Approach to conservation laws based on Bayes group generators and its geodesic flows

Manouchehr Amiri
Research institute for fundamental sciences,Tabriz ,Iran

Abstract

Abstract: On the basis of Bayes transformation group generators ,we introduce geodesic flows in non-Euclidean Bayes group manifold and derive the relation between conditional likelihoods and affine connections of this topological group manifold in four dimensional parametric space.Using this method , we show that field equations of log-likelihoods are similar to spinless particle field equations. Generalization to the two dimensional space-time leads to the well known Lorentz transformation.By introducing the relation between probability conservation and conservation of physical quantities, we derive momentum conservation law and solutions of one dimensional heat transfer.

Key-Words:Bayes group generators,Conservation law,Geodesic flows.

Introduction

Recent trends toward application of Bayesian analysis in diverse fields of astronomy [14,15],quantum physics interpretation[3],solid state physics[21], medical diagnosis [20] , equilibrium statistical physics [9]artificial intelligence,classical statistical mechanics[23] and quantum measurement [3,21] indicates that Bayesian approach accepted as an efficient tool for evolutive systems.

Exploring relation between *probability conservation* and *conservation of physical quantity* via the Bayes rule and its extension as Bayes group generators[9] regardless of traditional symmetry approach to conservation laws, is the main purpose of our paper.Similar concept is known in quantum mechanics as the probability density (squared amplitude of particle wave function) follows the well known schrodinger equation.These two *conservation laws* are far from each other, where the former governs abstract fields in mathematics and its integrity has been questioned frequently by philosophers [1] and "Fuzzists"[2], and the other is respectful fundamental concept in physics.

Obviously if one finds the connection between two separate concept of conservations , then will face to an interesting substitution ,i.e. the conditional probability density $\varphi(A\,|\,s)$ for a continuous state '*A*' and parameter 's' can be assumed as a real physical concept such as field function $\varphi(A,s)$. This assumption can be found out in Bayesian field theories[22]and allows us to consider the probability densities as physical quantity densities and to treat the probability laws as physics laws.On the basis of logical assumption one may consider the Bayes rule as a form of physical causality[18].We intend to show Bayes rule's likelihoods can be considered as the link between pre and post evolution of an isolated system in which conserved physical quantities replaced by probability conservation ,threrfor in specific problems the suitable choice of these likelihoods to satisfy the problem conditions results in desired solution.(see for example SEC.4).

Recent trends observed in quantum information theories states that in the Bayesian approach, probability measures a degree of belief for a *single trial* without connection to limiting frequencies and therefore is an acceptable technique in quantum physics [3,19,21] .on the other hand "deterministic physics laws" govern the results of a *single trial* [4], and hence there is a possible connection between "Bayesian approach "(i.e. Bayes rule) and "physics laws".althuogh all details of this connection is beyond the aim of our paper, but it conveys the possible relation between probability and physical quantity conservation.

Most applications of Bayesian approach focused on its statistical views [13-16] although some papers have revealed deep correlation with quantum mechanics interpretations [3,17].

In this paper we emphasize on the special features of Bayes analytic group and its generators [9] when parameteric space(or group manifold as demonstrated in appendix A)be considered generally as a (Pseudo-)Riemannian manifold with the dimension equal or greater than target manifold $\mathcal{X}$ and in a special case , as a *real* physical space-time. We apply the method (SEC.1) to describe spinless particles fields(Klein-Gordon equation), one dimensinal heat transfer,momentum conservation, wave propagation , derivation of Lorentz transformation and *light velocity invariance* (SEC.3) and homomorphism of its two dimensional group representation with *SL(2,C)* (SEC.2).At first glance,it seems to be similar to a series of papers [5], whose *characteristic hypersufaces* substituted *metric fields* to reformulate general relativity (GR). However our approach to this subject is essentially different and aforementioned characteristic hypersurfaces (and techniques) can be considered as special cases of present article.

The concept of metrics and differential geometry in statistics (i.e.information geometry), was first introduced by Fisher and later developed by Amari [6,7]. The application of *log- likelihoods* in Fisher information metric results in a new concept in information theory [7,8]. In present article the role of *log- likelihoods* has been deduced from the new concept of Bayes generators as "field functions".

These generators was first introduced for statistical equilibrium states(Boltzmann distribution) [9].

Extension of this concept from finite dimensonal space $\mathcal{X}$ to an infinite dimensional one can be done by fourier transformation. We introduce a novel method for solution of Cauchy–Dirichlet problem or heat transfer equation (and wave propagation) in one dimension .Finally concept of left invariant vector field σ_g in prarametric space leads to derivation of various free particle fields.

1. Bayes group generators formulation in Riemannian parametric space

We assign G_B as the Bayes transformation analytic group or an Abelian Lie group with its $\hat{g}_i$ generators (see Appendix A) [9]. Let the set $\mathcal{X}$ be considered as the target manifold (m-1 dimensional) for action group $G_B \times \mathcal{X} \to \mathcal{X}$ with a flat space structure in X_i Euclidean coordinates($1 \le i \le m$) and constraint $\sum_i X_i = 1$, which contains all possible probabilities X_i (or quantities governed by conservation law) of independent states $(A_1, A_2, \ldots A_m)$.

Parameteric space S is considered as a Riemannian positive definite metric manifold ,(i.e. $\sum v^{\alpha} v_{\alpha} > 0$) [12]. Topologically manifold of this group corresponds the points of parametric space S. Generally the action of G_B on $\mathcal{X}$ results in a diffeomorphism.

