A Comprehensive Integer Programming Model with Special Forms For Optimal Provision of Multiple Manufactures

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Abstract: - In a previous paper written by the same authors, the provision of manufactured components of only one type was presented, the provision can be done using three different methods, production, importing/storing, and subcontracting. The problem is to determine how many components should be provided using different method in order to accomplish the required demand and to minimize the total cost of provision. A mathematical model for such a problem was formulated and used to obtain the optimal solution.

In another paper for the same authors, a more generalized problem with multiple manufactured components was considered. The objective function is to minimize the total fixed and variable costs of provision of all needed types of components. A special (If-Then) form for the objective function is generated due to the addition of the fixed cost elements. Another special form for the objective function is generated due to the stepped unit prices. The main problem constraints are: The provision of the total demand, maximum available time for production, the limited area for storing, and the constraints corresponding to the subcontracting stepping prices conditions. The if-then conditions were transformed into conditional relations with the addition of auxiliary 0-1 variables.

In the previous papers, the interior design of the provision systems was determined, and it was taken as structure constraints. In this paper a comprehensive model is designed in order to take care of the optimal interior design of the provision systems in addition to all the previously mentioned system constraints. The design includes the determination of the optimal number of different machines for the production facilities, and the optimal area for the import/ storing facilities.

Key-Words: - Integer programming, Provision methods, If-then and Fixed cost conditions.

1 Introduction

In practice, it can be easy to evaluate different simple methods of provision of manufactured components, these methods are: production in normal and in overtimes, importing/ storing, and subcontracting with a third party. The total market demand from each type can be estimated, and also the total fixed and variable cost elements for each method can be evaluated. In such a case the total cost is compared and the decision maker will choose the method having the smallest one.

A more complicated problem will be raised when it is required to establish additional lines of production, importing and storing due to the large amount of the required demand. In such cases, the number of feasible solutions will be huge and cumbersome for evaluation.

In a previous paper written by the same authors [1], an Integer programming model was presented to represent the complete problem with the objective of minimizing the total provision cost, and to cover all possible choices of the provision mix for components of only one type. A real example of application was presented for the Power Transmission & Distribution (PTD) equipment for one of the most important and highly cost components in one of the famous companies in the kingdom of Saudi Arabia.

As an extension in the same subject, a second paper by the same authors is written [2], it describes a generalized Integer programming model to formulate the problem of provision of manufactured components of multiple types. The provision can be done by the four methods: production in normal time, production in overtime, import and storing, and subcontracting with an external supplier. Due to the amounts required, many production and storing lines may be required to satisfy the required demand. The decision variables represent the number of components of different types to be provided using various methods of provision, and the objective function coefficients concerning the fixed and variable costs corresponding to different methods of provision. The model contains some If-then constraints that are presented by a linear minimization model constructed without the conditional constraints by the addition of non-construction 0-1 variables.

In this paper, a new comprehensive Integer programming model is designed for the same types of problems. In the previous generalized model, the production and storing lines are already designed and their maximum capacities were considered as constraints for the problem. In this new model, the problem will includes also the optimum design for the production and import/ storing facilities, the number of production machines of various types, and the capacity for the storing area will be considered as additional decision variables.

The second part of this paper discusses the problem definition, the third part includes the definition of the decision variables, the fourth part for the formulation of the objective function, the fifth part is devoted to the problem constraints: fixed cost coefficients constraints, provision of the total demand, determination of the optimum number of production machines in both normal and over times, determination of the optimum storing capacity for the importing facility, If-then for overtime constraints, If-then conditions for subcontracting stepping prices, integral constraints, and the 0-1 constraints. In the sixth part, the steps of the used algorithm are summarized.

The seventh part gives the same example of illustration with the same data as that was given in the previous paper, but in this new case, the number of needed machines, and the space of the storing area for the imported components are considered as decision variables beside the previous ones for the number of manufactured components provided by different provision methods. The problem has two types of manufactured components, seven kinds of production machines, and three stepped subcontracting prices for each type of manufactures. The complete mathematical model is formulated, and the optimal solution is given using LINGO 9.0 computer program. The last part of the paper has the main conclusions and the proposed points for future researches.

2 Problem Definition

The problem is to make an optimum design for the production and importing facilities, and to determine how many components of each type of manufactures should be provided using individual method with the aim to minimize the total cost of provision of the whole requirements. The design of the production facility includes the determination of the optimum number of production machines of different kinds in normal and overtimes, and the design of the import/ storing facility means the determination of the optimum storing area so that beside other decision variables, the total cost of provision will be the minimum. Other decision variables are the number of manufactures from all types provided by different methods of provision. The provision methods that are: production in normal and / or overtime, importing/ storing, and subcontracting. The total cost comprises fixed and variable cost elements associated with different methods of provision.

