Numerical solutions to a two-dimensional Riemann problem for gas dynamics equations

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Abstract: We consider a Riemann problem for gas dynamics equations in two space dimensions, following the approach by Čanić, Keyfitz, Kim and Lieberman for studying shock reflection phenomena. The initial data consists of two sectors and each discontinuity results in a shock followed by a linear wave in the far field. We derive regimes, depending on the initial data, for which regular reflection occurs and we pose local free boundary problems describing the subsonic solution and the position of the reflected shock. Using the CLAWPACK software we compute numerical solutions modeling strong and weak regular reflection, and we confirm preliminary conjectures on the structure of the solutions to the above free boundary problems.

Key–Words: two-dimensional Riemann problems, shock reflection, gas dynamics

1 Introduction

We study shock reflection phenomena by considering a Riemann problem for gas dynamics equations. Our ideas follow the approach of Čanić, Keyfitz, Kim and Lieberman in analysis of solutions to particular two-dimensional Riemann problems for systems of conservation laws. The first idea in their approach is to reformulate the problem using self-similar coordinates and to obtain a mixed type system and a free boundary problem for the subsonic state and the position of the reflected shock. Using the theory of second order elliptic equations with mixed boundary conditions, developed in [9], and fixed point theorems, local existence of solutions to this free boundary problem was shown in the case of steady transonic small disturbance equation [5], unsteady transonic small disturbance equation [1, 3, 12] and the nonlinear wave system [4, 13]. For general ideas of this approach see [15], and for more details on the structure of systems that have been studied see [2]. For related work, we refer to [6, 7, 8, 23, 24]. Many numerical techniques have been designed for studying shock reflection (for example, [10, 11, 14, 16, 17, 18, 20, 21, 22]), while theoretical understanding of this phenomena is still incomplete.

In this paper we state a Riemann problem for gas dynamics equations giving rise to both strong and weak regular reflection by carefully selecting initial data and solving quasi-one-dimensional Riemann problems at the reflection point, we derive free boundary problems for the subsonic state and the reflected shock, and we compute approximate solutions using the CLAWPACK software (see [19]).

2 The Riemann problem

We consider system

$$\begin{aligned}
&\rho_t + (\rho u)_x + (\rho v)_y = 0,\\
&(\rho u)_t + (\rho u^2 + p)_x + (\rho u v)_y = 0,\\
&(\rho v)_t + (\rho u v)_x + (\rho v^2 + p)_y = 0,\\
&(\rho E)_t + (u(\rho E + p))_x + (v(\rho E + p))_y = 0.
\end{aligned} \tag{1}$$

Here, $(x, y, t) \in \mathbb{R}\times\mathbb{R}\times[0,\infty)$, $\rho : \mathbb{R}\times\mathbb{R}\times[0,\infty) \to (0,\infty)$ is the density, $u, v : \mathbb{R}\times\mathbb{R}\times[0,\infty) \to \mathbb{R}$ are the x- and y-velocity components, respectively, $p : \mathbb{R}\times\mathbb{R}\times[0,\infty) \to (0,\infty)$ is the pressure, and $E = (\gamma-1)^{-1}p/\rho + (u^2+v^2)/2$ is the total specific energy, where $\gamma > 1$ is the gas constant. As in [2, 6, 23], we rewrite system (1) using self-similar coordinates $\xi = x/t$ and $\eta = y/t$ and obtain

$$\begin{aligned}
&(\rho U)_\xi + (\rho V)_\eta + 2\rho = 0,\\
&(\rho U^2 + p)_\xi + (\rho U V)_\eta + 3\rho U = 0,\\
&(\rho U V)_\xi + (\rho V^2 + p)_\eta + 3\rho V = 0,\\
&(U(\rho\tilde{E} + p))_\xi + (V(\rho\tilde{E} + p))_\eta + 2(\rho\tilde{E} + p)\\
&\qquad\qquad + \rho(U^2 + V^2) = 0,
\end{aligned} \tag{2}$$