First we focus on the main condition for identity of the group (Appendix A ;equation A .11):

$$\prod_i f_{i\mu} = \prod_j f_{j\nu}$$

We set

$$F_\mu = \prod_i f_{i\mu}$$

Then:

$$F_\mu = F_\nu \quad \text{(for all } 1 \le \mu, \nu \le m) \qquad (1.1)$$

Each $f_{i\mu}$ is a function of corresponding single parameter s_i and μ (index determines its form). F_μ is function of n independent parameters $(s_1, s_2, \ldots, s_n)$ i.e. coordinates of parameteric space S and lower index μ. Therefore the set of points which satisfy the equations (1.1) constitute a hypersurface with a lower dimension ($< n$) in S, provided that $m \le n$. We assume this manifold as a submanifold of S, generally to be a Riemannian manifold with positive definite metrics. because all points on this manifold satisfy the condition of *identity*, we call it "Identity manifold K " and know that it is the Kernel of a homomorphic mapping $G_B \to sym(\mathcal{X})$[10], thus:

$$K = \{g \in G_B | g.p = p\} \quad \text{for every } p \in \mathcal{X} \qquad (1.2)$$

$sym(\mathcal{X})$ is a local group on $\mathcal{X}$, acting on it's local coordinate as a symmetry group, so it's dimension will be $m' = m - 1$. It has been proved that K is a submanifold of S of ($n - m'$) dimension [11], i.e.:

$$\dim K = n - m'$$

It can be verified that for every $g \in G_B$ there exist left *coset* of K (e.g gK) which constitutes the qoutient space G_B / K . Actions of all points of each left *coset* on $\mathcal{X}$ are the same. In the other words it can be shown that for every $g \in G_B$ there exist a set of points (or a submanifold) in manifold

G_B which their actions on $\mathcal{X}$ is the same as g (see below dicussion) .Therefore, the qoutient space G_B / K comprises an *equivalence class* of left *cosets* . Obviously there is an isomorphic mapping of this *factor group* G_B / K onto $sym(\mathcal{X})$. K constitutes a normal subgroup of G_B so due to a theorem in differential geometry for a given vector field with an *integral curve* passing through e (identity) each gK makes an *integral curve* σ_g passing through $g \in G_B$ in parameteric space S [10]. Therefore the initial integral curve σ on K extended to the whole manifold G_B as a flow of σ_g [10]. σ and set of all σ_g ($g \in G_B$) make flows of integral curves which corresponds to each $g \in G_B$ .This means that there exist a vector field (invariant) on S whose integral curves sweeps all the points of S . The curve passing through identity $e \in G_B$ is σ curve and one passing through $g \in G_B$ is σ_g curve. All the points on σ have the action of "e"(identity) and all points on σ_g have the action identical to $g \in G_B$. Regard to the property of K and hence σ for given $g, g' \in G_B$ and $\tau \in \sigma$ we have:

$$g = g\tau \text{ and } g' = g'\tau \qquad (1.3)$$

So $g\sigma$ is a point (g) , not a curve.To find σ_g we can define a curve of points g' (with coordinate s'_i), for which the action on $\mathcal{X}$ is the same as g (with coordinate s_i) , with repect to equation (A.6) for Bayes rule we have:

$$X'_\mu = \frac{F_\mu(\vec{s}) X_\mu}{\sum_\nu F_\nu(\vec{s}) X_\nu} = \frac{F_\mu(\vec{s}\,') X_\mu}{\sum_\nu F_\nu(\vec{s}\,') X_\nu} \qquad (1.4)$$

($\vec{s}$ is a vector in parametric space).
With *induction proof* it can be shown that this equation results in a system of equations whose number of independent equations is $m-1$ with n unknown variable s'_i . So in the case m=n the solution is a curve in G_B , this curve coincides σ_g .
Closure property of G_B may be verified also by this proof and equation (1.4).
Important property of these curves is that :
covariant derivative of vector field of Bayes group generators (Lie algebra bases) vanishes on these curves.i.e. σ_g as geodesic curves are integral curves of this vector field (see below).
If we limit our general case to the case of real physical space-time i.e. $n = 4$, hence from the equation (1.1), it can be understood that intersection of $Z = F_\mu$ hypersurfaces can be found only with the condition $m \geq n$. Therefore in this case, $m = 4$ is a necessary condition(internal coordinates is not considered). This means that the number of X_μ variables which obey conservation law $\sum_{i=1}^{m} X_i = 1$ *should be* of the same dimension of space-time.
Now let define a vector field by Lie algebra generators of G_B as follows:
from (1.1) it is easy to show that by introducing $A^i_{\bar{\mu}}$ as (μ replaced by $\bar{\mu}$ to show it has not index role in $\mathcal{X}$) :

$$A^i_{\bar{\mu}} = \frac{1}{F_{\bar{\mu}}} \frac{\partial F_{\bar{\mu}}}{\partial s_i} = \frac{1}{f_{i\bar{\mu}}} \frac{\partial f_{i\bar{\mu}}}{\partial s_i} \qquad (1.5)$$

(no summation on repeates $\bar{\mu}$ because in this paper we *do not* apply dummy summation on repeated indices, all summation accompanied by $\sum$)
Or

$$A^i_{\bar{\mu}} = \frac{\partial}{\partial s_i} \log_e F_{\bar{\mu}} = \frac{\partial}{\partial s_i} \log_e f_{i\bar{\mu}} . \qquad (1.6)$$

$A^i_{\bar{\mu}}$ is a contravariant vector field on parameter space S . Now consider the generator of Bayes Lie group G_B [9](see Appendix A):

$$\hat{g}^i = -\sum_{\bar{\mu}} X_{\bar{\mu}} \left(\frac{1}{f_{i\bar{\mu}}} \frac{\partial f_{i\bar{\mu}}}{\partial s_i} - \sum_{\bar{\nu}} \frac{1}{f_{i\bar{\nu}}} \frac{\partial f_{i\bar{\nu}}}{\partial s_i} X_{\bar{\nu}} \right) \frac{\partial}{\partial X_{\bar{\mu}}} \qquad (1.7)$$

or by (1.5) and (1.6):

$$\hat{g}^i = -\sum_{\bar{\mu}} X_{\bar{\mu}} \left(\frac{1}{F_{\bar{\mu}}} \frac{\partial F_{\bar{\mu}}}{\partial s_i} - \sum_{\bar{\nu}} \frac{1}{F_{\bar{\nu}}} \frac{\partial F_{\bar{\nu}}}{\partial s_i} X_{\bar{\nu}} \right) \frac{\partial}{\partial X_{\bar{\mu}}} \qquad (1.8)$$

Expression in bracket should be calculated on K (or σ) for each arbitrary point in $\mathcal{X}$ (in other words on the identity of G_B). Indeed we can conclude that $\frac{1}{F_{\bar{\mu}}}, \frac{\partial F_{\bar{\mu}}}{\partial s_i}$ have common values on this intersection manifold K for all hypersurfaces:

$$Z = F_{\bar{\mu}} \tag{1.9}$$

Therefore, $A_{\bar{\mu}}^{i} = \frac{1}{F_{\bar{\mu}}} \frac{\partial F_{\bar{\mu}}}{\partial s_i}$ is the same for all $\bar{\mu}$ on K.

