Evaluation of Orbit Determination Using Dual Ranging Method

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Abstract: - The paper provides both the theoretical and practical results for Geo-stationary satellites orbit determination using both dual Ranging from Two separate earth station locations and Classical orbit determination from an earth station. The orbit determination is performed through a generated Matlab program and is compared with a flight proven software tool. The program takes into account the dynamic model [1],[5],[6] of the satellite which takes orbit perturbations due to non_ spherical earth shape, the gravitational forces of the sun and moon, and the atmospheric drag. Acceptable results where foreseen in comparison to the flight proven software tool.

Key-Words: - Orbit, coordinate systems, orbit determination, dual ranging

1. Introduction

To determine a spacecraft's orbit, measurements such as range, angles are needed which can be obtained by a number of Earth-based systems tracking systems.

These measurements are the means for calculating the trajectory of a satellite with the help of filtering algorithms and models of orbit evolution which always compare the real measurements with a theoretical quantity calculated from a pre-assumed trajectory as seen in fig 1 [2].

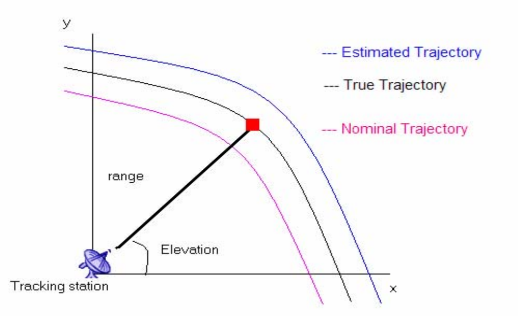


Fig 1: Orbit Determination Problem

Orbit determination could be determined by the utilization of both ranging and angular measurements or ranging measurements only, but using angular measurements have varies types of problems as:

1. The mechanical precession which is required for the large equipments leads to very high operational costs.
2. Azimuth-Elevation accuracy depends on the mechanical antenna system not on the electrical system as the ranging.
3. The errors on the measurements are often biases, that are slowly evolving, like the alignment of the mechanical axes, or cyclic (due to day-night temperature fluctuations) or variable like the deviation between the targeted radio- frequency direction and the mechanical direction, which may be due to wind or to the dynamics of the displacement (servo control).

Thus, due to these problems and due to the need for higher precession method, the paper provides a method for orbit determination based only on the ranging measurements.

The paper is divided into several sections. The first section contains some basic definitions. The second section contains the orbit determination algorithm and dual ranging algorithm. The third section contains the simulation results performed. Fourth section contains the conclusion of this paper. The last section contains the future work and reference for this work.

2. Basic Definitions

2.1 Orbital parameters

The elements of an orbit [1],[3] are the parameters needed to specify that orbit uniquely. Traditionally used set of orbital elements called the set of Keplerian elements. The Keplerian elements are six: Semi-major axis (a), eccentricity of the ellipse (e), inclination angle (i) , Right ascension of ascending node (Ω) , argument of perigee (ω), True anomaly (ϑ).

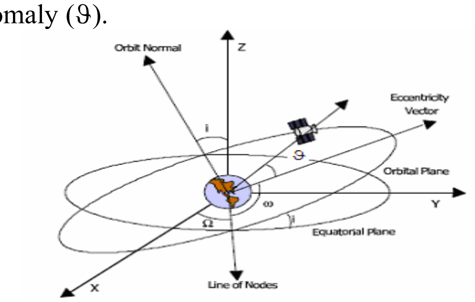


Fig 2 : Orbital angles

For geostationary orbit, the inclination angle (i) nearly equal to zero, so the values of ω and ϑ can not be given with sufficient accuracy, as the position of the ascending node is not determined accurately. The parameters in the kepelerian set are slightly modified to include implicitly the parameters (i, ω, ϑ). The new sets of modified orbital parameters are given by definition as:

Semi-major axis: a (1)

Eccentricity vector in the x , y directions:

$$\bar{e}_x = \bar{e}\cos(\omega+\Omega) \quad (2)$$
$$\bar{e}_y = \bar{e}\sin(\omega+\Omega)$$

Inclination vector in the x , y directions :

$$\bar{i}_x = \sin(i)\cos(\Omega) \quad (3)$$
$$\bar{i}_y = \sin(i)\sin(\Omega)$$

Longitude : $l = \omega + \vartheta + \Omega - GAST$ (4)

Where; GAST = Grinitch apparent sidereal time

2.2 Orbit Perturbations

The Keplerian orbit is ideal since it assumes that the earth is a uniform spherical mass.

