A Multistage Mean/Variance approach for Portfolio Management in the Mexican Market

MARIA A. OSORIO[1], ANA BALLINAS[2], ERIKA JIMÉNEZ[3], ABRAHAM SÁNCHEZ[4]
[1]Department of Chemical Engineering, [2,3,4]Department of Computing
Universidad Autónoma de Puebla
Ciudad Universitaria, San Manuel, Puebla. Pue.
MÉXICO

Abstract: - This paper describes the use of Mean/Variance multistage portfolio management for building efficient frontiers. The maximization of the returns yields the maximum and the variance minimization the minimum points in the efficient frontier. The efficient frontier is the graph describing all the optimal options between these two points. The intermediate points are obtained minimizing the variance (risk measure) subject to different percentages of the maximum utility expected. According to the investor's characteristics, a point in the graph, containing a complete set of investment strategies can be chosen. The stochastic quadratic and linear models use a scenario tree to represent the multistage discretization of the random returns. The examples are applied to the Mexican bursaries market.

Key-Words: - **Portfolio Management, Stochastic Programming, Mean/Variance Optimization.**

1 Introduction

In portfolio management, stochastic programming is used to get an efficient frontier with all possible combinations of returns (represented by the mean) and risk levels (represented by the variance) subject to constraints specified by the investor and taking into account the possible fluctuation of the assets return in the future (Trippy et al. [5]). The uncertainty on return values of instruments is represented by a discrete approximation in a scenario tree.

The Stochastic Linear Programming model (SLP maximizes the expected wealth at the end of the investment horizon, represented by the man of the scenario tree. Expected wealth is calculated as the total net redemption value at time period T. The solution of the pure SLP problem represents the maximum amount of money that an investor can obtain when the person is risk seeker and it is considered the 100% risk option.

The Stochastic Quadratic Programming model (SQP), named the Markowitz model, minimizes the variance in the scenario tree. The variance is used as a risk measure and the pure SQP model will yield the minimum return that the investor can expect, and it is considered the 0% risk level. The intermediate points are calculated with the stochastic quadratic model (SQP), but including a constraint forcing the net redemption value to be a percentage of the mean obtained with the SLP model. The graph containing all points is named the efficient frontier. The model is multistage because it uses the wealth generated in the previous period in order to represent the constraint in the next period and stochastic because uses a scenario tree, including all possibilities of the future, with its respective probability, instead of single return values.

The main concern of this paper is to obtain an efficient frontier that allows the investor to find the risk level appropriate to their age, characteristics and risk aversion/seeking level to choose the corresponding point in the efficient frontier in order to get the right investment policy. The randomized returns distribution is represented by a scenario tree generated with clustering and simulation and the linear and quadratic optimization models are solved with the models including specific constraints.

We present the Stochastic Programming models for Mean/Variance analysis in section 2, including the way the scenario tree was generated. In section 3, we describe the efficient frontier and the procedure to generate it. In section 4, we present two examples and the conclusions are exposed in section 5. Tables with data and results are presented at the end.

2 Stochastic Programming Portfolio Models

The main definitions used in the rest of the paper are:

Portfolio: A set of assets available for the investor.

Assets: The assets considered are equities in the Mexican bursaries market (BMV), available for the constitution of a portfolio distribution.

Returns: Percentage of returns in the form of dividends for equities.

Net Redemption Value: Total amount of money received at the end of the horizon when the investment is encashed.

The notation is described in Table 1.