If we set:

$$\gamma_{\bar{\mu}}^{i} = \left(\frac{1}{F_{\bar{\mu}}} \frac{\partial F_{\bar{\mu}}}{\partial s_i} - \sum_{\bar{\nu}} \frac{1}{F_{\bar{\nu}}} \frac{\partial F_{\bar{\nu}}}{\partial s_i} X^{\bar{\nu}}\right) \tag{1.10}$$

We,find:

$$\gamma_{\bar{\mu}}^{i} = (A_{\bar{\mu}}^{i} - \sum A_{\bar{\nu}}^{i} X^{\bar{\nu}}) = 0 \tag{1.11}$$

So $\gamma_{\bar{\mu}}^{i}$ is a contravariant vector field which vanishes on K (or σ) manifold (with exclusion the points on which $F_{\bar{\mu}} = 0$):

$$\gamma_{\bar{\mu}}^{i} = 0 \tag{1.12}$$

This is a natural result ,because the actions of group elements on K make no change on X manifold points**.** Thus this kernel corresponds the *stationary (equilibrium) state* of states in space X.

With respect to assumption that S is a Riemannian manifold with positive definite metric, $\gamma_{\bar{\mu}}^{i}$ is a *null vetor field on* K [12].

basically $\gamma_{\bar{\mu}}^{i}$ are mixed tensors of rank 2 and can be demonstrated as matrics form ,each it's μ th column can be considered as a column vector γ^{i}.

We can imagine situations where covariant deri vative of $\frac{\partial}{\partial s_i} \log_e F_{\bar{\mu}} = \frac{1}{F_{\bar{\mu}}} \frac{\partial F_{\bar{\mu}}}{\partial s_i} = A_{\bar{\mu}}^{i}$ vanishes. If we set $\varphi_{\bar{\mu}} = \log_e F_{\bar{\mu}}$ (as a log-likelihood) obviousely $\varphi_{\bar{\mu}}$ and $(\frac{\partial}{\partial s_i} \varphi_{\bar{\mu}})$ have common value for all $\bar{\mu}$'s at every point on K (or σ). The derivative $\frac{\partial}{\partial s_i} \varphi_{\bar{\mu}}$, as a directional derivative, is calculated on K and if the covariant derivative of $\frac{\partial}{\partial s_i} \varphi_{\bar{\mu}}$ vanishes on a curve σ on K, then σ will be a *null geodesics* in K or S, in such a case, considering the equation (1.11), covariant derivative of γ^{i} also disappears :

$$\gamma_{\bar{\mu};i}^{i} = 0 \qquad \text{(no summation on } i\text{)} \tag{1.13}$$

(obviously $\gamma_{\bar{\mu};j}^{i} = 0$ for $i \neq j$)

We can conclude the fact that vanishing of $\gamma_{\bar{\mu};i}^{i}$ means $\sigma \subset K$ is a *null geodesics*.(if parameteric space coincides space-time,and $m = n$ then $\sigma = K$). This means that if regular derivative of $\gamma_{\bar{\mu}}^{i}$ respect to s_i denoted by:

$$\gamma_{\bar{\mu},i}^{i} = A_{\bar{\mu},i}^{i} - \sum_{\bar{\nu}} A_{\bar{\nu},i}^{i} X^{\bar{\nu}} \tag{1.14}$$

the covariant derivative will become:

$$\gamma_{\bar{\mu};i}^{i} = A_{\bar{\mu},i}^{i} - \sum_{\alpha} \Gamma_{i\alpha}^{i} A_{\bar{\mu}}^{\alpha} - \sum_{\bar{\nu}} A_{\bar{\nu},i}^{i} X^{\bar{\nu}} + \sum_{\bar{\nu},\beta} \Gamma_{i\beta}^{i} A_{\bar{\nu}}^{\beta} X^{\bar{\nu}}$$

And can be written in the form:

$$\gamma_{\bar{\mu};i}^{i} = \frac{\partial^2 \varphi_{\bar{\mu}}}{\partial {s_i}^2} - \sum_{\alpha} \Gamma_{i\alpha}^{i} \frac{\partial \varphi_{\bar{\mu}}}{\partial s_\alpha} - \sum_{\bar{\nu}} \frac{\partial^2 \varphi_{\bar{\mu}}}{\partial {s_i}^2} X^{\bar{\nu}} + \sum_{\bar{\nu},\beta} \Gamma_{i\beta}^{i} \frac{\partial \varphi_{\bar{\nu}}}{\partial s_\beta} X^{\bar{\nu}} \tag{1.15}$$

The unique condition for vanishing $\gamma_{;i}^{i}$ (for arbitrary $X^{\bar{\mu}}$) will be :

$$\frac{\partial^2 \varphi_{\bar{\mu}}}{\partial {s_i}^2} - \sum_{\alpha} \Gamma_{i,\alpha}^{i} \frac{\partial \varphi_{\bar{\mu}}}{\partial s_\alpha} = C(s_i)\,. \tag{1.16}$$

Simply ,this equation is satisfied only on "*null geodesic*" in K ,i.e. σ curve(provided that $n - m \geq 2$).in the space-time coordinate K can be depicted as light cone surface and σ as light rays lied on this surface. $C(s_i)$ in this equations system is common for all $\varphi_{\bar{\mu}}$ s.

So in a 4-D space-time($n = 4$) , with $m = n = 4$ K the kernel of homomorphic mapping of ($G_B \rightarrow sym(X)$) is exactly the "ray path" in "Light cone" manifold and coincides σ as a light ray path , because: $\dim(K) = n - m' = 1$. Intuitively σ_g are timelike geodesics curves.