The problem contains many different constraints: If-then conditions associated with the fixed cost coefficients in the objective function, provision of the total demand, determination of the optimum number of production machines from different kinds in both normal and over times, determination of the optimum storing capacity for the importing/ storing facility, If-then conditions associated with the different ranges for subcontracting stepping prices, If-then conditions for overtime constraints, integral constraints, and the 0-1 constraints.

3 Decision Variables

Let: x_{tj} Integer variables represent the number of components of type t, to be provided using the method j, and j represents a provision method that can be: production line (p), overtime (o), and importing/ storing (i).

x_{ts}^{r} Integer variable representing the number of components of type t, to be provided by subcontracting, in the interval of stepping price r,

x_{m}^{p} Integer variables represent the number of machines of type m to work in the production system in normal time, m = 1, 2, …., n_m = number of types of machines,

x_m^o Integer variables represent the number of machines of type m to be included in overtime, m = 1, 2, …., n_m = number of types of machines, and

x^a Integer variable represents the area of the import/ storing facility.

4 Objective Function

4.1 Objective function coefficients

Let: f_j = the fixed cost coefficient corresponding to the method of provision j, (production, importing, and subcontracting), excluding the costs of machines for production provision method, and excluding the land cost for the import/ storing method.

c_m = the cost for one machine of type m,

c_a = the unit cost for the storing land (cost / m^2)

v_{tj} = represent the variable cost coefficients corresponding to the variable x_{tj} .

4.2 If-Then constraints for the fixed cost

To minimize the total costs of provision of all needed parts:

Min z=

$$\begin{cases} \sum_{t=1}^{n_t} \sum_{j} v_{tj} x_{tj} + \sum_{t=1}^{n_t} \sum_{r=1}^{n_s} v_{ts}^r x_{ts}^r + \sum_{m=1}^{n_m} c_m . x_m^p + \\ c_a . \ x^a + f_{j'} , \forall \ j ; j' = p, i, s; & x_{tj} > 0 \\ 0 , & x_{tj} = 0 \end{cases}$$

Where: n_t = number of types of needed manufactures,

n_s = number of stepping prices for subcontracting,

n_m = number of machines used in production.

This should be understood as:

1. For j' = p, and i: If any x_{tj} > 0, t = 1, 2, …… n_t, then a corresponding term f_j, corresponding to the total fixed cost for the products provided with the provision mean j, should be added, this term should be deleted in case when all x_{tj} = 0.

2. For j' = s: If: any x_{ts}^r > 0, r =1, 2, …., n_s , then a term f_s, corresponding to the total fixed cost for subcontracting, should be added, this term should be deleted in case when all x_{ts}^r = 0, r =1, 2, …., n_s.

For the first case: These are nonlinear in x_{tj} , because the discontinuity at the origin.

It is known that:

$0 \le x_{tj} \le N_t$ for all t, and j ,

Where: N_t = The demand for a manufacture of type t.

The nonlinearity can be transformed into linear minimization model as follows:

Min.

$$z = \sum_{j} \sum_{t=1}^{n_t} v_{tj} \ x_{tj} + \sum_{t=1}^{n_t} \sum_{r=1}^{n_s} v_{ts}^r x_{ts}^r + \sum_{m=1}^{n_m} c_m . x_m^p + c_a . x^a + \sum_{j'} f_{j'} y_{j'} + f_s y_s \ , j' = p, i,$$

Subject to:

$$\sum_{t=1}^{n_t} x_{tj'} \le \sum_{t=1}^{n_t} N_t \ . y_{j'} ,$$

$$y_{j'} = 0, 1 \quad , j' = p, \text{ and } i.$$

If any $x_{tj'}$ > 0, then the corresponding $y_{j'}$ is forced to be 1, to eliminate the infeasibility, and:

If all $x_{tj'}$ = 0, then no restrictions are imposed on $y_{j'}$, it can be equal to 0, or 1, but according to the minimization of the objective function, it will be forced to have the value of 0.

For the second case:

IF: (x_{ts}^1 OR x_{ts}^2 OR …. OR $x_{ts}^{n_{ts}}$ OR any combinations of them) > 0 , t = 1, 2, , n_t , then a term f_s, corresponding to the total fixed cost for subcontracting the components should be added to the objective function, this term should be deleted in case when all the x_{ts} .

Where:

n_{ts} = the number of price steps for subcontracting components of type t.