where $\tilde{E} := (\gamma-1)^{-1}p/\rho + (U^2+V^2)/2$, $U := u-\xi$ and $V := v - \eta$. Clearly, when system (2) is linearized about a constant state $\overline{U}_* = (\rho_*, u_*, v_*, p_*)$, the system changes type across the sonic circle C_* : $U_*^2 + V_*^2 = c^2(\rho_*)$, where $c^2(\rho) := \gamma\, p/\rho$. More precisely, system (2) is hyperbolic outside of C_*.

Using the Rankine-Hugoniot relations, a shock between states $\overline{U}_0 = (\rho_0, u_0, v_0, p_0)$ and $\overline{U} = (\rho, u, v, p)$, given by $\eta = \eta(\xi)$, satisfies

$$\sigma_\pm := \frac{d\eta}{d\xi} = \frac{U_0 V_0 \pm \sqrt{\overline{c}^2(U_0^2 + V_0^2 - \overline{c}^2)}}{U_0^2 - \overline{c}^2}, \quad (3)$$

$$\frac{d\eta}{d\xi} = -\frac{U - U_0}{V - V_0}, \quad (4)$$

$$\rho_0\,(U_0\,\sigma_\pm - V_0)\,(V - V_0) = p - p_0, \quad (5)$$

and

$$p - p_0 = \frac{c^2(\rho_0)}{B}(\rho - \rho_0), \quad (6)$$

where

$$B := \frac{\gamma+1}{2} - \frac{\gamma-1}{2}\frac{\rho}{\rho_0} \quad \text{and} \quad \overline{c}^2 = \frac{\rho}{\rho_0}\frac{c^2(\rho_0)}{B}.$$

Similarly, a linear discontinuity $\eta = \eta(\xi)$ between states $\overline{U}_0$ and U satisfies $d\eta/d\xi = V/U = V_0/U_0$ and $p = p_0$.

For simplicity, we assume $\gamma = 2$. We consider Riemann data for system (1) consisting of two sectors:

$$\overline{U}(x, y, 0) = \begin{cases} \overline{U}_1, & \text{if } -ky < x < ky, \\ \overline{U}_0, & \text{otherwise}, \end{cases} \quad (7)$$

where $\overline{U}_0 := (\rho_0, u_0, 0, p_0)$ and $\overline{U}_1 := (\rho_1, 0, 0, p_1)$. We assume that $p_0 > p_1 > 0$ and $\rho_1 > 0$ are fixed, and that ρ_0 and u_0 are given by

$$\rho_0 = \frac{p_1 + 3p_0}{3p_1 + p_0}\rho_1 \quad (8)$$

and

$$u_0 = \sqrt{\frac{2(1+k^2)}{\rho_1}}\frac{p_0 - p_1}{\sqrt{p_1 + 3p_0}}. \quad (9)$$

The parameter $k > 0$ will be specified in the next section, depending on p_0 and p_1.

3 Conditions on the parameter k

In this section we find a solution to the Riemann problem (1), (7) in the hyperbolic part of the domain and we investigate how to choose the parameter k so that the problem leads to regular reflection.

Far from the origin, the one-dimensional Riemann problem along the line $x = ky$, $y > 0$, with states $\overline{U}_1$ and $\overline{U}_0$, on the left and on the right, results in a shock $S_1 : x = ky + wt$, connecting $\overline{U}_1$ to an intermediate state $\overline{U}_a = (\rho_a, u_a, v_a, p_a)$, and a linear wave $l_1 : x = ky + \alpha t$, connecting $\overline{U}_a$ to $\overline{U}_0$. We find

$$\rho_a = \rho_0, \quad u_a = \frac{1}{\sqrt{1+k^2}}\sqrt{\frac{2}{\rho_1}}\frac{p_0 - p_1}{\sqrt{p_1 + 3p_0}},$$

$$v_a = -k u_a, \quad p_a = p_0,$$

$$w = \sqrt{\frac{1+k^2}{2\rho_1}(p_1 + 3p_0)} \quad \text{and} \quad \alpha = u_0 < w.$$