The dynamic model is introduced for a more realistic orbit. Thus we take into account orbit perturbations [2],[3] which are due to:

1. The forces due to the contribution of the non- spherical components of terrestrial attraction.
2. The attraction of the sun and the moon (Third-Body Perturbations)
3. Solar radiation pressure
4. Aerodynamic drag , which is negligible for altitudes above 3000 Km

2.3 State Vector

Another way to determine the orbit rather than the set of orbital parameters is the state vector (position, velocity), where the orbit is determined through the definition of the position and velocity in cartesian coordinate system (X, Y and Z – directions).

The state vector is shown in this section as the MATLAB program compute the optimum increment in the state vector – equation (25) – and add it to the initial state vector to produce a new state vector which is more precise. This new state vector is transferred back to the orbital parameters.

Transformation from the orbital parameters to the state vector [2]: position (X, Y, Z) and velocity (dX, dY, dZ), is done as shown:

$$X = r*[\cos(\omega+\vartheta)*\cos(\Omega) - \sin(\omega+\vartheta)*\sin(\Omega)*\cos(i)] \quad (5)$$
$$Y = r*[\cos(\omega+\vartheta)*\sin(\Omega) + \sin(\omega+\vartheta)*\cos(\Omega)*\cos(i)] \quad (6)$$
$$Z = r*[\sin(\omega+\vartheta)*\sin(i)] \quad (7)$$

$$dX = -\mu/H * [\cos(\Omega)*(\sin(\omega+\vartheta) + e*\sin(\omega)) + \sin(\Omega)*(\cos(\omega+\vartheta) + e*\cos(\omega))*\cos(i)] \quad (8)$$
$$dY = -\mu/H * [\sin(\Omega)*(\sin(\omega+\vartheta) + e*\sin(\omega)) - \cos(\Omega)*(\cos(\omega+\vartheta) + e*\cos(\omega))*\cos(i)] \quad (9)$$
$$dZ = \mu/H * [\cos(\omega+\vartheta) + e*\cos(\omega)]*\sin(i) \quad (10)$$

Where; H = Magnitude of the angular momentum.
r = Magnitude of the position vector (r) in the P-Q frame plane as shown in equation (11).
μ = Earth Gravitational constant $=3.986e^5$ km^3/sec^2.

2.4 Reference Frames

Co-ordinate transformation systems are needed in order to determine the computed measurement (Range, Azimuth and Elevation) from the given orbital parameters. Thus, three co-ordinate systems [1],[2] are introduced in this paper to define the satellite position relative to the ground tracking earth station taking into account the attraction effect of the motion of the sun and the moon. These coordinate systems are the following:

1. The Perifocal coordinate (*p*,*q*,*w*).
2. The Geocentric coordinate (*I*,*J*,*K*) , and
3. The Satellite in rotating frame coordinate (*X*,*Y*,*Z*),

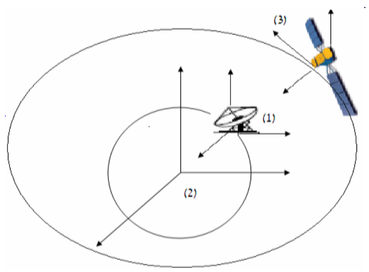


Fig 3: Coordinate systems transformations

Each of these reference frames are presented in details;

2.4.1 Perifocal coordinate (p , q ,w)

The position of a geostationary satellite as measured from the Earth station. The observer's horizon becomes the reference plane and his position, the origin.