The map $C(s_i)$ in (1.16) is independent of $\bar{\mu}$ on σ and because of the fact that $\varphi_{\bar{\mu}}, \frac{\partial \varphi_{\bar{\mu}}}{\partial s_i}, \frac{\partial^2 \varphi_{\bar{\mu}}}{\partial s_i^2}$ have common values for all $\bar{\mu}$ on σ, so naturally we choose $C(s_i)$ in general case as:

$$C(s_i) = L(\varphi_{\bar{\mu}}, \frac{\partial \varphi_{\bar{\mu}}}{\partial s_i}, \frac{\partial^2 \varphi_{\bar{\mu}}}{\partial s_i^2}, \ldots) \qquad (1.17)$$

In this case some convenient choices for $C(s_i)$ as a point function of parametric coordinates are:

$$C(s_i) = \kappa \varphi_{\bar{\mu}} + \lambda \qquad (a)$$

(κ is a constant)

$$C(s_i) = \eta \frac{\partial \varphi_{\bar{\mu}}}{\partial s_i} \qquad (b)$$

(η is a function of s_i)

On the other hand $C(s_i)$ on σ_g are not necessarily equal for different $\bar{\mu}$ and are dependent on $\bar{\mu}$. Thus the equation (1.16) on σ_g takes this form:

$$\frac{\partial^2 \varphi_{\bar{\mu}}}{\partial {s_i}^2} - \sum_{\alpha} \Gamma^i_{i,\alpha} \frac{\partial \varphi_{\bar{\mu}}}{\partial s_\alpha} = C_{\bar{\mu}}(s_i) \qquad (1.18)$$

Examples:

1)Applying (a) on σ, then from equation (1.16) we have:

$$\frac{\partial^2 \varphi_{\bar{\mu}}}{\partial {s_i}^2} - \sum_{\alpha} \Gamma^i_{i\alpha} \frac{\partial \varphi_{\bar{\mu}}}{\partial s_\alpha} - \kappa \varphi_{\bar{\mu}} + \lambda = 0 \qquad (1.19)$$

This equations determine relations between log-likelihoods $\varphi_{\bar{\mu}}$ and affine connection $\Gamma^i_{i\alpha}$.

For a flat parameteric manifold we have, $\Gamma^i_{i\alpha} = 0$ (no summation on *i*), therefore:

$$\frac{\partial^2 \varphi_{\bar{\mu}}}{\partial {s_i}^2} - \kappa \varphi_{\bar{\mu}} + \lambda = 0 \qquad (1.20)$$

λ are constants along σ.

2) Summation on index $'i'$ in (1.20) for real space-time parameteric space, gives us:

$$\Box \varphi_{\bar{\mu}} - n\kappa \varphi_{\bar{\mu}} = 0$$

or

$$\Box \varphi_{\bar{\mu}} = n\kappa \varphi_{\bar{\mu}} = C(s_i) \qquad (1.21)$$

Where, $C(s_i)$ can be considered as *Source* function, thus these equations are interpreted as *log-likelihood wave equations.* For κ=0 the equation is the same as for an scalar electromagnetic wave in vacuum and for $-n\kappa = M^2$ it turns to Klein-Gordon equation. On the other hand $\varphi_{\bar{\mu}}$ shows wave properties on geodesics σ on K. In this case $\varphi_{\bar{\mu}}$ acts as scalar potential of electromagnetic (EM) field that propagates on a null geodesic in space time coordinates. In other words K is a *light cone* that contains all null geodesics σ passing through a given point in the space-time coordinate(i.e. *S).*

3) As we mentiond above , gK with σ_g curves should be the integral curves of γ^i, consequently γ^i vectors on each σ_g in gK, are tangential vectors on these curves. This means that :

$$\gamma^i_{\bar{\mu};i} = 0 \qquad (1.22)$$

Covariant derivative has been calculated on a σ_g in gK .Such a curve should be a *geodesic* curve that is not a *null geodesic*. So these curves form a *flow of geodesics*. These geodesics may define the behaviour of free massive spinless particles in real space-time.

On these geodesics,the condition of equation (1.1) will not be necessary:

$$F_\mu \neq F_\nu \qquad (1.23)$$

In these cases (on gK curves or surfaces) $\varphi_{\bar{\mu}}$ s can be considered as independent components and $\bar{\mu}$ defines form of $\varphi_{\bar{\mu}}$. With respect to choice (b) we conclude the following equation ($\Gamma^i_{i\alpha}$=0):

$$\frac{\partial^2 \varphi_{\bar{\mu}}}{\partial {s_i}^2} - \eta \frac{\partial \varphi_{\bar{\mu}}}{\partial s_i} = 0 \qquad (1.24)$$

By summation on index “i” :

$$\Box \varphi_{\bar{\mu}} = \sum_i \eta \frac{\partial \varphi_{\bar{\mu}}}{\partial s_i} = \xi_{\bar{\mu}} \qquad (i = 1 \text{ to } 4) \qquad (1.25)$$

Here $\bar{\mu}$ has the index role in space –time.

As an example one may consider $\varphi_{\bar{\mu}}$ as four vector potential $A_{\bar{\mu}}$, we have**:**

$$\Box A_{\bar{\mu}} = \sum_i \eta \frac{\partial \varphi_{\bar{\mu}}}{\partial s_i} = \frac{4\pi}{c} J_{\bar{\mu}} = \xi_{\bar{\mu}} \qquad (1.26)$$

This equation is vector potential equation in 4 dimensional flat space-time.

4) Finally summation on “i”(from 1 to 4) in (1.19) because of the following identity:

$$\sum_i \Gamma^i_{i\alpha} = \frac{\partial}{\partial s_\alpha}\sqrt{g}$$

(g is determinent of metric tensor) (1.27)

Results in:

$$\Box \varphi_{\bar{\mu}} - \sum_{\alpha} \frac{\partial \sqrt{g}}{\partial s_\alpha} \frac{\partial \varphi_{\bar{\mu}}}{\partial s_\alpha} - n\kappa \varphi_{\bar{\mu}} = 0 \qquad (1.28)$$

For especial case $\kappa = 0$ this equation reduces to a form similar to *Laplace-Beltrami equation* in pure covariant form. otherwise it is compatible with *Helmholtz equation.*
All of these examples conveys us the idea that *log-likelihoods* $\varphi_{\bar{\mu}}$ could be assumed as *potentials* or generalized *fields* in parametric space.

