \cdots \prec \alpha_n$ implies that a list of $\alpha_1, \alpha_2, \cdots, \alpha_n$ has been rotationally sorted, and we can always use "$\prec$" to denote a sequence of any two angles in the list, if the condition of Definition 2 is satisfied. It should be important to note that even if $\alpha_1, \alpha_2, \cdots, \alpha_n$ are all distinct and unequal, the rotational sorting of the list is not unique.

**Definition 3** (movable unimodal): The finite series of real numbers $\alpha_1, \alpha_2, \cdots, \alpha_n$, is called a movable unimodal series if and only if there exits two element $\alpha_i, \alpha_j$, such that the series $\alpha_i, \alpha_{i+1}, \cdots, \alpha_{j-1}$ is a strictly decreasing series and $\alpha_j, \alpha_{j+1}, \cdots, \alpha_{i-1}$ is a strictly increasing series, where $\alpha_{i+n}$ is denoted as $\alpha_i$.

**Observation 1**: Suppose $\beta_{i1}, \beta_{i2}$ and $\beta_{i3}$ are distinct and pair-wise unequal elements of the vector $\beta = (\beta_1, \beta_2, \cdots, \beta_N)'$, satisfying $\beta_{i1} \prec \beta_{i2} \prec \beta_{i3}$, and $\hat{\beta}_{j1}, \hat{\beta}_{j2}$, $\hat{\beta}_{j3}$ are distinct and pair-wise unequal elements of the vector $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2, \cdots, \hat{\beta}_N)'$ satisfying $\hat{\beta}_{j1} \prec \hat{\beta}_{j2} \prec \hat{\beta}_{j3}$. If C is the assignment matrix satisfying $[C]_{j1i1} = [C]_{j3i2} = [C]_{j2i3} = 1$ then C can not be a best assignment matrix

The Lemma 1 is needed to prove Observation 1.

**Lemma 1**: Let C be an assignment matrix between vector $\beta = (\beta_1, \beta_2, \cdots, \beta_N)'$ and $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2, \cdots, \hat{\beta}_N)'$, satisfying $[C]_{j2i1} = [C]_{j1i2} = 1$, $\beta_{i1} \ne \beta_{i2}$ and $\hat{\beta}_{j1} \ne \hat{\beta}_{j2}$. And suppose that there is a rotational sort of the angles $\beta_{i1}, \beta_{i2}$ and $\hat{\beta}_{j1}, \hat{\beta}_{j2}$, which satisfies property 1, property 2 or 2', and property 3 or 3' below:

1 $\hat{\beta}_{j1} \prec \hat{\beta}_{j2}$ and $\beta_{i1} \prec \beta_{i2}$

2 min{$\beta_{i1}, \hat{\beta}_{j1}$}$= \beta_{i1}$, $\hat{\beta}_{j2} - \beta_{i1} \ge 0$, and $\hat{\beta}_{j2} - \hat{\beta}_{j1} \ge 0$;

2'min{$\beta_{i1}, \hat{\beta}_{j1}$}$= \hat{\beta}_{j1}$, $\beta_{i2} - \hat{\beta}_{j1} \ge 0$, and $\beta_{i2} - \beta_{i1} \ge 0$;

3max{$\beta_{i2}, \hat{\beta}_{j2}$}$= \beta_{i2}$, $\beta_{i2} - \hat{\beta}_{j1} \ge 0$, and $\hat{\beta}_{j2} - \hat{\beta}_{j1} \ge 0$;

3'max{$\beta_{i2}, \hat{\beta}_{j2}$}$= \hat{\beta}_{j2}$, $\hat{\beta}_{j2} - \beta_{i1} \ge 0$, and $\beta_{i2} - \beta_{i1} \ge 0$;

Then C can not be a best assignment matrix.