Symmetrically, in the far field, the one-dimensional solution along the line $x = ky$, $y < 0$, with states $\overline{U}_0$ and $\overline{U}_1$ on the left and on the right, consists of a linear wave $l_2 : x = -ky + \alpha t$, between $\overline{U}_0$ and an intermediate state $\overline{U}_b = (\rho_a, u_a, -v_a, p_a)$, and a shock $S_2 : x = -ky + wt$, between $\overline{U}_b$ and $\overline{U}_1$.

We denote the sonic circles for the states $\overline{U}_0$, $\overline{U}_1$, $\overline{U}_a$ and $\overline{U}_b$ by C_0, C_1, C_a and C_b, respectively. Further, we denote the projected point of intersection of the shocks S_1 and S_2 by $\Xi_s := (\xi_s, \eta_s)$, and we note that $\xi_s = w$ and $\eta_s = 0$. Suppose that $\rho_1 > 0$ and $p_0 > p_1 > 0$ are fixed and let ρ_0 and u_0 be as in (8)-(9). We distinguish four regions depending on the value of the parameter k.

Region A: The value of k is such that the point Ξ_s is inside at least one of the circles C_1, C_a or C_b. In this case, shocks S_1 and S_2 do not intersect at ξ-axis and regular reflection does not occur. We claim that there exists a value k_A, depending on p_0 and p_1, such that for $k \in (0, k_A)$ we have $\Xi_s \in C_1 \cup C_a \cup C_b$.

Region B: The value of k is such that $\Xi_s \notin C_1 \cup C_a \cup C_b$. Therefore, Ξ_s is hyperbolic with respect to the states $\overline{U}_a$ and $\overline{U}_b$. We further assume that for this value of k the quasi-one-dimensional Riemann problem at Ξ_s with states $\overline{U}_b$ and $\overline{U}_a$, on the left and on the right, respectively, does not have a solution consisting of two shocks. We claim that there exists $k_C > 0$, depending on p_0 and p_1, such that this situation occurs if $k \in (k_A, k_C)$.

Note that the quasi-one-dimensional Riemann problem at point Ξ_s has a solution consisting of two shocks if the shock polars $S^+(\overline{U}_b)$ and $S^-(\overline{U}_a)$ intersect. We recall equations (3)-(6) and note that shock polars for state $\overline{U}_0 = (\rho_0, u_0, v_0, p_0)$ with respect to a point $\Xi_* = (\xi_*, \eta_*)$ can be parametrized as

$$p(\rho) = p(\rho_0) + \frac{c^2(\rho_0)}{B}(\rho - \rho_0),$$

$$v(\rho) = v(\rho_0) + \frac{p(\rho) - p(\rho_0)}{\rho_0((u_0 - \xi_*)\sigma_\pm - (v_0 - \eta_*))},$$

$$u(\rho) = u_0 - \sigma_\pm(v(\rho) - v_0),$$

where $\sigma_\pm$ is a function of ρ given by (3). Note that in our case $\xi_* = w$ and $\eta_* = 0$. We consider projections of shock polars $S^\pm(\overline{U}_b)$ and $S^\pm(\overline{U}_a)$ in (ρ, v)-plane.