2. Derivation of Lorentz transformation

By the fact that $\varphi_1 - \varphi_2$ is constant along σ_g , we have $d(\varphi_1 - \varphi_2) = 0$ and consider $\varphi_1 - \varphi_2$ as level curves.
Consequently the corresponding vector field is:

$$\nabla(\varphi_1 - \varphi_2) = \nabla \log \frac{F_1}{F_2} \qquad (2.1)$$

It is clear that $\varphi_1 - \varphi_2$ on σ as the identity of G_B, vanishes($F_1 = F_2$):

$$\psi = \varphi_1 - \varphi_2 = 0 \qquad (2.2)$$

Respect to (1.20) solutions for $\varphi_{\bar{\mu}}$ is:

$$\varphi_{\bar{\mu}} = \lambda_{\bar{\mu}} s_1^2 + \eta_{\bar{\mu}} s_2^2 + \zeta_{\bar{\mu}} s_1 + \xi_{\bar{\mu}} s_2 + \chi \qquad (2.3)$$

As described before $\lambda_{\bar{\mu}}, \eta_{\bar{\mu}}$ are the same constants for all $\bar{\mu}$.
in the two dimensional space-time with $s_1 = x$ and $s_2 = ct$, ψ , on each σ_g should be a function of velocity *v* (corresponding velocity of σ_g) :

$$\psi = \varphi_1 - \varphi_2 = \zeta\ s_1 + \xi\ s_2 + \chi = f(v) \qquad (2.4)$$

Regardless of conditions for elimination of *x,ct* in left side , finally we have:

$$\psi = \chi = f(v) \qquad (2.5)$$

Obviousely $\varphi_1 - \varphi_2$ depends on corresponding velocity of each σ_g and remains constant($f(v)$) along σ_g .
σ as a null geodesic corresponds the velocity *c* (light velocity) and identity of group G_B , then $\varphi_1 - \varphi_2$ will be zero on σ (because $F_1 = F_2$) and solution in this case indicates that we should have:

$$\varphi_1 - \varphi_2 = f(c) = 0 \qquad (2.6)$$

And in general form on σ_g :

$$\psi = \varphi_1 - \varphi_2 = f(v) \qquad (2.7)$$

Consequently ψ changes its sign through σ .

If for this group manifold we consider a generator ***g*** we can generate group members in this manifold by exponentiating map in the form:

$$g = \exp(\theta\, \boldsymbol{g}) \qquad (2.8)$$

In other words for each σ_g there is a corresponding exponential map in which, θ the angle between σ_g and σ in the manifold ,is a parameter to be used for representation of group.this exponential representation reveals a symmetry property in the group manifold ,i.e.changing the sign of parameter θ shifts group member to its inverse.consequently σ is the symmetry axis for groups member and their inverses.this simply means that for each σ_g with $\frac{v}{c}$ as its tangent and constant $\psi = f(v)$,we have a $\sigma_{g'}$ which is mirror image of σ_g respect to σ for which the constant and tangent are $\psi = -f(v)$ and $\frac{c}{v}$ respectively.(fig 1).On the other hand regard to the definition of $\varphi_1 - \varphi_2$ and (2.2) by replacement of F_1 and F_2 we arrive to the inverse transformation and the sign of $\varphi_1 - \varphi_2$ will be changed .Consequently by replacement of F_1 and F_2 in corresponding transformation of σ_g , the sign of constant $f(v)$ changes and the new transformation path is mirror image of σ_g respect to σ and therefore its tangent is $\frac{c}{v}$ .This means that for each group element $g \in G_B$ and its σ_g with tangent $\frac{v}{c}$, the corresponding path for inverse element g^{-1} is $\sigma_{g'}$ with tangent $\frac{c}{v}$ .as a result the axes s_1 and s_2 (i.e. *x* and *ct*) are *inverse path* .
Now consider a shift of reference frame from stationary reference frame to a moving one with velocity v .this transformation is equivalent to change of time geodesic (*ct* axis) to a geodesic σ_g (time axis of new reference frame) with tangent $\frac{v}{c}$ and similarly acts on whole of geodesics , so s_1 (*x* axis) as the inverse of time axis should be transformed to geodesic $\sigma_{g'}$ (new x axis) with

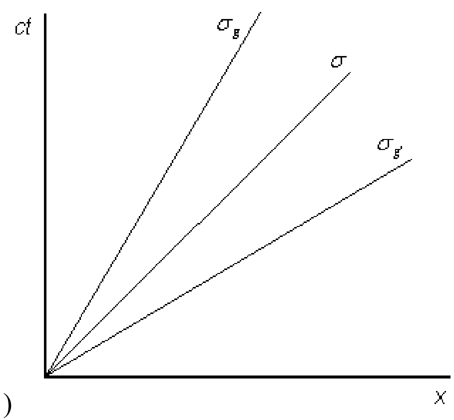
tangent $\frac{c}{v}$ which acts as the inverse of σ_g to construct new reference frame with new space and time axis $\sigma_{g'}$ and σ_g.obviousely σ as identity of the group coincides itself and acts as a symmetry axis in group manifold.this is equivalent to *light velocity invariance*.

In summary shift from stationary reference frame to moving one results in shifting of x and ct to σ_g and $\sigma_{g'}$ as new axes with tangents $\frac{v}{c}$ and $\frac{c}{v}$ respectively.

It is well known that the only transformation that coincides with such a changes in space-time axes(geodesics) is *Lorentz Transformation* with $\beta = \frac{v}{c}$.

In (fig.1)"*ct*" and "*x*" axes after a Lorentz transformation rotate and coincides on σ_g and $\sigma_{g'}$ respectively.

This is an interesting conclusion that manifold structure and geodesics of parameteric space of G_B, coincides with Lorentz transformation.



Fig(1)

3.linear momentum conservation(another example)

As a simple example in flat metric space , and by assumption of *conservation law of mass* ,which can be replaced by *probability conservation* , and first Newton law, we derive linear momentum conservation law. in our sense first Newton law means that a particle m_μ is in equilibrium if it's velocity vector remains constant with time.

Suppose an isolated macroscopic system $\mathcal{S}$ of n body with masses $X_\nu = m_\nu$ and spatial coordinate x_ν (one dimensional) to be considered. Based on the conservation law of mass:

$$\sum_\nu m_\nu = c \qquad (3.1)$$

with comparison of this conservation law and conservation of probability:

$$\sum_\nu X_\nu = c = 1 \qquad (3.2)$$

This equation states that the mass transfer between masses is possible (with collisions) while total mass is preserved.generally speaking, in all collisions in a system of masses interchange of energy and hence mass is inevitable.

We substitute mass conservation by probability conservation and will deduce linear momentum conservation via the basic assumption concerned with the Bayes transformation generators.