Table 1 Notation

Symbols and Input Data	
1	$\equiv (1,1,1, \ldots, 1)'$
$\mathbf{p}'\mathbf{q}$	$\equiv p_1q_1 + p_2q_2 + \ldots + p_nq_n$ (Inner product)
$\mathbf{e} \equiv (s,t)$	index denoting an event (a node of the scenario tree)
$a(\mathbf{e})$	ancestor of event **e** (parent in the scenario tree)
$\Lambda_{\mathbf{r}} \in \Re^{nxn}$	covariance matrix of returns
N_t	set of nodes of the scenario tree at time t
$p_{\mathbf{e}}$	branching probability of event **e**: $p_{\mathbf{e}} = \text{Prob}[\mathbf{e} \mid a(\mathbf{e})]$
$P_{\mathbf{e}}$	probability of event **e: if e** $= (s,t)$, then $P_{\mathbf{e}} = \prod_{i=1\ldots t} p_{(s,i)}$
n	number of investment assets
M	amount of initial investment
T	investment planning horizon
TW_t	total withdrawal at time t
ic_i	percentage paid in initial cost for asset i
ac_i	percentage paid in annual cost for asset i
$\mathbf{r}_{i\mathbf{e}}$	dividends or income returns for asset i at node **e**
tc	transaction cost
$\mathbf{w}^u_{i\mathbf{e}}$	upper bound for asset i
	Decision Variables
NR	net redemption value
$\mathbf{w}_*$	amount of money held in each asset
$\mathbf{h}_*$	withdrawal
$\mathbf{b}_*$	amount bought of each asset
$\mathbf{s}_*$	amount sold of each asset

2.1 Scenario Trees and Uncertainty Representation

Generating scenario trees is important for the performance of the multistage stochastic programming. The root node of the scenario tree represents the decision "today" and the nodes further on represent conditional decisions at later stages. The arcs linking the nodes represent various realizations of the uncertain variables. The dynamics of decision making is thus captured as decisions and adjusted according to realizations of uncertainty. The discretization of the random values and the probability space leads to a framework in which a random variable takes finitely many values. At each time period, new scenarios branch from the old, creating a scenario tree. Scenario trees can be generated based on different probabilistic approaches as simulation or optimization as presented in Gülpinar et al. [2].

Due to the recourse nature of the multistage problem, decision variables $\mathbf{w}_t$, $\mathbf{b}_t$, and $\mathbf{s}_t$ are influenced by previous stochastic events ρ^t, and hence $\mathbf{w}_t = \mathbf{w}_t(\rho^t)$, $\mathbf{b}_t = \mathbf{b}_t(\rho^t)$ and $\mathbf{s}_t = \mathbf{s}_t(\rho^t)$. However, for simplicity, we shall use the terms $\mathbf{w}_t$, $\mathbf{b}_t$, and $\mathbf{s}_t$, and assume their implicit dependence on ρ^t. Thus, the factors driving the risky events are approximated by a discrete set of scenarios or a sequence of events. Given the event history up to a time t, ρ^t, the uncertainty in the next period is characterized by finitely many possible outcomes for the next observation ρ^{t+1}. Each node $\mathbf{e} \in N_t$ at a level $t = 1, \ldots, T$ corresponds to a decision $\{\mathbf{w_e}, \mathbf{b_e}, \mathbf{s_e}\}$ which must be determined at time t, and depends in general on ρ^t and the past decisions $\{\mathbf{w}_j, \mathbf{b}_j, \mathbf{s}_j\}$, for $j = 1, \ldots, t-1$. The scenario tree is the input to the financial optimization problem. The We used a binary tree generated with a clustering and simulation procedure; the main steps needed to generate the scenario are described in Osorio et al. [3].

2.2 Stochastic Linear Programming model (SLP)

The Stochastic Linear Programming (SLP) model maximizes the expected wealth at the end of the investment horizon. Expected wealth is calculated as the total net redemption value at time period T.

