Many-Valued Logic in an Intelligent Tutoring System

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Abstract: This paper describes relations between a trilattice and corresponding meet-distributive lattices. The three meet-distributive lattices illustrate the five information levels, five logical levels and five levels of constructivity respectively. While the trilattice shows connections among the sexteen truth values in general, the three meet-distributive lattices visualize specific information about the sexteen truth values with respect to information, logic and constructivity.

Key–Words: sixteen-valued logic, meet-distributive lattices

1 Introduction

Intelligent tutoring systems apply a variate of methods for automated grading of students' knowledge. Most systems are forsing the student to point the correct answer (if the system suggests exactly one correct answer), to find all the correct answer (if the system suggests several correct answers), or to provide a solution (if the system suggests open ended questions). If a question is unanswered than it is treated as a wrongly answered question with respect to both grading and providing further guidance and help. Some systems applying negative marking and assign a zero to an unanswered question.

In this paper we propose use of sixteen-valued logic for dealing with incomplete and inconsistent information. The five information levels, five logical levels and five levels of constructivity in trilattice of sixteen truth values are arranged in three meet-distributive lattices. One of the interesting observations of involves computation of '*a sentence is constructively refuted* $\wedge$ *a sentence is rejectable*'. The conclusion is based on the rule that a conjunction is true if and only if both conjuncts are true. Since there is no truth value such that both *a sentence is constructively refuted* and *a sentence is rejectable* have it, the result is the empty set.

The rest of the paper is organized as follows. Related work and statements from many-valued logic are discussed in Section 2 and Section 3 respectively. The main results of the paper are placed in Section 4. The paper ends with a conclusion in Section 5.

2 Related Work

Inspired by the Aristotle writing on propositions about the future - namely those about events that are not already predetermined, Lukasiewicz devised a three-valued calculus whose third value, $\frac{1}{2}$, attached to propositions referring to future contingencies [8]. The meaning of the third truth value can be 'intermediate' or 'neutral' or 'indeterminate' [13].

Another three-valued logic, known as Kleene's logic is developed in [7] and has three truth values, truth, unknown and false, where unknown indicates a state of partial vagueness. These truth values represent the states of a world that does not change.

Two kinds of negation, weak and strong negation are discussed in [15]. Weak negation or negation-as-failure refers to cases when it cannot be proved that a sentence is true. Strong negation or constructable falsity is used when the falsity of a sentence is directly established.

The semantic characterization of a four-valued logic for expressing practical deductive processes is presented in [1]. In most information systems the management of databases is not considered to include neither explicit nor hidden inconsistencies. In real life situation information often come from different contradicting sources. Thus different sources can provide inconsistent data while deductive reasoning may result in hidden inconsistencies. The idea in Belnap's approach is to develop a logic that is not that dependable of inconsistencies. The Belnap's logic has four truth values 'T, F, Both, None'. The meaning of these values can be described as follows:

- an atomic sentence is stated to be true only (T)

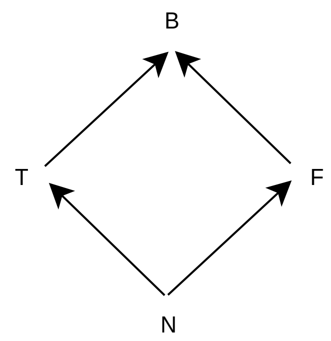

Figure 1: Approximation lattice

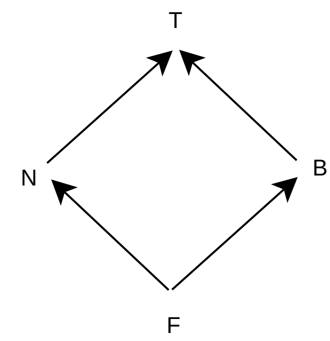

Figure 2: Logical lattice

- an atomic sentence is stated to be false only (F)
- an atomic sentence is stated to be both true and false, for instance, by different sources, or in different points of time (Both), and
- an atomic sentences status is unknown. That is, neither true, nor false (None).

Sixteen generalized truth values obtained as a power set of the initial truth values in Belnap's logic are arranged in a trilattice [12].

3 Preliminaries

Let P be a non-empty ordered set. If $sup\{x, y\}$ and $inf\{x, y\}$ exist for all $x, y \in P$, then P is called a *lattice* [3].

A complete lattice is a partially ordered set in which all subsets have both a supremum (join) and an infimum (meet). A lattice L is meet-distributive if every coatomic interval is Boolean [4].

A billatice is a set equipped with two partial orderings $\leq_t$ and $\leq_k$. The t partial ordering $\leq_t$ means that if two truth values a, b are related as $a \leq_t b$ then b is at least as true a. The k partial ordering $\leq_k$ means that if two truth values a, b are related as $a \leq_k b$ then b labels a sentence about which we have more knowledge than a sentence labeled with a.