First we represent a symmetry on Bayes transformation group.behaviour of classical systems of masses can be obtained with initial x_0, p_0.

Obviously the translation:

$x_\nu \to x_\nu + \Delta x_\upsilon$

as asymmetry group leaves the system $\mathcal{S}$ invariant, (x_μ is time dependent). Bayes transformation under this translation should also be invariant:

$$\frac{f_{i\mu}(x_\mu)X_\mu}{\sum_\nu f_{i\nu}(x_\nu)X_\nu} = \frac{f_{i\mu}(x_\mu + \Delta x_\mu)X_\mu}{\sum_\nu f_{i\nu}(x_\nu + \Delta x_\nu)X_\nu} \qquad (3.3)$$

(i *index* depends on the involved coordinate , in this case suppose $i=1$)

The most probable solution for $f_{i\mu}$ with arbitrary X_μ will be:

$$f_{i\mu} = \kappa_i \exp(-\alpha x_\mu) \qquad (3.4)$$

κ_i is a coefficients independent respect to x_i and dependent on other parameters .

α is a very small constant respect to x_i because experimentally mass does not depend on usual x_i (even in usual astronomical distance).

This choice guarantees the translation invariance. Now we use the Bayes generator (see appendix A).

$$\hat{g}^i = -\sum_{\bar{\mu}} X_{\bar{\mu}} \left(\frac{1}{f_{i\bar{\mu}}}\frac{\partial f_{i\bar{\mu}}}{\partial s_i} - \sum_{\bar{\nu}} \frac{1}{f_{i\bar{\nu}}}\frac{\partial f_{i\bar{\nu}}}{\partial s_i} X_{\bar{\nu}}\right)\frac{\partial}{\partial X_{\bar{\mu}}} \qquad (3.5)$$

(here $s_i = t$; $i=1$) Action of this generator on $\mathcal{X}$ manifold is a vector field with components along each X_μ :

$$X_{\bar{\mu}}(\frac{1}{f_{i\bar{\mu}}}\frac{\partial f_{i\bar{\mu}}}{\partial s_i}-\sum_{\bar{\nu}}\frac{1}{f_{i\bar{\nu}}}\frac{\partial f_{i\bar{\nu}}}{\partial s_i}X_{\bar{\nu}}) \quad i=1 \qquad (3.6)$$

To meet stability or equilibrium state of m_μ (or X_μ) ,a particle with negligible mass which is in an equilibrium state(without collision) , this component (the expression in the bracket) should be vanished,because zero component results in no change along X_μ:

$$(\frac{1}{f_{i\bar{\mu}}}\frac{\partial f_{i\bar{\mu}}}{\partial s_i}-\sum_{\bar{\nu}}\frac{1}{f_{i\bar{\nu}}}\frac{\partial f_{i\bar{\nu}}}{\partial s_i}X_{\bar{\nu}})=0 \qquad (3.7)$$

By variable replacement: $\pi_{i\mu}=\frac{1}{f_{i\mu}}\frac{\partial f_{i\mu}}{\partial s_i}$

After substitution of $f_{i\mu}$ from (3.4) we have:

$$\pi_{i\mu}=-\alpha\dot{x}_\mu \quad : \quad i=1 \qquad (3.8)$$

And:

$$(\pi_{i\mu}-\sum_{\bar{\nu}}\pi_{i\nu}X_\nu)=\alpha(-\dot{x}_\mu+\sum\dot{x}_\nu X_\nu)=0 \qquad (3.9)$$

Or:

$$\dot{x}_\mu=\sum\dot{x}_\nu X_\nu \qquad (3.10)$$

But we have put:

$$\sum_\mu X_\mu=\sum_\mu m_\mu=c=1 \qquad (3.11)$$

Then:

$$\dot{x}_\mu=\frac{\sum_\nu \dot{x}_\nu X_\nu}{\sum_\mu X_\mu}=\frac{\sum_\nu m_\nu\dot{x}_\nu}{\sum_\mu m_\mu} \qquad (3.12)$$

Therefore $\dot{x}_\mu$, the velocity of the μ th mass ,due to this equation is the velocity of *centre of mass* of the system $\mathbb{S}$, since m_μ is a negliginle mass and has been considered to be in equilibrium state (i.e. motion on a straight line and constant velocity) thus $\dot{x}_\mu$ should be a constant and we have proved that center of mass in an isolated system always is in equilibrium(constant direction and velocity) and also the expression $\sum_\nu m_\nu\dot{x}_\nu=\dot{x}_\mu\sum_\mu m_\mu$ is a constant (i.e.the linear momentum conservation law).this proof can be repeated for other coordinates similarly.

So based on Bayes assumption first Newton law and mass conservation, we arrived to linear momentum conservation laws which is equivalent with third Newton law in a closed system regardless of mutual collisions or interactions.so Bayes rule application indicates the first and third Newton laws are not independent rules.

Angular momentum conservation also can be proved by similar method.

4. Generalization to infinite dimensional space X (continuous states)

We have already worked out the previous subjects based on discrete dimensional spaces of states. In this section our aim is to develop Bayes transformation on a continuous dimensional space for a spectrum of continuous states.

As shown in appendix A, regular Bayes transformation group acts on discrete and independent states, so for finding out the exact form of this transformation for continuous states $A(x)$ (one dimensional case with only one parameter "t" to be considered),we should project $A(x)$ onto an orthogonal basis functions (i.e. e^{ikx} by Fourier transformation)and then after performing Bayes transformation on the components in this orthogonal basis, by reverse Fourier obtain the Bayes transformation of $A(x)$.(i.e. transition from "x" coordinate to an infinitely orthogonal coordinates "k"). In other words if φ_B (seeA.8) assumed as the usual mapping of Bayes transformation group G_B then the actual form acts on continuous function $A(x)$ with the condition:

$$\int A(x)dx=cte$$

Takes the form:

$$\varphi'_B[A(x)]=\mathcal{F}^{-1}\varphi_B\,\mathcal{F}[A(x)] \qquad (4.1)$$

Where $\mathcal{F}$ is Fourier transformation.