Example 1. *Let the Riemann data* (7) *be given by* $\rho_1 = p_1 = 1$, $p_0 = 2$ *and* $k = 0.8$. *We find that the point* Ξ_s *is hyperbolic with respect to the states* $\overline{U}_a$ *and* $\overline{U}_b$, *however the projected shock polars* $S^+(\overline{U}_b)$ *and* $S^-(\overline{U}_a)$ *do not intersect (Fig. 1).*

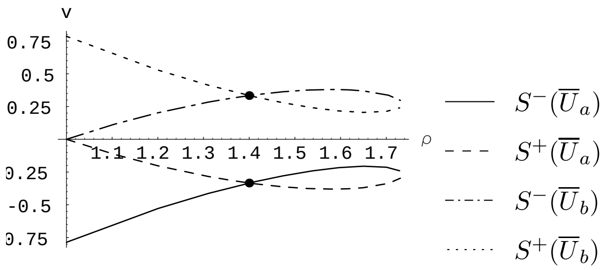

Figure 1: The projected shock polars $S^+(\overline{U}_b)$ and $S^-(\overline{U}_a)$ in (ρ, v)-plane do not intersect.

By symmetry, it is clear that $S^+(\overline{U}_b)$ and $S^-(\overline{U}_a)$ intersect at points where $v = 0$. From the above equations describing shock polars, we have that intersections of projected shock polars $S^+(\overline{U}_b)$ and $S^-(\overline{U}_a)$ in the (ρ, v)-plane correspond to intersections of polars in the (ρ, u, v, p)-space.

Example 2. *The initial data* (7) *is given by* $\rho_1 = p_1 = 1$, $p_0 = 2$ *and* $k = 1.1665$. *We find* $\overline{U}_a = (1.4, 0.34806, -0.40566, 2)$, $\overline{U}_b = (1.4, 0.34806, 0.40566, 2)$ *and* $\Xi_s = (2.87304, 0)$. *The point* Ξ_s *is hyperbolic with respect to states* $\overline{U}_a$ *and* $\overline{U}_b$, *and the shock polars* $S^+(\overline{U}_b)$ *and* $S^-(\overline{U}_a)$ *are tangent (Fig. 2). Their intersection is*

$$\overline{U}_R = (2.04, 0.95024, 0, 4.37037).$$

We also find that Ξ_s *is inside the sonic circle* C_R *for state* $\overline{U}_R$.

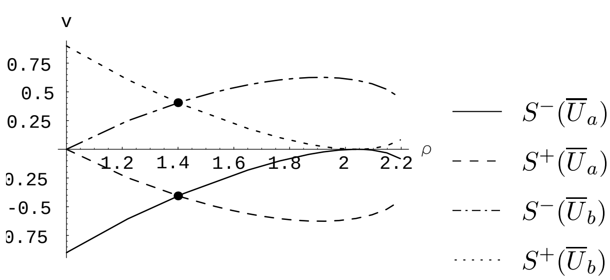

Figure 2: The projected shock polars $S^+(\overline{U}_b)$ and $S^-(\overline{U}_a)$ in (ρ, v)-plane are tangent.

Example 3. *We consider the Riemann initial data with* $\rho_1 = p_1 = 1$, $p_0 = 2$ *and* $k = 1.172$. *We find that the point* $\Xi_s = (2.88228, 0)$ *is hyperbolic with respect to states* $\overline{U}_a = (1.4, 0.34695, -0.40662, 2)$ *and* $\overline{U}_b = (1.4, 0.34695, 0.40662, 2)$. *Moreover, the shock polars* $S^+(\overline{U}_b)$ *and* $S^-(\overline{U}_a)$ *intersect at two points* $\overline{U}_R = (1.99748, 0.89314, 0, 4.17018)$ *and* $\overline{U}_F = (2.09186, 1.02142, 0, 4.4255)$ *(see Fig. 3). We further note that* Ξ_s *is subsonic with respect to both* $\overline{U}_R$ *and* $\overline{U}_F$.