The four truth values in Belnap's logic are elements of an approximation lattice [2] in Fig. 1. The information about the truth-value of a sentence can have values from None to Both.

The four truth values are arranged in a logical lattice [2] in Fig. 2. A logical conjunction and logical disjunction are related to the meet operation and to the join operation respectively.

4 Meet-Distributive Lattices

The generalized constructive truth-value space has as a base a set $\mathcal{I} = \langle T, F, t, f \rangle$ containing the initial truth values

- *T* - a sentence is constructively proven
- *F* - a sentence is constructively refuted
- *t* - a sentence is acceptable
- *f* - a sentence is rejectable

The power set of $\mathcal{I}$ gives sixteen generalized truth values. The empty multivalue is denoted by N and A represents the set that contains the initial truth values T, F, t, f.

$\mathcal{P}(\mathcal{I})$ = [{ },{ T }, { F },{ t}, { f}, { T, F }, { T, t}, { T, f}, { F, t}, { F, f}, { t, f}, { T, F , t}, { T, F , f}, { T, t , f}, { F, t , f}, { T, F , t, f}]

The truth table for the 16 truth values is Table 2. A trilattice is a structure $\mathcal{T} = (S, \leq_i, \leq_t, \leq_c)$ such that S is a nonempty set and $(S, \leq_i)$, $(S, \leq_t)$ and $(S, \leq_c)$ are complete lattices. The three partial orderings $\leq_i, \leq_t, \leq_c$ arrange elements according to the possessed degree of information, truth and constructivity respectively. The bounds relative to the three partial orderings are shown in Table 1. Accomplished constructions (proofs and disproofs) are presented by constructive values. Non-constructive truth values do not imply any completed construction.

The lattice on Fig. 3 has five information levels, five logical levels and five levels of constructivity shown in Table 3, [12]. The sixteen truth values from the five information levels are arranged in a meet-distributive lattice Fig. 4. In such a lattice the meet of two elements of $\mathcal{P}(\mathcal{I})$ illustrates the relations between these elements that can be seen in the trilattice Fig. 3 with the advantage that if there is a meaning like $F \wedge f$ is N for two elements than this meaning is valid for all the elements at that level in Fig. 4.

Relative ordering	Bounds	Elements in $\mathcal{P}(\mathcal{I})$ being most and least
$\leq_i$	A, N	informative
$\leq_t$	Tt, Ff	true
$\leq_c$	TF, tf	constructive

Table 1: The bounds relative to the three partial orderings

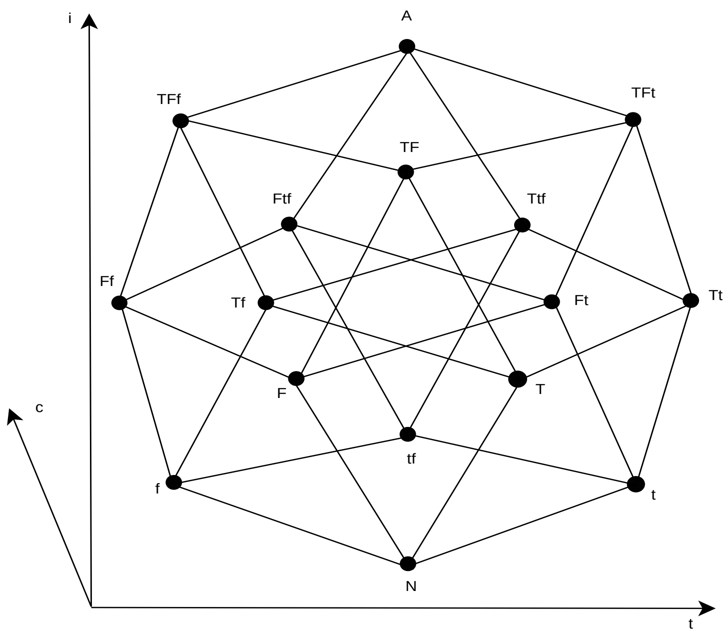

Figure 3: Trilattice (projection i-t)