Assume that probability density of states $A(x)$ at the moment "t" be $u(x,t)$, therefore the normalization condition will be:

$$\int u(x)dx=\int u(x,t)dx=1 \qquad (4.2)$$

This normalization condition in new coordinate will become:

$$\int\tilde{u}(k)dk=1$$

Let: $u(x)\xrightarrow{f}\tilde{u}(k)$ and $u(x,t)\xrightarrow{f}\tilde{u}(k,t)$ are the Fourier transformations of $u(x)$ and $u(x,t)$ then we can act on it by $\chi(g)$ (or equivalence action of Bayes transformation) clearly:

$$u(x)=\int\tilde{u}(k)e^{ikx}dk$$

And

$$\tilde{u}(k) = \int u(x) e^{-ikx} dx \tag{4.3}$$

Then:

$$\int \tilde{u}(k)dk = \iint u(x) e^{-ikx} dxdk = \int u(x)(\int e^{-ikx} dk)dx$$

$$= \int u(x)\delta(-x)\, dx = u(0) \tag{4.4}$$

In a similar manner:

$$\int \tilde{u}(k,t)dk = u(0,t)$$

Instead of, $\tilde{u}(k,t)$ respect to normalization condition, to find out Bayes transformation in new coordinate we will use: $\tilde{U}(k) = \frac{\tilde{u}(k)}{u(0)}$

And

$$\tilde{U}(k,t) = \frac{\tilde{u}(k,t)}{u(0,t)} \tag{4.5}$$

Finally the Bayes equation after Fourier transformation takes the form:

$$\tilde{U}(k \mid t) = \frac{\tilde{U}(k)\tilde{f}(t \mid k)}{\int \tilde{U}(k) f(t \mid k) dk} \tag{4.6}$$

After substitution from (5.5):

$$\tilde{u}(k \mid t) = \frac{\tilde{u}(k)\tilde{f}(t \mid k)}{\frac{1}{u(0,t)}\int \tilde{u}(k)\tilde{f}(t \mid k)dk} \tag{4.7}$$

This is Bayes transformation in new coordinate after performing Fourier transformation. In this equation $\tilde{f}(t \mid k)$ determines the conditional probability in Fourier base.

Denominator is an explicit function on t, $\varphi(t)$:

$$\tilde{u}(k \mid t) = \frac{\tilde{u}(k)\tilde{f}(t \mid k)}{\varphi(t)} \tag{4.8}$$

After performing convolution and reverse Fourier transformation on both sides of (5.8), this equation takes the form:

$$u(x \mid t) = \frac{\int u(x') f(t \mid x - x')dx'}{\varphi(t)} \tag{4.9}$$

Regard to normalization condition we have:

$$\varphi(t) = \iint u(x') f(t \mid x - x')dxdx'$$

Therefore:

$$u(x \mid t) = \frac{\int u(x') f(t \mid x - x')dx'}{\iint u(x') f(t \mid x - x')dxdx'} \tag{4.10}$$

This is the Bayes transformation for *continuous states* case (one dimensional) with single parameter (t).

This equation may be rewritten in the compact form:

$$u(x \mid t) = \frac{u(x') * f(t \mid x')}{\int u(x') * f(t \mid x')dx'} \tag{4.11}$$

Instead of usual multiplication, the convolution substituted in Bayes rule.

Example 1:

Suppose a linear distribution of temperature $u(x,t)$ diffuses along a linear bar (without energy loss) with initial conditions:

$$u(x) = u(x,0) \tag{4.12}$$

$$\int u(x)dx = E = 1$$

Naturally due to energy conservation after t interval:

$$\int u(x,t)dx = E = 1 \tag{4.13}$$

(For the sake of simplicity we equate energy content $"E"$ with unit and ignore the details of heat energy calculation).

These equations satisfy the necessary condition for Bayes transformation in continuous manifold. From equation (5.10) we can replace $u(x \mid t)$ by $u(x,t)$:

$$u(x,t) = \frac{\int u(x') f(t \mid x - x')dx'}{\iint u(x') f(t \mid x - x')dxdx'} \tag{4.14}$$

But at $t = 0$ this equation should reduce to: $u(x,0) = u(x)$ hence $f(t \mid x - x')$ at the limit $t \to 0$ should be $\delta(x - x')$, and therefore the only choice for $f(t \mid x - x')$ after normalization condition should be a normal distribution:

$$f(t \mid x - x') = \frac{1}{2\sqrt{\pi.t}} \exp\left(-\frac{(x - x')^2}{4t}\right) \tag{4.15}$$

Substitution of this likelihood into (5.14) results in:

$$u(x,t) = \frac{\int u(x') \exp\left(-\frac{(x - x')^2}{4t}\right)dx'}{\sqrt{4\pi.t}} \tag{4.16}$$

This is the solution of linear heat transfer or *Cauchy-Dirichlet* problem.

It should be noted that this approach accomplished without involving by heat differential equation problem.

Appendix A:

Let A_i`s $(1 \le i \le m)$ form a set of independent states so that the posterior probability of A_i for a given set of *independent* parameters $S = \{s_1, s_2, ..., s_j, ..., s_n\}$ is described by Bayes rule [4]:

$$P(A_i|S) = \frac{P(A_i)F(S|A_i)}{\sum_k P(A_k)F(S|A_k)} \quad \text{(A.1)}$$

$F(S|A_i)$ Equals to the production of $f(s_j|A_i)$:

$$F(S|A_i) = \prod_j f(s_j|A_i) \quad \text{(A.2)}$$

if we replace $P(A_i|S)$, $P(A_i)$ by X_i' , X_i then:

$$X_i' = P(A_i|S) = \frac{X_i \prod_j f(s_j|A_i)}{\sum_k X_k \prod_l f(s_l|A_k)} \quad \text{(A.3)}$$

$f(s_j|A_i)$ And X_i' obey normalization condition:

$$\int f(s_j|A_i)ds_j = \sum_j X_j = 1 \quad \text{(A.4)}$$

For convenience we can replace $f(s_j \mid A_i)$ by f_{ji} in (A.1):

$$f(s_j \mid A_i) = f_{ji} \quad \text{(A.5)}$$

$$X_i' = \frac{X_i \prod_j f_{ji}}{\sum_k X_k \prod_l f_{lk}} = \frac{X_i F_i}{\sum_k X_k F_k} \quad \text{(A.6)}$$

We call (A.6) *Bayesian transformation*.assuming $\mathcal{X}$ be a $(n-1)$ manifold so that;

$$\mathcal{X} = \left\{ m(x_1, x_2, ...x_m) \in R_+^m \middle| \sum_i x_i = 1, x_i \ge 0 \right\} \quad \text{(A.7)}$$

$\mathcal{X}$ is a differentiable compact manifold.