Proof: Without loss of generality, we suppose that the property 2 is applicable. Then we will need to consider only the three cases of figure 2; (1) $\beta_{i1} \prec \beta_{i2} \prec \hat{\beta}_{j1} \prec \hat{\beta}_{j2}$, (2) $\beta_{i1} \prec \hat{\beta}_{j1} \prec \beta_{i2} \prec \hat{\beta}_{j2}$ and (3) $\beta_{i1} \prec \hat{\beta}_{j1} \prec \hat{\beta}_{j2} \prec \beta_{i2}$ separately. For the case of (2) and (3), we see immediately that

$(\hat{\beta}_{j2} \div \beta_{i2})^2 + (\hat{\beta}_{j1} \div \beta_{i1})^2 < (\hat{\beta}_{j2} \div \beta_{i1})^2 + (\hat{\beta}_{j1} \div \beta_{i2})^2$ is obvious, For case (1), we have

$(\hat{\beta}_{j2} \div \beta_{i2})^2 + (\hat{\beta}_{j1} \div \beta_{i1})^2$

$= ((\hat{\beta}_{j2} \div \beta_{i1}) - (\beta_{i12} \div \beta_{i1}))2 + (\hat{\beta}_{j1} \div \beta_{i1})^2$

$< (\hat{\beta}_{j2} \div \beta_{i1})^2 + ((\hat{\beta}_{j1} \div \beta_{i1}) - (\beta_{i1} \div \beta_{i1}))2$

$= (\hat{\beta}_{j2} \div \beta_{i1})^2 + (\hat{\beta}_{j1} \div \beta_{i2})^2$

The conclusion of the lemma is now immediate.

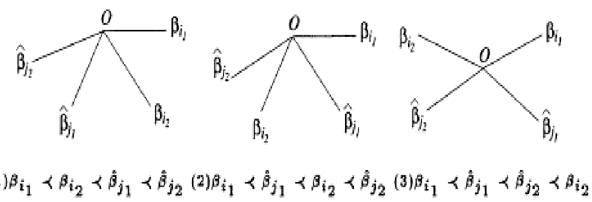


**Figure 2**. An Example of the 4 Angles in Lemma 1

**Theorem 1**: Suppose that the cumulative bearing estimate measurement vector for sensor i at time j ( $0 \le j \le k$ ) is given by $(\hat{\beta}^i(j) = (\hat{\beta}^{1,i}(j), \hat{\beta}^{2,i}(j), \cdots, \hat{\beta}^{N,i}(j))'$, and that the real bearing data of sensor i is given by $(\beta^i(j) = (\beta^{1,i}(j), \beta^{2,i}(j), \cdots, \beta^{N,i}(j))'$. Given two 0-1 distinct matrices C1 and C2, each having just one 1 element in each row and each column, such that $C_1\beta^i(j) = (a_1, a_2, \cdots, a_N)'$, $C_2\hat{\beta}^i(k) = (b_1, b_2, \cdots, b_N)'$, where $a_1 \prec a_2 \prec \cdots \prec a_N$ and $b_1 \prec b_2 \prec \cdots \prec b_N$, then the best data assignment matrix $C^i(j)$ for minimizing ASE with fixed $\hat{X}$ should be chosen from the set $\{C_2' P_r C_1 \mid P_r = [p_{r1,r2}]_{N\times N}, r = 0,1,2,\cdots N-1\}$, where $p_{r1,r2}$ is defined as

$$p_{r1,r2} = \begin{cases} 1 & if \cdot r_2 - r - r_1 \equiv 0 \bmod N. \\ 0 & otherwise \end{cases}$$

Proof: If $C_r^i(j) = C_2' P_r C_1$, we have ASE= $\frac{1}{skN}\sum_{i=1}^{s}\sum_{j=0}^{k} E_{ij}(r)$. We write:

$E_{ij}(r) = [C^i(j)\beta^i(j) \div \hat{\beta}^i(j)]' R_i^{-1}[C^i(j)\beta^i(j) \div \hat{\beta}^i(j)]$.

Noting that $C_2' = C_2^{-1}$, we have the following relation:

$E_{ij}(r) = [C_2' P_r C_1\beta^i(j) \div \hat{\beta}^i(j)]' R_i^{-1}[C_2' P_r C_1\beta^i(j) \div \hat{\beta}^i(j)]$

$= [P_r C_1\beta^i(j) \div C_2\hat{\beta}^i(j)]' R_i^{-1}[P_r C_1\beta^i(j) \div C_2\hat{\beta}^i(j)]$