The redemption value is basically defined as the amount of money received at time T when the investment is encashed. The basic LP model only includes constraints to express the wealth return and cash balance. We added annual bank fees, transaction costs for purchase operations, the withdrawal variable in the wealth return equation, the total withdrawal (TW_t) equation in the model and the upper bounds on the assets amount in a diversification constraint in order to obtain a more complete and descriptive model. The constraints in the SLP model are:

Net Redemption Value of every asset. (**1**)
Initial Allocation. (2)
Cash Balance Equations. (**3**)
Wealth for asset i in node **e**. (**4**)
Total Withdrawal at time t. (**5**)
Diversification constraints (**6**)

The objective function is the sum of the net redemption values of every asset at the end of the complete horizon, i.e. the net amount of money that the investor can obtain when the total investment is encashed. The general expression for the multistage portfolio optimization model is:

$$\max \Sigma_{i=1,n} NR_i \, .$$

Subject to

$$NR_i = \Sigma_{\mathbf{e}\in NT} P\mathbf{e}\, [\mathbf{1}'\, \mathbf{w}_{i\mathbf{e}}] \quad i=1,\ldots,n \qquad \textbf{(2)}$$

$$\Sigma_{i=1,n}\mathbf{1}'\mathbf{w}_{i0} = M \qquad \textbf{(2)}$$

$$\mathbf{1}'\mathbf{b}_{i\mathbf{e}} - \mathbf{1}'\mathbf{s}_{i\mathbf{e}} = 0 \qquad \textbf{(3)}$$

$$\mathbf{w}_{i\mathbf{e}} = (1-ac_i)[(1+\mathbf{r}_{i\mathbf{e}})\mathbf{w}_{ia(\mathbf{e})}] - \mathbf{h}_{i\mathbf{e}} + (1-tc)\mathbf{b}_{i\mathbf{e}} - \mathbf{s}_{i\mathbf{e}} \ \textbf{(4)}$$

$$TW_t = \Sigma_{\mathbf{e}\in Nt} P\mathbf{e}\, \Sigma_{i=1,n}\, \mathbf{1}'\mathbf{h}_{i\mathbf{e}}, t=1,\ldots,T \qquad \textbf{(5)}$$

$$\Sigma_{i=1,n}\, \mathbf{w}_{i\mathbf{e}} \le \mathbf{w}^{u}_{i\mathbf{e}}\, \Sigma_{i=1,n}\, (\mathbf{1}'\mathbf{w}_{i\mathbf{e}})\mathbf{e}\in N_t, t=1,\ldots,T \qquad \textbf{(6)}$$

$$NR_i \ge 0, i=1,\ldots,n \qquad \textbf{(7)}$$

$$\mathbf{w}_{i\mathbf{e}}, \mathbf{b}_{i\mathbf{e}}, \mathbf{s}_{i\mathbf{e}} \ge 0, \mathbf{e}\in N_t, t=1,\ldots,T, i=1, \ldots,n \qquad \textbf{(8)}$$

Notice that the annual bank fees deducted by term $(1-ac_i)$ for $i = 1, \ldots, n$ must be augmented by the bank's initial setup fees in the first year. For children of the root scenario node, $\mathbf{e} \in N_1$, the term becomes $(1 - ic_i - ac_i)$, and is imposed on all constraints. The wealth in every period t for asset i, is $\Sigma_{\mathbf{e}\in Nt}\ P\mathbf{e}\ (\mathbf{1}'\mathbf{w}_{i\mathbf{e}})$, for $i=1,\ldots,n$, and $t=1,\ldots,T$. The total wealth in for every period can be evaluated as $\Sigma_{\mathbf{e}\in Nt}\ P\mathbf{e}\ \Sigma_{i=1,n}\ \mathbf{1}'\mathbf{w}_{i\mathbf{e}}$, for $t=1,\ldots,T$.

2.3 Stochastic Quadratic Programming model (SLP)

The SQP approach attempts to inject risk aversion into the optimization model. It incorporates the quadratic variance term and permits the minimization of the variability of the terminal wealth given observed statistics. This ensures a degree of risk aversion by the investor by relaxing the certainty of the return values along a given leaf of the scenario tree.