For example in the case $n = 3$,$\mathcal{X}$ is a triangle (3 simplex) with vertices at (1,0,0) ,(0,1,0) and (0,0,1). In (A.3) X_i' and X_i are posterior and prior probabilities, let them be considered as points on the flat manifold $\mathcal{X}$ .now transformation (A.6) acts on *manifold* $\mathcal{X}$ as a diffeomorphism:

$$\varphi_B : \mathcal{X} \to \mathcal{X} \quad \text{(A.8)}$$

For each s_j we have a sequence of functions $f(s_j|A_i)$, which define the distribution of each A_i and their ranges in a positive real interval.

Now consider a point in parametric space defined by:

* $\vec{s} = s_1, s_2, ..., s_n$

To find out the set of points s' with the same transformation we need to solve :

$$X_\mu' = \frac{F_\mu(\vec{s})X_\mu}{\sum_\nu F_\nu(\vec{s})X_\nu} = \frac{F_\mu(\vec{s}')X_\mu}{\sum_\nu F_\nu(\vec{s}')X_\nu} \quad \text{(A.9)}$$

In the case $m = n$ it can be shown by *induction proof* that this equation results in a system of equations whose number of independent equations is $m-1$ with n unknown variable s_i'. So in this case the solution is a curve in G_B,we name this curve σ_g and conclude that all points on this curve show identical Bayes transformation φ_B. This guarantees the *closure* property of group.

Some examples of the likelihoods which satisfy the group properties of Bayes transformation are the well known distributions e.g:

$$f = c\exp(-\mu s^2) \qquad s \in R$$

$$,0 \le c \le 1, \mu > 0$$

and

$$f = c\exp(-\lambda s) \qquad s \in R^+$$

$$,0 \le c \le 1, \lambda > 0$$

Briefly *closure* property can be verified by (1.9) *Associativity* and *inverse* property can be verified easily using equation(A.9) and also have been demonstrated in[9].

***Bayes Generators*:**

Because of continuous property of Bayes transformation ,we define it generally as:

$$X_\mu' = F_\mu(X_1, X_2, ...X_m, s_1, s_2, ...s_n) \quad \text{(A.13)}$$

The generators can be obtained as follows:

$$\hat{g}^i = -\sum_\mu \left[\frac{\partial X_\mu'(\vec{X}, \vec{s})}{\partial s_i} \right]_{S=I} \frac{\partial}{\partial X_\mu} \quad \text{(A.14)}$$

Right hand side of the above equation is calculated at the identity.

After performing partial derivatives and setting:

$$Q_{i\mu} = \frac{\prod_j f\left(s_j \middle| A_\mu\right)}{f\left(s_i \middle| A_\mu\right)} \quad \text{and}$$

$$f_{i\mu} = f\left(s_i \middle| A_\mu\right) \; ; \; \dot{f}_{i\mu} = \frac{\partial f_{i\mu}}{\partial s_i}$$

$$\text{and} \quad \pi_{i\mu} = \frac{\dot{f}_{i\mu}}{f_{i\mu}} \qquad \text{(A.15)}$$

we have:

$$\hat{g}^i = -\sum_\mu X_\mu \frac{\dot{f}_{i\mu} Q_{i\mu} \sum_l X_l \prod_k f_{kl} - \left(\sum_l X_l \dot{f}_{il} Q_{il}\right)\left(\prod_j f_{j\mu}\right)}{\left(\sum_l X_l \prod_k f_{kl}\right)^2} \frac{\partial}{\partial X_\mu}$$

(A.16)

Again the right side of the above equation is calculated at the identity.as mentioned above the unique condition for existence of identity in group is [9]:

$$\prod_j f_{ji} = \prod_k f_{kl} = c \text{ (for all } i,l\text{)} \qquad \text{(A.17)}$$

Then:

$$\hat{g}^i = -\sum_\mu X_\mu \frac{\dot{f}_{i\mu} \frac{c}{f_{i\mu}} \sum_l X_l c - \left(\sum_l X_l \dot{f}_{il} \frac{c}{f_{il}}\right) c}{\left(\sum_l X_l c\right)^2} \frac{\partial}{\partial X_\mu}$$

(A.18)

With the condition $\sum_l X_l = 1$ it becomes:

$$\hat{g}^i = -\sum_\mu X_\mu \left(\frac{\dot{f}_{i\mu}}{f_{i\mu}} - \sum_l X_l \frac{\dot{f}_{il}}{f_{il}}\right) \frac{\partial}{\partial X_\mu}$$

(A.19)

It's easy to prove Abelian commutative property of algebra [9] :

$$[\, \hat{g}^i \, , \hat{g}^j \,] = 0 \qquad \text{(A.20)}$$

Interestingly by equating (A.19) to zero to find out zero modes for equilibrium states, we arrive to the solution :

$$\frac{\dot{f}_{i\mu}}{f_{i\mu}} = \lambda \Rightarrow f_{i\mu} = C\left(A_\mu\right)\exp\left(\lambda s_i\right)$$

If one assumes that states of a system can be determined by a single energy parameter $\varepsilon = s$ we will conclude:

$$f_\mu = C\left(A_\mu\right)\exp\left(\lambda\varepsilon\right)$$

This coincides the Boltzmann energy distribution in equilibrium statistical mechanics.

Conclusion

We have used an extension of Bayes continuous (Analytical) group with a certain family of probability densities and related generators to achieve:

1) A probability conservation law to interpret some physical conservation laws such as linear momentum conservation and heat transfer solutions.
2) A Lie group action transformation equipped with a group manifold to explain light velocity invariance in inertial reference frames and to explore similarities of related likelihoods densities with spinless scalar fields through defining its geodesics flow.
3) Approach to equilibrium states(like Boltzmann distribution) by finding the solutions for vanishing Bayes generators.

These approaches facilitated by introducing Bayes rule as a continuous group action on a probability space manifold and its generators.Bayes rule meets the group conditions for especial likelihood distributions such as exponential, Cauchy and normal distributions. We hope to extend this method successfully in other fields of physics when likelihood distributions act as generalized functions and or covariance matrices.

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