	T	F	t	f	TF	Tt	Tf	Ft	Ff	tf	TFt	TFf	Ttf	Ftf	TFtf	N
T	T	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
F	N	F	N	N	N	N	N	N	N	N	N	N	N	N	N	N
t	N	N	t	N	N	N	N	N	N	N	N	N	N	N	N	N
f	N	N	N	f	N	N	N	N	N	N	N	N	N	N	N	N
TF	N	N	N	N	TF	T	T	F	F	N	T	F	T	F	N	N
Tt	N	N	N	N	T	Tt	T	t	N	t	t	T	T	t	t	N
Tf	N	N	N	N	T	T	Tf	N	f	f	T	f	T	f	T	N
Ft	N	N	N	N	F	T	N	Ft	F	N	t	t	t	F	F	N
Ff	N	N	N	N	F	N	f	F	Ff	f	F	f	f	f	f	N
tf	N	N	N	N	N	t	f	N	f	tf	t	f	t	f	N	N
TFt	N	N	N	N	T	t	T	t	F	t	TFt	TF	Tt	N	Ft	N
TFf	N	N	N	N	F	T	f	t	f	f	TF	TFf	Tf	Ff	Ff	N
Ttf	N	N	N	N	T	T	T	t	f	t	Tt	Tf	Ttf	ft	ft	N
Ftf	N	N	N	N	F	t	f	F	f	f	N	Ff	tf	Fft	ft	N
TFtf	N	N	N	N	N	t	T	F	f	N	Ft	Ff	tf	tf	TFtf	N
N	N	N	N	N		N	N	N	N	N	N	N	N	N	N	N

Table 2: Truth table

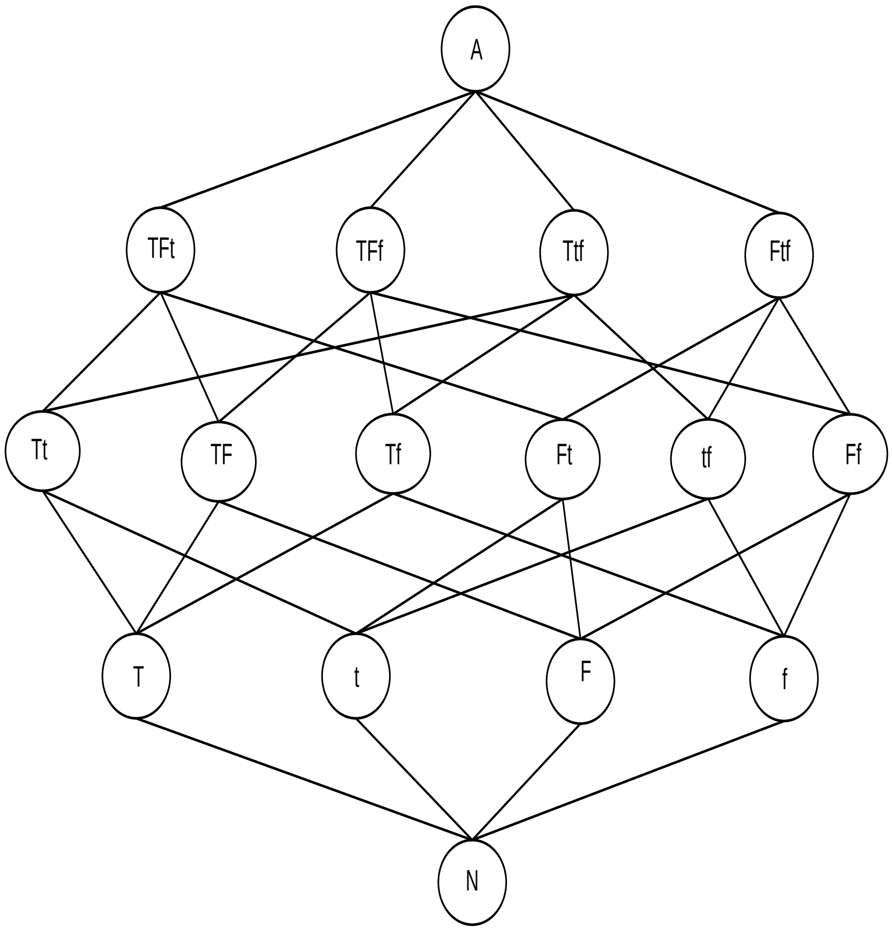

Figure 4: Information meet-distributive lattice

	Information	Truth	Constructivity
1	N	Ff	tf
2	T, F , t, f	F, f, TFf, Ftf	t, f, Ttf, Ftf
3	TF, Tt, Tf, Ft, Ff, tf	A, TF, Tf, Ft, ft, N	A, Tt, Tf, Ft, Ff, N
4	TFt, TFf, Ttf, Ftf	T, t, TFt, Ttf	T, F, TFt, TFf
5	A	Tt	TF