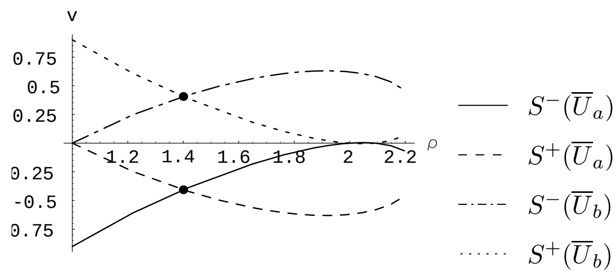

Figure 3: The projected shock polars $S^+(\overline{U}_b)$ and $S^-(\overline{U}_a)$ in (ρ, v)-plane.

Region C: We claim that for $\rho_1 > 0$, $p_0 > p_1 > 0$ and ρ_0 and u_0 as in (8)-(9), there exists k_*, depending on p_0 and p_1, such that for $k \in (k_C, k_*)$ we have
(a) $\Xi_s \notin C_1 \cup C_a \cup C_b$,
(b) the quasi-one-dimensional Riemann problem at the point Ξ_s has two solutions (each consisting of two shocks and an intermediate state), and
(c) Ξ_s is subsonic for both intermediate states in these two solutions.

Example 4. *Let the Riemann data* (7) *be given with* $\rho_1 = p_1 = 1$, $p_0 = 2$ *and* $k = 2$. *We note that the point* $\Xi_s = (4.1833, 0)$ *is supersonic with respect to the states* $\overline{U}_a = (1.4, 0.23905, -0.47809, 2)$ *and* $\overline{U}_b = (1.4, 0.23905, 0.47809, 2)$. *Further, the shock polars* $S^+(\overline{U}_b)$ *and* $S^-(\overline{U}_a)$ *intersect at two points* $\overline{U}_R = (1.86264, 0.46785, 0, 3.58345)$ *and* $\overline{U}_F = (3.07493, 2.33792, 0, 13.9099)$ *(see Fig. 4). We notice that* Ξ_s *is supersonic with respect to the state* $\overline{U}_R$ *and subsonic with respect to the state* $\overline{U}_F$.

Region D: We claim that, for $\rho_1 > 0$, $p_0 > p_1 > 0$ and ρ_0 and u_0 as in (8)-(9), if $k > k_*$, then
(a) $\Xi_s \notin C_1 \cup C_a \cup C_b$,
(b) the quasi-one-dimensional Riemann problem at the point Ξ_s has two solutions, each consisting of two shocks and an intermediate state (we denote these two intermediate states by $\overline{U}_R = (\rho_R, u_R, 0, p_R)$ and $\overline{U}_F = (\rho_F, u_F, 0, p_F)$ and assume $\rho_R < \rho_F$), and
(c) the point Ξ_s is supersonic with respect to the state $\overline{U}_R$ and subsonic with respect to the state $\overline{U}_F$.

We graph curves $k_A(p_0/p_1)$, $k_C(p_0/p_1)$ and $k_*(p_0/p_1)$, in the case $\rho_1 = 1$, in Fig. 5.

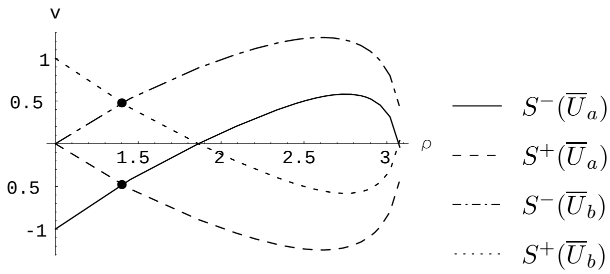

Figure 4: The projected shock polars $S^+(\overline{U}_b)$ and $S^-(\overline{U}_a)$ in (ρ, v)-plane.

