An Investigation of Wave Propagation over Irregular Terrain and Urban Streets using Finite Elements

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Abstract: A method to model electromagnetic wave propagation in troposphere on irregular terrain in the presence of height dependant refractivity is presented using finite element analysis. In this work, the helmholtz equation applied on radiowave propagation properly manipulated and simplified using pade approximation is solved using finite element method. Paraxial form of helmholtz equation is also called wide angle formulation of parabolic equation (WPEM) and is used because of its accuracy and behavior on large propagation angles. By using this method, horizontal and vertical tropospheric characteristics are assigned to every element, and different refractivity and terrain profiles can be entered at different stages. We also consider wave propagation on an urban street to demonstrate the effectiveness of our method.

Key–Words: Troposphere, finite element method, parabolic equation method.

1 Introduction

Radio coverage in the troposphere and urban areas has been a challenging problem for many years [1]. Researchers in wave propagation area have been searching for efficient mathematical models for describing the problem of electromagnetic wave propagation in troposphere. Numerous methods are available for predicting electromagnetic wave propagation in the atmosphere [2]. However, the presence of vertical refractivity stratification in the atmosphere complicates the application of some methods. To model refractivity variations in the horizontal as well as vertical direction, geometric optics, coupled-mode analysis, or hybrid methods have been employed [3].

In the past, emphasis was given to geometrical optics (GO) techniques and modal analysis. GO provide a general geometrical description of ray families, propagating through the atmosphere. Ray tracing methods present many disadvantages; for example the radiowave frequency is not accounted for and it is not always clear whether the ray is trapped by the specific duct structure [4]. An alternative approach for tropospheric propagation modelling was developed by Baumgartner [5] and later extended [6], normally known as Waveguide Model or Coupled Mode Technique. The main disadvantage of coupled mode technique lie in the complexity of the root finding algorithms and large computational demands, especially when higher frequencies and complicated ducting profiles are involved.

The solution of electromagnetic propagation problems in the presence of an irregular terrain is a complicated matter. Three-dimensional variations in refraction and terrain make the full vector problem extremely difficult to solve in a reasonable time. Many existing prediction models are based on simplified Deygout solution for multiple knife-edge diffraction [7]. An important class of propagation models over irregular terrain is based on integral equation formulation which in general can be simplified by using paraxial approximation [8]. If one chooses to simplify the problem by assuming symmetry in one or more of the coordinate directions, the vector problem can be decoupled into scalar problems.However the solution of two dimensional scalar problem is still difficult for realistic environments. Some approximations and numerical schemes for the solution are used to reduce the solution of the full two-way equation to one-way equation. Benefits of one-way propagation are the simple numerical implementation of range dependencies in the medium and the avoidance of prohibitive numerical aspects of solving elliptic equations associated with implementing two range-dependant boundary conditions.

One of the most reliable and widely used technique in literature is parabolic equation (PE) method, initially developed for the study of underwater acoustic problems and later on extended to tropospheric propagation ones [9]. The PE is based on the solution of the two dimensional differential parabolic equation, fitted by homogenous or inhomogeneous refractive profiles. Models based on the parabolic approximation of the wave equation have been used extensively for modelling refractive effects on tropospheric propagation [10, 11], in last decade. The biggest advantage to using the PE method is that it gives a full-wave solution for the field in the presence of range-dependent environments. Solution of PE in complex environments require some numerical method like finite element method.

In this work wide angle parabolic equation is used to model radio wave propagation over irregular terrain in troposphere and on urban streets using finite element method (FEM). Parabolic equation in wide angle formulation is used to make sure that every single ray should be accounted in the solution. The main advantage of using WPEM over irregular terrain is that it combines the effects of both terrain diffraction and atmospheric refraction while remaining straightforward to implement [12]. The FEM has been applied previously to model wave propagation over smooth ground [13]. In the present work, we extend this approach to the solution of wide angle parabolic equation in the presence of irregular terrain and on urban streets. In this paper, helmholtz equation is properly manipulated and simplified using pade approximations [14], the solution of which is achieved by using finite element method. Vertical tropospheric profile characteristics are assigned to every mesh element, while solution advances in small variable range steps, each excited by solution of the previous step.

The remainder of the paper is organized as follows. In section 2, wave propagation modelling using paraxial approximation of wave equation is described along with boundary conditions and initial field. Section 3 describes finite element formulation of the problem. Results and discussions are given in section 4 and section 5 concludes the paper.