This can be reduced to:

IF: $x_{ts}^{1} > 0$, $t = 1, 2, \ldots, n_t$ then a term corresponding to the total fixed cost for subcontracting the panels f_s, should be added, this term should be deleted in case when all $x_{ts}^{1} = 0$. Since for any t, none of the other variables x_{ts}^{r}, $r > 1$, can have nonzero values unless $x_{ts}^{1} > 0$.

This could be done by adding the following constraint:

$$\sum_{t=1}^{n_t} x_{ts}^{1} \le y_s^1 \sum_{t=1}^{n_t} N_{ts}^{1} ,$$

$$y_s^1 = 0 , 1 .$$

4.3 Objective function for the problem

$$\text{Min.z}=\sum_{j} \sum_{t=1}^{n_t} v_{tj} x_{tj} + \sum_{t=1}^{n_t} \sum_{r=1}^{n_s} v_{ts}^{r} x_{ts}^{r} +$$

$$\sum_{m=1}^{n_m} c_m . x_m^p + c_a . x^a + \sum_{j'} f_{j'} y_{j'} \ , j = p, \text{o}, i, j' = p, i, s, \ldots\ldots\ldots \quad (1)$$

5 Constraints

5.1 Fixed cost coefficients constraints

For the production, and importing/ storing facilities:

$$\sum_{t=1}^{n_t} x_{tp} \le \sum_{t=1}^{n_t} N_t \ y_p , \ \ldots\ldots\ldots \quad (2)$$

$$y_p = 0, 1 \ \ldots\ldots\ldots \quad (i.1)$$

$$\sum_{t=1}^{n_t} x_{ti} \le \sum_{t=1}^{n_t} N_t \ y_i , \ \ldots\ldots\ldots \quad (3)$$

$$y_i = 0, 1 \ \ldots\ldots\ldots \quad (i.2)$$

Where:

N_t = The total No. of required components of type t, $t = 1, 2, \ldots, n_t$.

For the stepped price subcontracting, we have:

$$\sum_{t=1}^{n_t} x_{ts}^{1} \le \sum_{t=1}^{n_t} N_{ts}^{1} . y_s \ , \ldots\ldots\ldots \quad (4)$$

$$y_s = 0 , 1 \ \ldots\ldots\ldots \quad (i.3)$$

5.2 Provision of the total demand

$$\sum_{j} x_{tj} + \sum_{r=1}^{n_s} x_{ts}^{r} = N_t , j = p, o, \text{ and } i, t = 1, 2, \ldots, n_t , \ \ldots\ldots\ldots \quad (5)$$

5.3 Optimum number of production machines

The total needed time for production of components in each machine kind for both the normal and overtime should not exceed the maximum available time for that machine kind, the maximum available time is a function of the number of machines of that kind, so for normal time:

$$\sum_{t=1}^{n_t} x_{tp} . \tau_t^m \le x_m^p . T_m^p \ , \ m = 1, 2, \ldots, n_m , \ \ldots\ldots\ldots \quad (6)$$

Where:

τ_t^m = needed time to produce one component of type t in machine no. m,

T_m^p = Total available annual time for production in one machine of type m in normal time.

n_m = number of kinds of machines.

And for overtime:

$$\sum_{t=1}^{n_t} x_{to} . \tau_t^m \le x_m^o . T_m^o \ , \ m = 1, 2, \ldots, n_m , \ \ldots\ldots\ldots \quad (7)$$

Where:

T_m^o = Total available annual time for production in machine m r in overtime.

5.4 Needed area for importing/ storing

The occupied area by different components of all types is calculated by knowing the total number of components provided by importing/ storing:

$$\sum_{t=1}^{n_t} x_{ti} \cdot a_t = x^a \text{ , } \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{ (8)}$$

Where:

a_t = area occupied by one unit of manufactures of type *t*,

5.5 Stepped subcontracting constraints

The number of provided components in each stepped range of prices, should not exceed the corresponding maximum range:

$$x_{ts}^{r} \leq N_{ts}^{r} \quad \forall t, r \text{ , } \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(9)}$$

Where:

N_{ts}^{r} = maximum possible quantity by subcontracting for product of type t in level price r.

5.6 Overtime constraints

5.6.1 If-then for overtime constraints

The overtime can not be performed unless the production facility is already established. The If-Then constraint is as follows:

If : $\sum_{t=1}^{n_t} x_{tp} = 0$, then $\sum_{t=1}^{n_t} x_{to} = 0$,

To transform this conditional constraint into an equivalent set of constraints without the conditional relation, then knowing that:

$x_{to} \leq N_t$, $t = 1, 2, \ldots, n_t$.