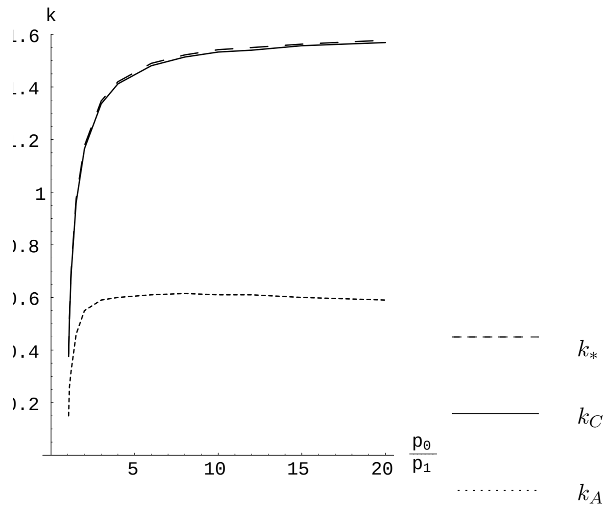

Figure 5: Curves k_A, k_C and k_*.

4 The free boundary problems

4.1 Strong regular reflection

We consider Riemann problem (1), (7) where $\rho_1 > 0$, $p_0 > p_1 > 0$ and $k \in (k_C, \infty)$. In this case the point Ξ_s of intersection of shocks S_1 and S_2 is outside of the sonic circles C_1, C_a and C_b, and the quasi-one-dimensional Riemann problem at Ξ_s with states $\overline{U}_b$ and $\overline{U}_a$, on the left and on the right, respectively, has two solutions, each consisting of two shocks (we call these shocks "reflected shocks"). We denote the intermediate states for these two solutions by $\overline{U}_R = (\rho_R, u_R, 0, p_R)$ and $\overline{U}_F = (\rho_F, u_F, 0, p_F)$, where $\rho_R < \rho_F$. Recall that the point Ξ_s is subsonic with respect to $\overline{U}_R$ if $k \in (k_C, k_*)$ and that Ξ_s is subsonic with respect to $\overline{U}_F$ for all $k \in (k_C, \infty)$. We assume that the value of our solution at the point Ξ_s is given by

$$\overline{U}_s := \overline{U}(\Xi_s) = \begin{cases} \overline{U}_R, & \text{if } k \in (k_C, k_*), \\ \overline{U}_F, & \text{if } k \in (k_*, \infty). \end{cases}$$

Hence, Ξ_s is subsonic with respect to the state $\overline{U}_s$ and the reflected shocks become transonic. Note that by causality they cannot exit the sonic circle C_s for the state $\overline{U}_s$ and, hence, they are curved. Since our configuration is symmetric, we will formulate the free boundary value problem in the upper half-plane. We denote the transonic shock by S_1' and we note that S_1' separates the constant states $\overline{U}_0$ and $\overline{U}_a$ from the nonuniform state $\overline{U} = (\rho, u, v, p)$ behind the reflected shock. Further, we define $\Sigma := \{(\xi, \eta(\xi) : \xi < \xi_s\}$ and $\Sigma_0 := \{(\xi, 0) : \xi < \xi_s\}$, and we denote the domain between Σ and Σ_0 by Ω.

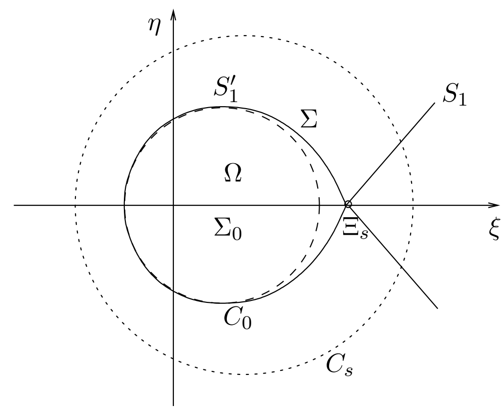

Figure 6: Strong regular reflection.