$= \frac{1}{\sigma_i}\sum_{t=1}^{N}(a_t \div b_{t+r-1 \bmod N+1})^2$

We should note that the set $\{C_2' P_r C_1 \mid r = 0,1,2,\cdots N-1\}$ has been obtained after rotational enumeration all the following matches:

$$\begin{pmatrix} \beta^{1,i}(j) & \leftrightarrow & \hat{\beta}^{1,i}(j) \\ \beta^{2,i}(j) & \leftrightarrow & \hat{\beta}^{2,i}(j) \\ \vdots & & \vdots \\ \beta^{N,i}(j) & \leftrightarrow & \hat{\beta}^{N,i}(j) \end{pmatrix}, \begin{pmatrix} \beta^{1,i}(j) & \leftrightarrow & \hat{\beta}^{N,i}(j) \\ \beta^{2,i}(j) & \leftrightarrow & \hat{\beta}^{1,i}(j) \\ \vdots & & \vdots \\ \beta^{N,i}(j) & \leftrightarrow & \hat{\beta}^{N-1,i}(j) \end{pmatrix}, \ldots, \begin{pmatrix} \beta^{1,i}(j) & \leftrightarrow & \hat{\beta}^{2,i}(j) \\ \beta^{2,i}(j) & \leftrightarrow & \hat{\beta}^{3,i}(j) \\ \vdots & & \vdots \\ \beta^{N,i}(j) & \leftrightarrow & \hat{\beta}^{1,i}(j) \end{pmatrix}$$

The conclusion of Theorem 1 is immediate from Observation 1 which has been proved.

Theorem 1 shows that an optimal local matrix $C^i(j)$ for a fixed $\hat{X}$ can be obtained first by sorting $\beta^i(j)$ and $\hat{\beta}^i(j)$ by any of the sorting algorithm with the complexity of $O(N\log N)$, and then choosing the one satisfying Theorem 1 from among N potential candidates. This can be obviously done with the complexity of $O(N^2)$. The complexity of computing the present assignment matrix $C^i(j)$ is far more efficient than the Hungarian method type algorithm used by most of previous works having the complexity of $O(N^3)$.

Now according to the rotational enumeration matches, we define that:

$$T_1(i,j) = ((\beta^{1,i}(j) \div \hat{\beta}^{1,i}(j))^2 + (\beta^{2,i}(j) \div \hat{\beta}^{2,i}(j))^2 + \cdots + (\beta^{N,i}(j) \div \hat{\beta}^{N,i}(j))^2)$$
$$T_2(i,j) = ((\beta^{1,i}(j) \div \hat{\beta}^{N,i}(j))^2 + (\beta^{2,i}(j) \div \hat{\beta}^{1,i}(j))^2 + \cdots + (\beta^{N,i}(j) \div \hat{\beta}^{N-1,i}(j))^2)$$
$$\vdots$$
$$T_N(i,j) = ((\beta^{1,i}(j) \div \hat{\beta}^{2,i}(j))^2 + (\beta^{2,i}(j) \div \hat{\beta}^{3,i}(j))^2 + \cdots + (\beta^{N,i}(j) \div \hat{\beta}^{1,i}(j))^2) \quad (7)$$

So, $T(i,j) = (T_1(i,j), T_2(i,j), \cdots, T_N(i,j))$

Then minimizing T(i, j) in (6) equal to selecting the minimum component in vector T(i, j) above. Furthermore, we found an interesting fact as follows.

**Observation 2**: $T_1$, $T_2$,…,$T_N$ form a movable unimodal series, if both of $\hat{\beta}_1, \hat{\beta}_2, \cdots, \hat{\beta}_N$ and $\beta_1, \beta_2, \cdots, \beta_N$ have been rotational sort. Where $T_1$, $T_2$,…,$T_N$ definite by $\hat{\beta}_1, \hat{\beta}_2, \cdots, \hat{\beta}_N$ and $\beta_1, \beta_2, \cdots, \beta_N$ as above.

Although we have found the rule in observation 2 which describes the perfect property of cost functions through experiments (Fig. 3), it is yet to be proved theoretically. But this rule is always correct in our experiments and it is so important that it can reduce the computational complexity from $O(N^2)$ to $O(\log N)$. Before we finally introduce the complete algorithm, a Dichotomous Search method should be given as below,

**Dichotomous Search:** This finds the minimum of a unimodal discrete function $T_1$, $T_2$,…,$T_N$ on an interval, [1, N], by evaluating points placed near the center, approximating the bisection method.