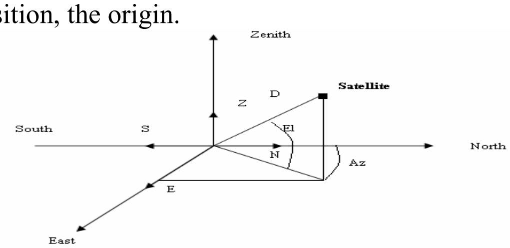


Fig 4: Perifocal coordinates

From this coordinate system the magnitude of the position vector r in the P-Q frame plane is computed;

$$r = a *(1-e*\cos(E)); \quad (11)$$

$$r_p = r*\cos(\vartheta); \; r_q = r*\sin(\vartheta); \quad (12)$$

2.4.2 Geocentric coordinate (I, J, K)

The general Geocentric Equatorial Coordinate System (*IJK*) is also known as the Earth-Centered Inertial (ECI) system. ECI's origin is at Earth's center, and its fundamental plane is the equator.

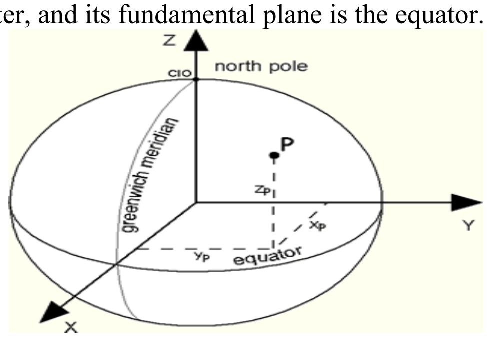


Fig 5: Geocentric coordinate

The *I*-axis (or +*X*-axis) points towards the vernal equinox; the *J*-axis (or +*Y*-axis) is 90^o to the east in the equatorial plane; and the *K*-axis (or +*Z*-axis) points towards the North Pole.

Computing the position components X, Y, Z,

$$r_i= [\cos(\Omega)*\cos(\omega)-\sin(\Omega)*\cos(i)*\sin(\omega))*r_p + (-\cos(\Omega)*\sin(\omega)- \sin(\Omega)*\cos(i)*\cos(\omega)]*r_q; \quad (13)$$

$$r_j=[\sin(\Omega)*\cos(\omega)+ \cos(\Omega)*\cos(i)*\sin(\omega))*r_p +(-\sin(\Omega)*\sin(\omega) + \cos(\Omega)*\cos(i)*\cos(\omega)]*r_q; \quad (14)$$

$$r_k= [\sin(i)*\sin(\omega))*r_p + (\sin(i)*\cos(\omega)]*r_q; \quad (15)$$

This coordinate system is considered inertial, but the equinox and plane of the equator move over time. Thus in order to take into account the relative motion of the satellite with respect to the earth, introduce the following coordinate system;

2.4.3 Satellite in rotating frame coordinate (X,Y,Z)

Known as Satellite Radial coordinate system (*RSW*), moves with the satellite. The radial, *R*-axis points from Earth's center along the radius vector to the satellite as it moves through an orbit. The along-track *S*-axis points in the direction of the velocity vector, and is perpendicular to the radius vector. The cross-track, *W*-axis is fixed along the direction normal to the orbital plane.

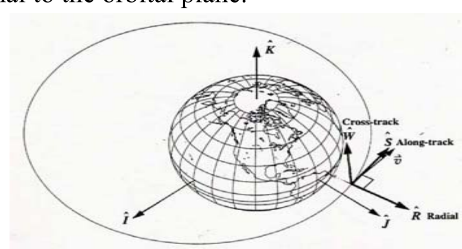


Fig 6: Rotating frame coordinate

Computing the rotating system coordinates:

$$X_r = \cos(GAST)*r_i+ \sin(GAST)*r_j \quad (16)$$

$$Y_r = -\sin(GAST)*r_i+ \cos(GAST)*r_j \quad (17)$$

$$Z_r = 1*r_k \quad (18)$$

From this point, we calculate the sub-satellite points (sub-satellite longitude and latitude) as shown in equations (19) and (20) respectively:

$$L_s= \quad \pi/2 - \cos^{-1}[Z_r/(X_r^2+Y_r^2+Z_r^2)^{0.5}] \quad (19)$$

$$l_s= \begin{cases} -\tan^{-1} (Y_r / X_r) & Y_r >0 ,X_r >0 \\ \pi - \tan^{-1} (Y_r / I\, X_r\, I) & Y_r >0 ,X_r <0 \\ \pi/2 +\tan^{-1} (I\, X_r\, I / I\, Y_r\, I) & Y_r <0 ,X_r <0 \\ \tan^{-1} (I\, Y_r\, I / X_r) & Y_r <0 ,X_r >0 \end{cases} \quad (20)$$