Formulation of the free boundary problem:
Given $\rho_1 > 0$, $p_0 > p_1 > 0$ and $k \in (k_C, \infty)$, find ρ, u, v, p such that

(a) system (2) holds in Ω,

(b) jump conditions (3)-(6) hold on $\Sigma : \eta = \eta(\xi)$,

(c) $\rho_\eta = u_\eta = p_\eta = v = 0$ hold on Σ_0, $\overline{U}(\Xi_s) = \overline{U}_s$ and $\eta(\xi_s) = 0$.

4.2 Weak regular reflection

We consider Riemann problem (1), (7) with $\rho_1 > 0$, $p_0 > p_1 > 0$ and assume $k \in (k_*, \infty)$. Hence, the point Ξ_s is supersonic with respect to the states $\overline{U}_a$ and $\overline{U}_b$ and, moreover, the quasi-one-dimensional Riemann problem at Ξ_s has two solutions with intermediate states $\overline{U}_R$ and $\overline{U}_F$, where $\rho_R < \rho_F$. We assume that the solution at the point Ξ_s is given by $\overline{U}_s := \overline{U}(\Xi_s) = \overline{U}_R$. Hence, the point Ξ_s is supersonic with respect to $\overline{U}_s$. Again, because our configuration is symmetric, we will pose the free boundary value problem in the upper half-plane. We denote the reflected shock by S_1' and note that it is rectilinear in a finite neighborhood of the point Ξ_s separating constant states $\overline{U}_a$ and $\overline{U}_R$. Let us denote its intersection with the sonic circle C_s for the state

$\overline{U}_s$ by $\Xi_R = (\xi_R, \eta_R)$, and further let us denote intersection of C_s and the ξ-axis by $\Xi'_R = (\xi'_R, 0)$. The part of the reflected shock inside the sonic circle C_s is transonic and curved. Let us denote its equation by $\eta = \eta(\xi)$ and let us note that S'_1 separates the constant states $\overline{U}_0$ and $\overline{U}_a$ from the nonuniform state $\overline{U} = (\rho, u, v, p)$ behind S'_1. We further denote closed part of the sonic circle C_s between points Ξ_R and Ξ'_R by σ_0, $\Sigma := \{(\xi, \eta(\xi)) : \xi < \xi_R\}$, $\Sigma_0 := \{(\xi, 0) : \xi < \xi'_R\}$ and the domain between Σ_0, Σ and σ_0 by Ω.

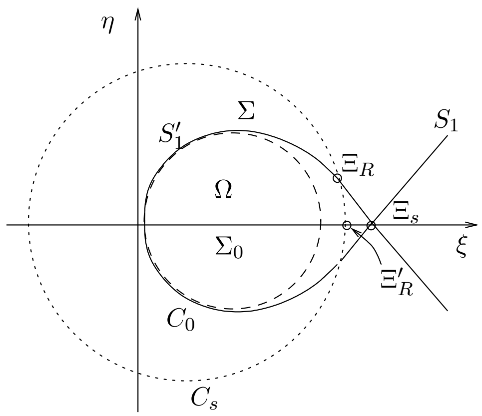

Figure 7: Weak regular reflection.

Formulation of the free boundary problem:
Given $\rho_1 > 0$, $p_0 > p_1 > 0$ and $k \in (k_*, \infty)$, find ρ, u, v, p such that

(a) system (2) holds in Ω,

(b) jump relations (3)-(6) hold on $\Sigma : \eta = \eta(\xi)$,

(c) $\rho_\eta = u_\eta = p_\eta = v = 0$ hold on Σ_0, $\overline{U} = \overline{U}_s$ holds on σ_0 and $\eta(\xi'_R) = \eta'_R$.