The variance of wealth at time t of a particular asset i can be calculated as

$$\begin{aligned}
\mathrm{Var}[\mathbf{1}'\mathbf{w}^i_t] &= \mathrm{Var}\,[(r^i\mathbf{r}_t)'\mathbf{w}^i_{t-1}] \\
&= \mathrm{E}\,[((r^i\mathbf{r}_t)'\mathbf{w}^i_{t-1})^2] - (\mathrm{E}[(r^i\mathbf{r}_t)'\mathbf{w}^i_{t-1}])^{2} \\
&\equiv \mathrm{E}\,[((r^i\mathbf{r}_t)'\mathbf{w}^i_{t-1})^{2}] \\
&= \mathrm{E}\,[\mathbf{w}^{i\,'}_{t-1}\,(r^i\mathbf{r}_t)\,(r^i\mathbf{r}_t)'\,\mathbf{w}^i_{t-1})] \\
&= \mathrm{E}\,[\mathbf{w}^{i\,'}_{t-1}\,((r^i)^2\,\Lambda_r)\,\mathbf{w}^i_{t-1})] \\
&= \Sigma_{\mathbf{e}\in Nt}\, P_{\mathbf{e}}\,[\mathbf{w}^{i\,'}_{a(\mathbf{e})}\,((r^i)^2\,\Lambda_r)\,\mathbf{w}^i_{\ a(\mathbf{e})})]
\end{aligned}$$

Where r^i is a scalar factor to returns. Notice that for any random vector $\mathbf{y}$, $\mathrm{E}[\mathbf{yy}']$ is equivalent to the covariance matrix of $\mathbf{y}$.

The SQP problem whose optimum is the efficient (least risky) multistage investment strategy can be outlined as the following optimization problem.

$$\min \mathrm{Var} = \Sigma_{i=1,n}\, \Sigma_{\mathbf{e}\in Nt}\, P_{\mathbf{e}}\, [\mathbf{w}^{k\,'}_{a(\mathbf{e})}\, ((r^k)^2\, \Lambda_r)\, \mathbf{w}^k_{\ a(\mathbf{e})})]$$

Subject to

Constraints (1) … (8)

3 Efficient Frontiers

$\mathbf{e}\in N_t, t=1,\ldots,T,\ i=1,\ldots,n$ **(3)**

$\mathbf{e}\in N_t, t=1,\ldots,T, i=1,\ldots,n$ **(4)**

Financial reality dictates that the highest-performing portfolio strategy is also the most risky efficient strategy available. In order to obtain other points on the Markowitz efficient frontier, it is necessary to consider risk (variance), in conjunction with the mean return. In this case, the required expected net redemption value can be provides as constant W_T.

The statistics measures that control the risk and the maximum income are the variance and the mean. As stated above, for this problem, they are defined as:

$$Mean = \Sigma_{i=1,n}\, \Sigma_{\mathbf{e}\in NT}\, P\mathbf{e}\, [\mathbf{1}'\, \mathbf{w}_{i\mathbf{e}}]$$

$$\mathbf{e}\in N_t, t=1,\ldots,T, i=1,\ldots,n$$

$$Variance = \Sigma_{i=1,n}\, \Sigma_{\mathbf{e}\in Nt}\, P_{\mathbf{e}}\, [\mathbf{w}^{i\,'}_{a(\mathbf{e})}\, ((r^i)^2\, \Lambda_r)\, \mathbf{w}^k_{\ a(\mathbf{e})})]$$

$$\mathbf{e}\in N_t, t=1,\ldots,T, i=1,\ldots,n$$

The optimal investment strategy yields the expected wealth, W_T subject to the linear constraints (1) to (8). The solution to the SLP problem that maximizes the mean is W_{MAX}, the maximum amount of money, and the solution to the SQP problem that minimizes the variance subject to the same constraints (1) to (8) is W_{MIN}. The W_T can take values between W_{MIN} and W_{MAX}. The intermediate values W_T values can be obtained solving the following quadratic problem,

Minimize *Variance*

Subject to

$Mean \ge W_T$

\+

Constraints (1) … (8)

By varying W_T from a risk-seeking strategy (obtainable by solving the SLP) to a risk-averse strategy (obtainable by optimizing the above SQP without the performance constrain, $Mean \ge W_T$), the efficient frontier can be generated.