And finally, computing the Azimuth, Elevation (Look angles) and ranging data from the satellite coordinates (X_r, Y_r, Z_r) using the geographical coordinates of the sub-satellite point as intermediaries;

As, distance from center of earth to the satellite (orbital radius) is;

$r_s = (X_r^2+Y_r^2+Z_r^2)^{0.5}$ (21)

Thus, Azimuth (Az), Elevation (El) and Ranging (Rg) equations are:

$EL = \cos^{-1} [\sin(\delta) / (1+(Re/r_s)^2 - 2*(Re/r_s)*\cos(\delta))^{0.5}]$ (22)

$R_g = r_s*[1+(Re/r_s)^2 - 2*(Re/r_s)*\cos(\delta)]^{0.5}$ (23)

Where; L_e, l_e are Earth station latitude and longitude respectively, & Re = Earth radius = 6378.13649 Km.

$\delta = \cos^{-1} [\cos(L_e)*\cos(L_s)*\cos((l_s-l_e))+\sin(L_e)*\sin(L_s)]$ (24)

And the Azimuth is computed depending on position of the earth station latitude with respect to the satellite latitude.

Thus, it is clear from the above equations that there exists a need for co-ordinate transformations in order to obtain the computed Azimuth and Elevation angles.

3. Dual Ranging Algorithm

3.1 Orbit Determination Algorithm

The algorithm presented in this paper is based on statistical orbit determination method - least squares (LS) method [9].

Based on the Goodyear relations [2], [3], [6], [8] the best estimate increment in the state vector is given by:

$\Delta x_k = (H^T H)^{-1} H^T y$ (25)

Where; y = difference between actual and computed measurements, and $(H^T H)^{-1}$ = Covariance matrix. The observation sensitivity matrix (H) equals:

$H = H''*\Phi$ (26)

Where; Φ = state transition matrix (linear transformation of the state vector) [1] = d(state vector)/d(orbital parameter) | at t_0 * [d(state vector)/d(orbital parameter)]$^{-1}$ | at t

And, H″ (Observation-state mapping matrix) [1] is the relation between variation of the measurements and the state vector.

3.2 Dual Ranging

Orbit determination using only ranging measurements [10], [11] approach is established to be independent from the Angular measurements.

Introducing in this work the precession and accuracy of Two station ranging as seen in fig(7) [1] , and compare it with the standard single station configuration (range, Angular measurements).

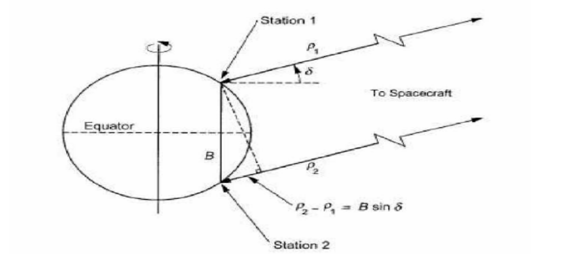


Fig 7: Dual ranging

The next flow chart [1] shows the orbit determination algorithm for dual ranging measurement method.