5 Numerical examples

We consider system (1) with $\gamma = 2$ and the Riemann data consisting of two sectors:

$$\overline{U}(x, y, 0) = \begin{cases} \overline{U}_1 & \text{in the first quadrant,} \\ \overline{U}_0 & \text{otherwise,} \end{cases}$$

where $\overline{U}_0 = (\rho_0, u_0, v_0, p_0)$ and $\overline{U}_1 = (\rho_1, 0, 0, p_1)$. We assume $\rho_1, p_1 > 0$, we take

$$\rho_0 = \frac{p_1 + 3p_0}{3p_1 + p_0}\rho_1, \quad u_0 = v_0 = \sqrt{\frac{2}{\rho_1}\frac{p_0 - p_1}{\sqrt{p_1 + 3p_0}}},$$

and we choose p_0 so that regular reflection occurs. Each of the two initial discontinuities results in a shock and a linear wave. The numerical solutions below are found using the CLAWPACK software (http://www.amath.washington.edu/ claw/).

Example 5. *Let $\overline{U}_1 = (1, 0, 0, 1)$ and $p_0 = 1.55$. Therefore, $\rho_0 = 1.24176$ and $u_0 = v_0 = 0.32723$. We find that the shock polars $S^+(\overline{U}_b)$ and $S^-(\overline{U}_a)$ are tangent, intersecting at the point $\overline{U}_R$. Further, Ξ_s is subsonic with respect to the state $\overline{U}_R$ and we have strong regular reflection (Fig. 8).*

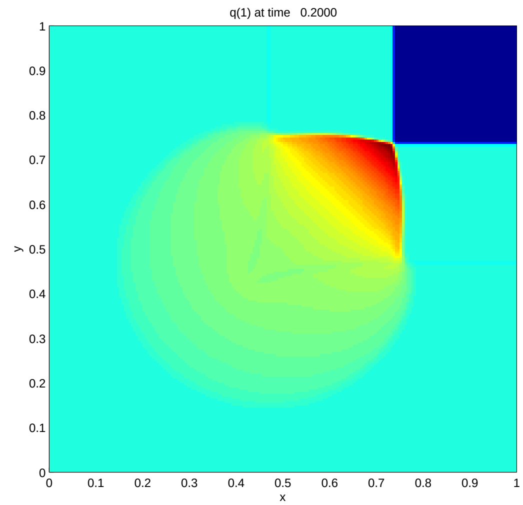

Figure 8: Pressure at $t = 0.2$.

Example 6. *Let $\overline{U}_1 = (1, 0, 0, 1)$ and let $p_0 = 1.2$. We find $\rho_0 = 1.09524$ and $u_0 = v_0 = 0.131876$. Shock polars through states $\overline{U}_a$ and $\overline{U}_b$ intersect at two points, $\overline{U}_R$ and $\overline{U}_F$, with $\rho_R < \rho_F$. We note that Ξ_s is supersonic with respect to the state $\overline{U}_R$. The corresponding solution results in a weak regular reflection (Fig. 9).*

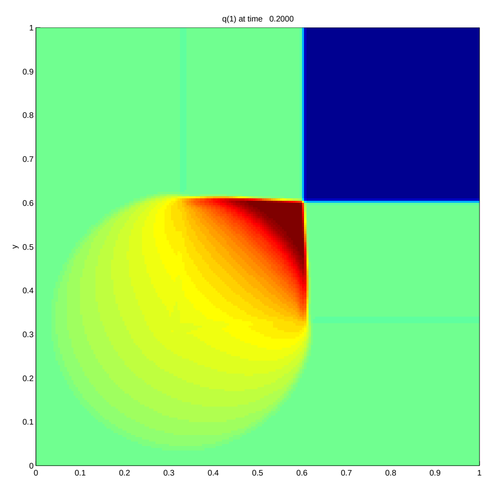

Figure 9: Pressure at $t = 0.2$.

6 Conclusion

In this paper we set up the stage for analysis of two-dimensional Riemann problems for gas dynamics equations using the approach by Čanić, Keyfitz, Kim and Lieeberman. We derive regimes in which regular reflection occurs and we pose free boundary problems for the subsonic state and the reflected shock. Our numerical results confirm the expected structure of solution which is analogous to studies in [1, 3, 4, 12, 13].

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