In general terms, the efficient frontier is obtained as follows: The maximum-mean SLP problem is first solved to find the risk-seeking strategy; that is also the maximum expected net redemption value, W_{MAX}. The minimum expected net redemption value, is then computed by solving the SQP problem. Its optimal represents the risk-averse strategy. Finally, for a number of equally-spaced points, the intermediate W_T

values can be obtained solving the quadratic problem SQP with the corresponding W_T value in the constraint $Mean \geq W_T$. The efficient frontiers for examples with 5 and 10 assets can be seen in Figs. 1 and 2.

4 Examples

The procedure was tested with two examples. In both cases, 50 monthly periods (2002-2006) were used to build a scenario tree with four future stages. The scenario tree has two branches in each node. We considered five assets in the first example and ten assets in the second one. The data correspond to real assets in the BMV (Mexican bursaries market) and were obtained from Econom@tica (financial database). The examples were tested in a Pentium IV with 1.7 GHz and 256 Mb. The initial amount M was of 100 money units for both examples and we considered a withdrawal of $TW_t=0$, for $t=1,\ldots,T$. The scenario trees used for the example with 5 assets and the example with 10 assets are showed in Tables 2 and 3 and were introduced in Osorio [4].

In both examples, the cash flow for every stage was calculated for 0% (using the SQP model), and 100% (using SLP model) risk levels, and values for 0%, 50% and 100% risk levels can be seen in Tables 4 and 5. We considered 100 intermediate intervals of risk including the constraint *Mean* ≥ (*percentage*)(*SLP optimal value*), in order to build the efficient frontiers.. The efficient frontiers for both examples, including all mean values for different levels of risk (represented by the variance values) are shown in figures 1 and 2. Models were solved with CPLEX.

5 Conclusions

Portfolio selection gives rise to difficult optimization problems when realistic side constraints and variables are added to the basic model.

Portfolio selection gives rise to difficult optimization problems when realistic side constraints and variables are added to the basic model. The number of variables and constraint in the SLP and SQP models are increased by the number of assets and the topology of the scenario tree. The size of the scenario tree depends on the depth and branching at each time period. Our computational results show that even for large scenario trees it is possible to find solutions near to the optimal in a reasonable amount of time.

The different level risk offer different investment options for different investors with different attitude to risk (Green [1]). The risk-averse investors will choose a risk level with a less variance value (for example, less than 50%) and the risk-seeker investor will choose greater levels of risk (for example, greater than 50%). The expected utility in every case, represented by the mean will be different.

References:

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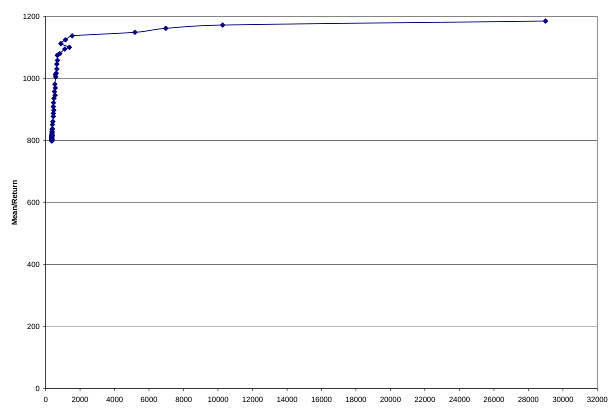

Fig. 1. Risk Frontier for the 5 assets example.