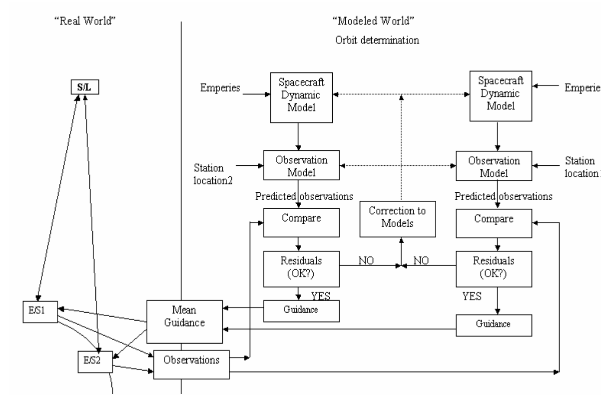


Fig 8 : Dual ranging Algorithm

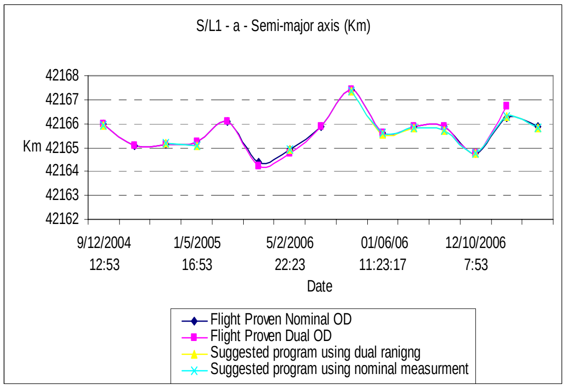
4. Simulation

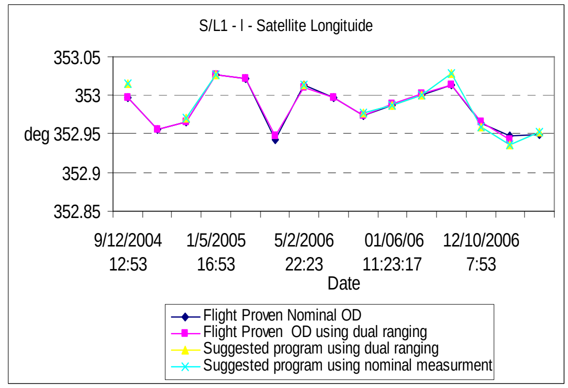
In this section we will provide simulation results, which utilized two earth stations one located in Cairo with coordinates 29.5 N latitude, 31.2 E longitudes, and the other is located in Alexandria with coordinates 31.4 N latitude, 29.46 E longitudes. The implemented system utilizes modern statistical method, using least square approach for minimizing the error.

A comparison study for two spacecraft between both nominal and dual ranging orbit determinations methods performed using a flight proven orbitoagraphy tool and using suggested program generated by MATLAB software.

Varies orbit determinations have been performed for around three years, each determination was based on two complete days of tracking data.

The shown results provide the final modified orbital parameters (semi-major axis, eccentricity vector in the x and y directions, inclination vector in the x and y directions and longitude) for varies methods used, and show comparable results with each others.


S/L1 - a - Semi-major axis (Km)
Km
42168
42167
42166
42165
42164
42163
42162
9/12/2004 12:53
1/5/2005 16:53
5/2/2006 22:23
01/06/06 11:23:17
12/10/2006 7:53
Date
Flight Proven Nominal OD
Flight Proven Dual OD
Suggested program using dual ranigng
Suggested program using nominal measurment


S/L1 - l - Satellite Longituide
deg
353.05
353
352.95
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9/12/2004 12:53
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5/2/2006 22:23
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Flight Proven Nominal OD
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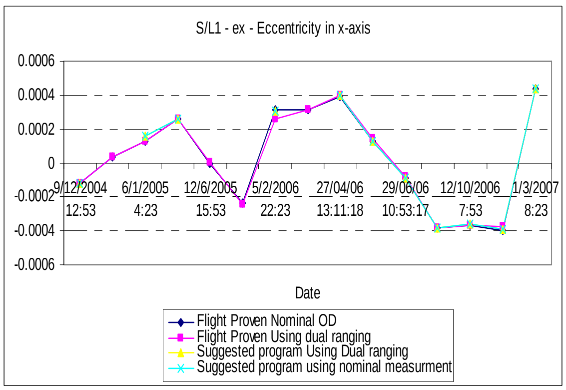
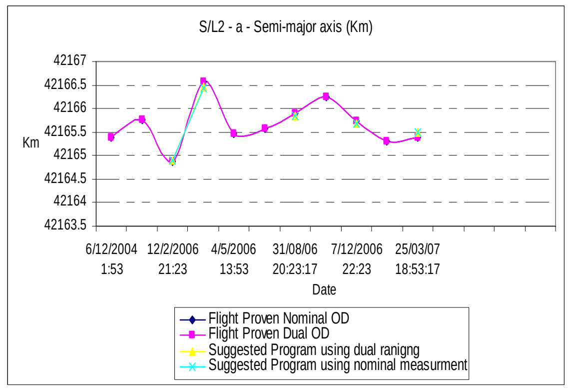
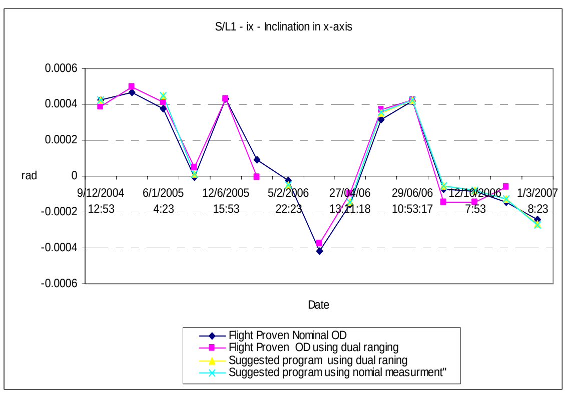
Suggested program using dual ranging
Suggested program using nominal measurment