The following formulation can be used:

$$\sum_{t=1}^{n_t} x_{to} \leq \sum_{t=1}^{n_t} N_t \cdot \sum_{t=1}^{n_t} x_{tp} \text{ , } \ldots\ldots\ldots\ldots \text{ (10)}$$

If all x_{tp} = 0, then accordingly all x_{to}^{r} will be equal to 0, and if any one of the x_{tp} is greater than 0, then no constraints are imposed for any of the variables x_{to} .

5.6.2 Number of machines in overtime

Another overtime constraints should be considered, the number of a certain type of machines working in overtime, should not exceed the number of machines of the same type working in normal time; these constraints can be accomplished as follows:

$$x_m^o \leq x_m^p \text{, } m = 1, 2, \ldots, n_m \text{ , } \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(11)}$$

5.7 Subcontracting stepping prices constraints

Consider the following decision variables x_{ts}^{r} representing the subcontracting amount with unit price = v_{ts}^{r}, r = 1, n_s = number of steps of unit prices. The unit price for the subcontracting variant are not constant, it has the shape of stepping prices related to the amount of components subcontracted x_{ts} as shown in Figure No. 1 drawn for three steps prices for a product of type *t*.

The subcontracting prices are:

v_{ts}^{1} for subcontracting quantities $0 \leq x_{ts}^{1} \leq N_{ts}^{1}$,

v_{ts}^{2} for subcontracting quantities $N_{ts}^{1} < x_{ts}^{1} + x_{ts}^{2} \leq N_{ts}^{1} + N_{ts}^{2}$,

v_{ts}^{3} for subcontracting quantities $N_{ts}^{1} + N_{ts}^{2} < x_{ts}^{1} + x_{ts}^{2}\ x_{ts}^{3} \leq N_{ts}^{1} + N_{ts}^{2} + N_{ts}^{3}$.

The constraints are as follows:

If $x_{ts}^{r} < N_{ts}^{r}$, then $x_{ts}^{w} = 0$, $t = 1, 2, \ldots, n_t$, $r = 1, 2, \ldots\ldots, n_s - 1$, $w = r+1, r+2, \ldots, n_{ts}$.

Knowing that: $x_{ts}^{r} \leq N_{ts}^{r}$, $t = 1, 2, \ldots, n_t$, $r = 1, 2, \ldots\ldots, n_{ts}$,

The conditional constraints can be transformed as follows:

$$x_{ts}^{r} \geq N_{ts}^{r} \lambda_{ts}^{r} \text{, } r = 1, 2, \ldots\ldots, n_{ts} - 1,\ t = 1, 2, \ldots, n_t \text{ , } \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{ (12)}$$

$$x_{ts}^{w} \leq N_{ts}^{w} \lambda_{ts}^{r} \text{, } r = 1, 2, \ldots\ldots, n_{ts} - 1,\ t = 1, 2, \ldots, n_t \text{, } w = r+1, r+2, \ldots, n_{ts} \text{ , } \ldots\ldots\ldots \text{ (13)}$$

$$\lambda_{ts}^{r} = 0, 1 \text{ , } t = 1, 2, \ldots, n_t \text{ , } r = 1, 2, \ldots\ldots, n_{ts} - 1, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{ (i.4)}$$

5.8 Integral constraints

All the construction decision variables are integrals:

x_{tj}, x_{ts}^{r}, x_m^p, x_m^o, x^a $\forall\ t, r, m$; $j = p, o, i$

Integers. ……………………..…………….……. (14)

5.9 0-1 Constraints

y_p , y_i , y_s λ_{ts}^{r} , $t = 1, 2, \ldots, n_t$, $r = 1, 2, \ldots\ldots$,

n_{ts} -1 = 0, 1 . ……….....………………….……. (15)

6 Conclusions and Points for Future Researches

6.1 Conclusions

This research has the following conclusions:

1- The problem of designing a production and an import/ storing facilities for provision of multiple types of components, and then to determine the optimum number of components to be provided by the designed facilities in addition to subcontracting with a third party with the objective of minimizing the total cost can not solved by the intuition or by try and error methods.

2- An integer 0-1 programming mathematical model is formulated to represent such problems. The formulation has some special forms such as: If-Then, combined If-Then/Or, addition of fixed costs to the objective function, and stepped variable unit costs.

6.2 Points for future researches

1. To give an illustrative example to clarify the advantages of the proposed algorithm over the intuitive and the try and error approaches.

2. To develop a decision support system for complete modeling and solving large scale problems using a commercial modeling and/or general purpose languages.

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