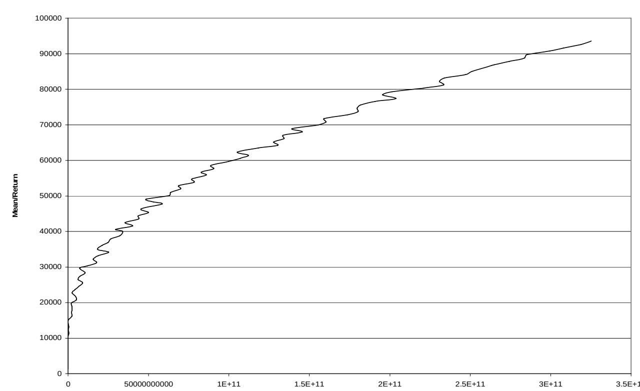

Fig. 2 Efficient frontier for ten assets example

Table 3. Scenario Tree for 5 Assets

Id node	Asset1	Asset2	Asset3	Asset4	Asset5	Probability	Id Father node	Stage
0	0.747	0.684	0.769	0.673	0.696	1.000	-1	0
1	0.856	0.703	1.104	0.691	0.741	0.346	0	1
2	0.655	0.667	0.485	0.657	0.658	0.654	0	1
3	0.897	0.710	1.229	0.698	0.757	0.290	1	2
4	0.774	0.689	0.853	0.677	0.707	0.710	1	2
5	0.914	0.713	1.284	0.701	0.764	0.595	2	2
6	0.964	0.722	1.438	0.709	0.785	0.405	2	2
7	0.687	0.673	0.584	0.663	0.672	0.237	3	3
8	0.797	0.791	0.692	0.904	0.680	0.763	3	3
9	0.736	0.682	0.736	0.671	0.692	0.559	4	3
10	0.504	0.641	0.020	0.632	0.597	0.441	4	3
11	0.797	0.693	0.924	0.681	0.717	0.805	5	3
12	0.695	0.675	0.610	0.664	0.675	0.195	5	3
13	0.716	0.678	0.673	0.668	0.683	0.499	6	3
14	0.757	0.685	0.798	0.674	0.700	0.501	6	3

Table 4. Scenario Tree for 10 Assets

id_node	Benchmarks_182D	Benchmarks_28D	Benchmarks_364D	Benchmarks_7D	Benchmarks_91D	America_Movil_A	America_Movil_L	Ara_Con_A31sorcio	Arca_Embotelladora	Asureste_B	probability	id_father_node	Stage
0	0.67	0.60	0.69	0.59	0.61	3.34	3.39	1.94	1.00	2.62	1.00	-1	0
1	1.08	0.14	0.98	1.12	0.96	3.18	3.41	0.93	0.50	1.82	0.23	0	1
2	1.14	1.17	0.91	0.72	0.46	1.53	4.21	1.07	1.53	1.56	0.77	0	1
3	1.24	0.02	0.37	1.79	1.35	1.87	4.24	0.43	0.83	3.61	0.44	1	2
4	0.05	0.21	0.40	2.18	0.43	4.97	0.60	0.72	0.71	2.69	0.56	1	2
5	0.29	2.16	0.78	1.44	0.31	0.82	4.54	0.80	0.17	2.11	0.77	2	2
6	1.17	0.83	0.28	1.36	0.89	0.73	5.24	0.55	2.85	2.78	0.23	2	2
7	1.87	0.00	0.48	3.46	1.17	1.99	6.57	0.33	0.81	0.00	0.96	3	3
8	0.09	0.01	0.70	3.58	2.56	0.13	2.19	0.82	0.93	5.34	0.04	3	3
9	0.01	0.27	0.29	3.92	0.76	7.86	1.11	0.23	1.40	4.94	0.42	4	3
10	0.06	0.38	0.31	3.11	0.21	6.88	0.52	0.25	0.75	5.33	0.58	4	3
11	0.14	3.28	0.51	2.37	0.07	0.89	3.44	1.26	0.17	0.71	0.01	5	3
12	0.19	2.39	0.29	1.39	0.61	0.83	7.25	1.05	0.26	4.08	0.99	5	3
13	0.09	0.77	0.47	0.18	1.33	1.43	8.54	0.49	3.06	1.12	0.35	6	3
14	1.67	1.66	0.47	2.65	1.19	0.90	3.83	0.25	3.57	1.47	0.65	6	3