Table 3: Levels

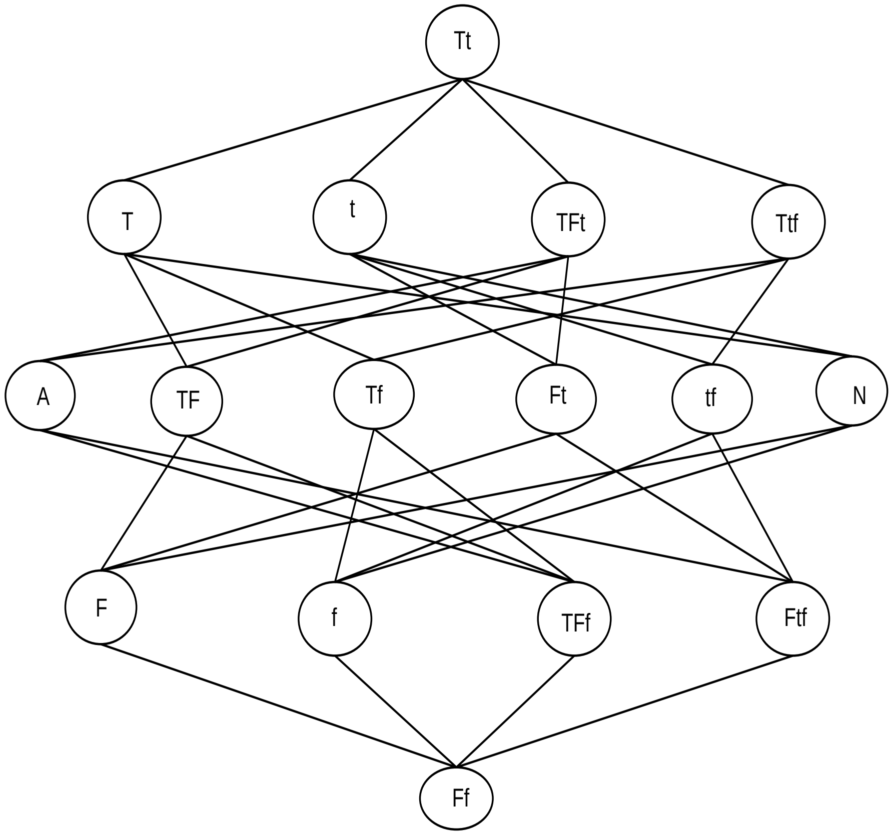

Figure 5: Truth meet-distributive lattice

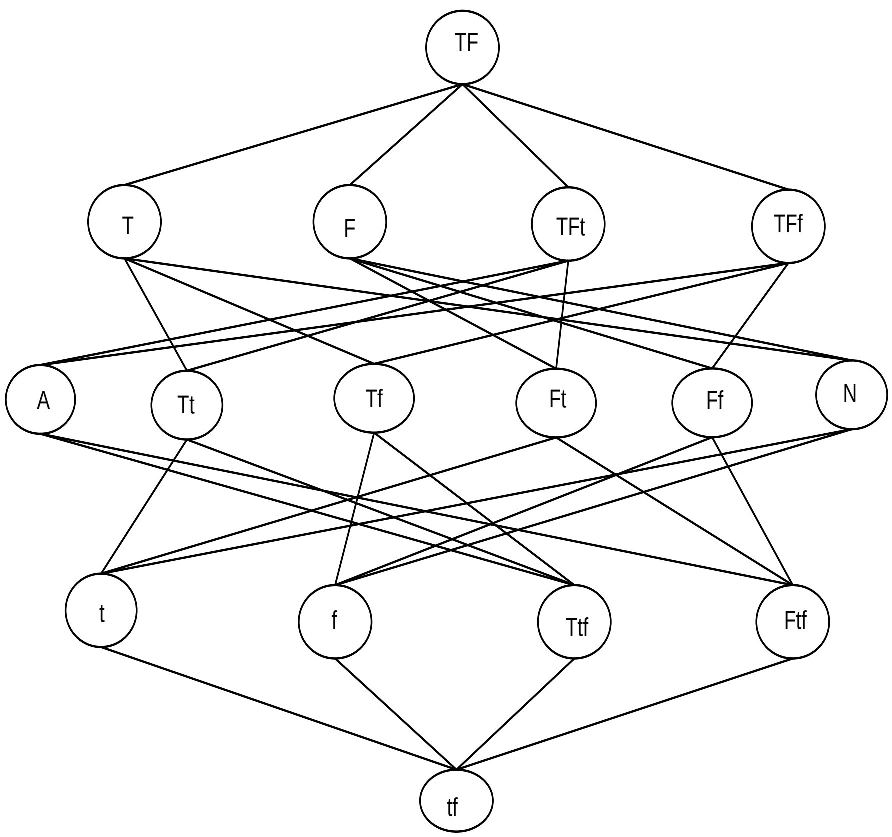

Figure 6: Constructivity meet-distributive lattice

5 Conclusion

In this paper we propose use of sixteen-valued logic for dealing with incomplete and inconsistent information in intelligent tutoring systems. The five information levels, five logical levels and five levels of constructivity in trilattice of sixteen truth values are arranged in three meet-distributive lattices.

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