S/L1 - ex - Eccentricity in x-axis
0.0006
0.0004
0.0002
0
-0.0002
-0.0004
-0.0006
9/12/2004 12:53
6/1/2005 4:23
12/6/2005 15:53
5/2/2006 22:23
27/04/06 13:11:18
29/06/06 10:53:17
12/10/2006 7:53
1/3/2007 8:23
Date
Flight Proven Nominal OD
Flight Proven Using dual ranging
Suggested program Using Dual ranging
Suggested program using nominal measurment


S/L2 - a - Semi-major axis (Km)
Km
42167
42166.5
42166
42165.5
42165
42164.5
42164
42163.5
6/12/2004 1:53
12/2/2006 21:23
4/5/2006 13:53
31/08/06 20:23:17
7/12/2006 22:23
25/03/07 18:53:17
Date
Flight Proven Nominal OD
Flight Proven Dual OD
Suggested Program using dual ranging
Suggested Program using nominal measurment


S/L1 - ix - Inclination in x-axis
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9/12/2004 12:53
6/1/2005 4:23
12/6/2005 15:53
5/2/2006 22:23
27/04/06 13:11:18
29/06/06 10:53:17
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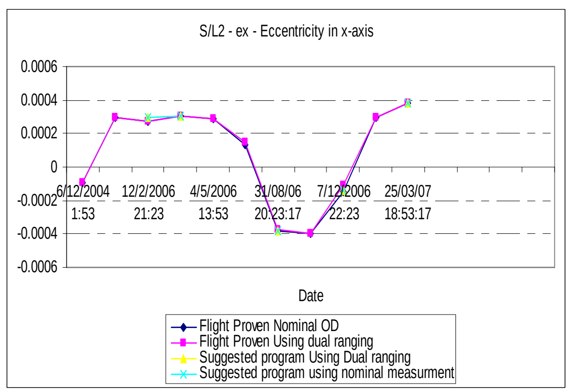
S/L2 - ex - Eccentricity in x-axis
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6/12/2004 1:53
12/2/2006 21:23
4/5/2006 13:53
31/08/06 20:23:17
7/12/2006 22:23
25/03/07 18:53:17
Date
Flight Proven Nominal OD
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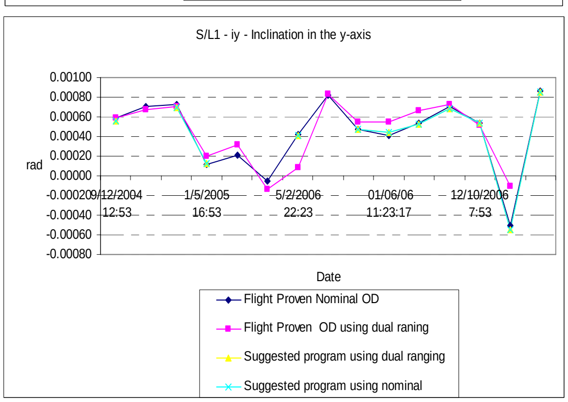
Suggested program using nominal measurment