Table 3. Five Assets

0% Risk	Stage 0	Stage 1	Stage 2	Stage 3	Stage 4
Asset 1	0.00	0.00	0.00	0.00	0.00
Asset 2	26.39	15.26	54.08	354.42	597.79
Asset 3	0.00	0.00	0.00	0.00	0.00
Asset 4	73.61	145.74	225.88	0.00	0.00
Asset 5	0.00	6.57	0.00	121.21	204.54
TOTAL	100	167.57	279.96	475.63	802.33

50% Risk	Stage 0	Stage 1	Stage 2	Stage 3	Stage 4
Asset 1	99.08	76.14	18.87	0	0
Asset 2	0	0	67.22	514.64	867.95
Asset 3	0	35.91	159.89	0	0
Asset 4	0.92	0	31.90	0	0
Asset 5	0	62.59	19.99	77.94	130.84
TOTAL	100	174.64	297.87	592.58	998.79
100% Risk	Stage 0	Stage 1	Stage 2	Stage 3	Stage 4
Asset 1	0	176.90	0	189.22	328.54
Asset 2	0	0	0	149.42	248.58
Asset 3	100	0	305.02	260.01	484.31
Asset 4	0	0	0	73.36	135.50
Asset 5	0	0	0	0	0
TOTAL	100	176.9	305.02	672.01	1196.93

Table 4. Ten Assets

0% Risk	Stage 0	Stage 1	Stage 2	Stage 3	Stage 4
Benchmarks_182D	5.00	0.00	38.12	78.79	130.38
Benchmarks_28D	8.49	47.24	48.31	117.34	335.00
Benchmarks_364D	24.64	42.85	35.48	97.25	131.74
Benchmarks_7D	0.00	14.07	25.28	117.35	348.65
Benchmarks_91D	48.51	38.30	54.86	139.37	245.93
America_Movil_A	0.00	0.11	73.59	0.00	0.00
America_Movil_L	0.00	0.00	0.00	0.00	0.00
Ara_Con_A31sorcio	0.00	25.88	36.75	0.79	0.98
Arca_Embotelladora	13.36	0.00	0.00	0.00	0.00
Asureste_B	0.00	0.00	0.00	0.08	0.19
TOTAL	100	168.45	312.39	550.97	1192.87
50% Risk	Stage 0	Stage 1	Stage 2	Stage 3	Stage 4
Benchmarks_182D	0.00	0.00	0.00	282.26	514.10
Benchmarks_28D	1.41	0.00	0.00	317.25	758.82
Benchmarks_364D	0.00	0.00	0.50	19.98	29.81
Benchmarks_7D	1.37	0.00	70.32	369.94	1098.24
Benchmarks_91D	0.00	1.36	0.00	332.55	573.74
America_Movil_A	42.24	72.80	272.83	619.53	2024.76
America_Movil_L	2.58	289.77	1052.51	4032.41	31284.61
Ara_Con_A31sorcio	0.00	6.29	0.01	207.88	322.31
Arca_Embotelladora	0.00	5.28	12.21	361.83	1037.81
Asureste_B	52.40	13.55	321.35	2016.12	10261.69
TOTAL	100	389.05	1729.73	8559.75	47905.89
100% Risk	Stage 0	Stage 1	Stage 2	Stage 3	Stage 4
Benchmarks_182D	0.00	0.00	0.00	0.00	0.00
Benchmarks_28D	0.00	0.00	0.00	0.00	0.00
Benchmarks_364D	0.00	0.00	0.00	0.00	0.00
Benchmarks_7D	0.00	0.00	0.00	0.00	0.00
Benchmarks_91D	0.00	0.00	0.00	0.00	0.00
America_Movil_A	0.00	0.00	450.79	1,498.64	12,428.14
America_Movil_L	100.00	439.44	1,755.93	10,594.33	81,967.64
Ara_Con_A31sorcio	0.00	0.00	0.00	0.00	0.00
Arca_Embotelladora	0.00	0.00	0.00	0.00	0.00
Asureste_B	0.00	0.00	0.00	0.00	0.00
TOTAL	100	439.44	2,206.72	12,092.97	94,395.78