Let $x = \left[\frac{N}{2}\right] - 1$, $y = \left[\frac{N}{2}\right]$, $z = \left[\frac{N}{2}\right] + 1$

Accordingly we got $T_x$, $T_y$, $T_z$ from unimodal series as well as the $K_{xy}$, $K_{yz}$, where $K_{xy}=T_y-T_x$, $K_{yz}=T_z-T_y$. Then if $K_{xy}\times K_{yz}<0$, the minimum point is (y, $T_y$), if $K_{xy}\times K_{yz}>0$, the new interval of uncertainty is [1, [N/2]] or [[N/2]+1, N] depending on (1) $K_{xy}>0$, $K_{yz}>0$ or (2) $K_{xy}<0$, $K_{yz}<0$. We continue this process searching for at most Log$N$ times, we can reach the minimum point.

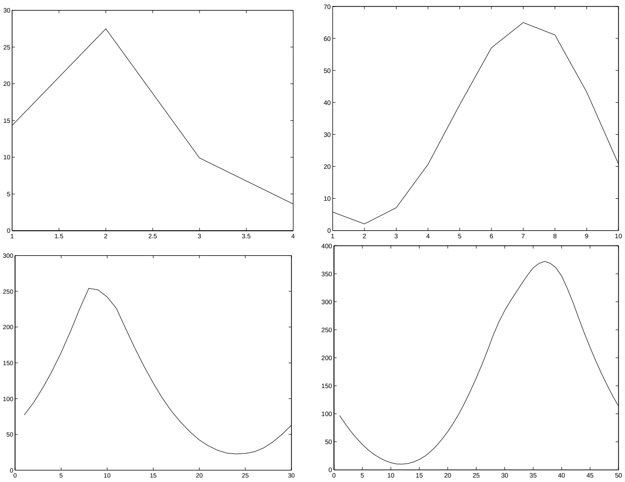

Figure 3 $T_1, T_2,\ldots,T_N$ form a movable unimodal series

It should be important to note that even if $T_1, T_2,\ldots,T_N$ is not a unimodal series but a movable unimodal series, Dichotomous Search method also can be valid. In fact, if $T_1, T_2,\ldots,T_N$ is a movable unimodal series, we can connect (1,$T_1$) and (N, $T_N$) by the line $f$, and then we have (1, $f$(1)), (2, f(2)),…,(N, $f$(N)), then we will get a new unimodal series, that is min{$T_1$, $f$(1)}, min{$T_2$, $f$(2)},…, min{$T_N$, $f$(N)}, and the minimum must be in those unimodal series. So we can use Dichotomous Search method to find the minimum. It is obvious that an rotational sorting algorithm can be used for the data assignment process of multiple targets tracking problem with O(log$N$) computational complexity. The algorithm is illustrated in the table 1

**Table 1** Routine for finding an optimal assignment matrix

| | |
|---|---|
| 1. | Sort the terms in $\beta^i(j)$ into increasing order |
| 2. | Sort the terms in $\hat{\beta}^i(j)$ into increasing order |
| 3. | Obtain $T_1, T_2,\ldots,T_N$ using (7) |
| 4. | Obtain $f(1), f(2),.., f(N)$ |
| 5. | Obtain min{$T_1$,f(1)}, min{$T_2$,f(2)},…, min{$T_N$,f(N)}, |
| 6. | Dichotomous Search method to find the minimum, then decide the optimal assignment matrix. |

# 4 Conclusions

In this paper, we presented an improved solution of a maximum likelihood(ML)-based multiple targets tracking problem. We have proved that a new O($N^2$) data assignment algorithm based on the rotational sorting of angles maximizes the conditional likelihood function with respect to data association for fixed target states. Further more a faster algorithm based Observation 2 was developed, the performance of the ML-based relaxation algorithm has been significantly improved. It would not be difficult to show that the present data assignment algorithm can also be converted to solve a generalized version of the n-Tokyoites' loop line commuter problem[11].

Further study would prove Observation 2 theoretically, and find applications in multiple targetS tracking problem with missing or cluttering sensor data.


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