S/L1 - iy - Inclination in the y-axis
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0.00100
0.00080
0.00060
0.00040
0.00020
0.00000
-0.00020
-0.00040
-0.00060
-0.00080
9/12/2004 12:53
1/5/2005 16:53
5/2/2006 22:23
01/06/06 11:23:17
12/10/2006 7:53
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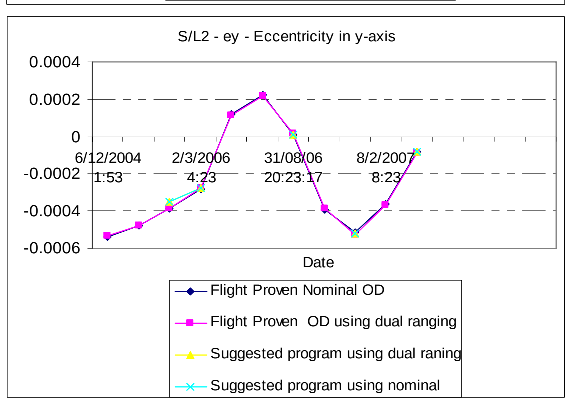
S/L2 - ey - Eccentricity in y-axis
0.0004
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6/12/2004 1:53
2/3/2006 4:23
31/08/06 20:23:17
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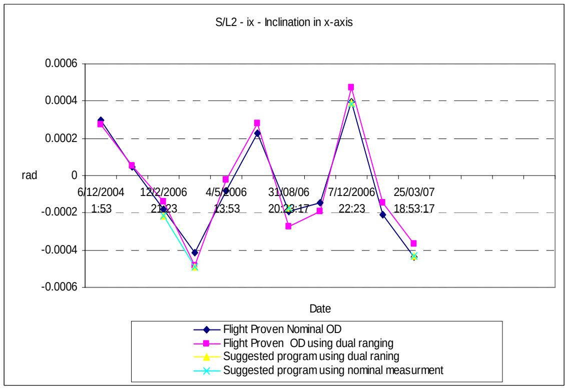

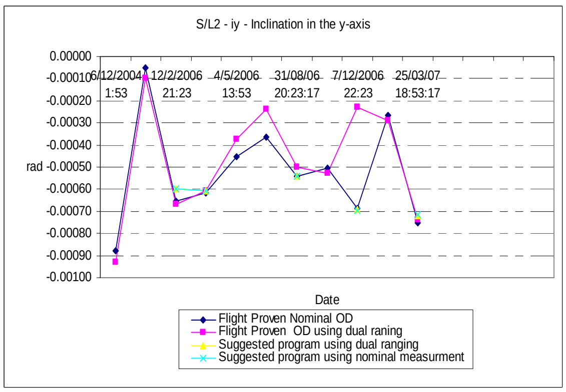

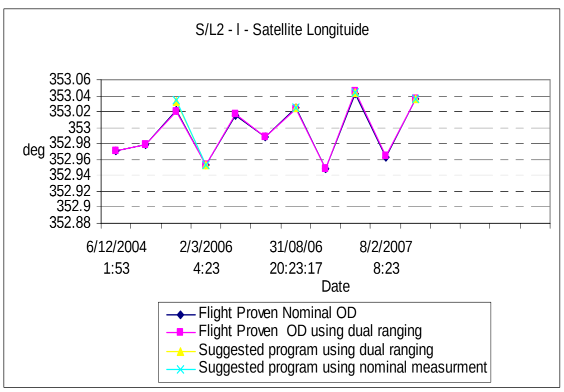

5. Conclusion

From the above complete Three (3) year analysis study, it appears that an Orbit determination using ranging data from two stations apart by around 250 Km and located within different longitudes from the tracking satellite, is feasible and produce acceptable results.

This conclusion lead to accepting the orbit computed from only the ranging data using two stations, thus in case of problems in the limited motion antenna campaign could be performed only using the fixed motion antenna and provide accurate and acceptable results.

6. Future Work

Next we will try to introduce different approach for the orbit determination by using spread spectrum technique instead of the normal Pseudo-range technique, discussing its capabilities and potentials.

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