2 Wave propagation modelling by paraxial form of helmholtz equation

The paraxial form of Helmholtz equation is:

$$\frac{\partial^2 u(x,z)}{\partial x^2} + j2k_0\frac{\partial u(x,z)}{\partial x} + \frac{\partial^2 u(x,z)}{\partial z^2} + k_0^2\left(n^2(x,z)-1\right)u(x,z) = 0 \quad (1)$$

where k_0 is the free space wave number, $u(x,z)$ is the unknown electric or magnetic field depending on polarization, $n(x,z)$ is the refractive index of troposphere, x is the propagation direction and z is the transverse direction. Equation (1) is also called wide angel representation of parabolic equation method. In long-distance propagation scenarios the effect of earth's curvature must be considered. In earth flattening transformation refractive index is replaced with modified refractive index $m(x,z)$ [2]:

$$m(x,z) = n(x,z)\left(1+\frac{z}{R}\right) \quad (2)$$

where R is the radius of earth. After substituting equation (2) in (1), it becomes,

$$\frac{\partial^2 u(x,z)}{\partial x^2} + j2k_0\frac{\partial u(x,z)}{\partial x} + \frac{\partial^2 u(x,z)}{\partial z^2} + k_0^2\left(n^2(x,z)-1+\frac{2z}{R}\right)u(x,z) = 0 \quad (3)$$

Wave propagation model based on WPEM as defined in equation (3) is subject to terrain boundary condition, which represents the relationship that must hold between the field $u(x,z)$ and terrain. Note from equation (3) that the earth curvature enters only through the $\frac{2z}{R}$ term; if this term is ignored, the equation describes propagation over a flat earth.

2.1 Boundary and Initial Conditions

In two dimensional wave propagation problem, there are two boundaries, one at the starting height, $z = z_{min}$ which in fact is the earth surface at some height depending on terrain profile, and at the maximum altitude considered, $z = z_{max}$. An impedance type boundary condition can be used at lower heights to account for the finite conductivity and permittivity of the surface of the earth. The entrance boundary conditions are expressed by the equation [9]:

$$\left[\frac{\partial u(x,z)}{\partial z} + jk_0 q u(x,z)\right]_{z=z_{max}} = 0 \quad (4)$$

where,

$$q = q_v = \frac{jk_0}{\sqrt{\varepsilon_r - j60\sigma\lambda}} \quad (5)$$

$$q = q_h = jk_0\sqrt{\varepsilon_r - j60\sigma\lambda} \quad (6)$$

for vertical and polarization respectively. ε_r is the complex relative permittivity and σ is the conductivity of ground whereas λ is the wavelength of radio waves in meters.

When numerical propagation simulations are performed, infinite propagation domains cannot be realized and the size of the propagation domain must be truncated. This is accomplished numerically by implementing absorbing boundary conditions or Perfectly Matched Layer (PML) on the upper boundary at $z = z_{max}$.

For initial field a normalised gaussian antenna pattern is used:

$$u(0, z) = [A(z - H_0) - \bar{A}(z + H_0)] \tag{7}$$

where H_0 represents the antenna height and bar denotes complex conjugate. The Fourier transform of $A(z)$ is given as,

$$a(\zeta) = e^{-\zeta^2 w^2/4} \tag{8}$$

where w is defined as,

$$w = \frac{\sqrt{2 \ln 2}}{k_0 \sin \frac{\theta_{bw}}{2}} \tag{9}$$

where $\sin \frac{\theta_{bw}}{2}$ is the sine of 3dB beamwidth.

3 Finite element formulation problem

In this section finite element formulation of the model is developed. Apply the finite element method over the domain $z_{min} \le z \le z_{max}$, choose an interpolation function and than by using weak formulation of Galerkin method, matrix equation for the system can be defined as,

$$[M]\frac{\partial^2\{u\}}{\partial x^2} + j2k_0[M]\frac{\partial\{u\}}{\partial x} + [K]\{u\} - k_0^2[M]\{u\} = \{0\} \tag{10}$$

where matrices $[M]$ and $[K]$ are defined as,

$$[M] = \sum_e \int_{\hat{z}} \{N\}^T \{N\}\, d\hat{z} \tag{11}$$

and,

$$[K] = \sum_e \int_{\hat{z}} k_0^2 \left[n^2 + \frac{2z}{R}\right] d\hat{z} - \sum_e \int_{\hat{z}} \{N\}^T \{N\}\, d\hat{z} - \sum_e \int_{\hat{z}} \{\acute{N}\}^T \{\acute{N}\}\, d\hat{z} \tag{12}$$

and $\{N\}$ is the shape function matrix, $\{\acute{N}\} = \frac{\partial\{N\}}{\partial z}$, superscript T denotes transpose function and $\{0\}$ is the zero matrix. Now, rewrite equation (10) as,

$$j2k_0[M]\frac{\partial\{u\}}{\partial x} = -\frac{[K] - k_0^2[M]}{1 + \frac{1}{j2k_0}\frac{\partial}{\partial x}}\{u\} \tag{13}$$

Using Pade approximation [14],

$$\frac{\partial}{\partial x} \approx \frac{1}{j2k_0}\left\{[K] - k_0^2[M]\right\} \tag{14}$$

Substituting equation (14) in (13) and after simplification it gives,

$$-j2k_0\Big([M] - \frac{[K] - k_0^2[M]}{4k_0^2}\Big)\frac{\partial\{u\}}{\partial x} = \Big([K] - k_0^2[M]\Big)\{u\} \tag{15}$$

Define,

$$[\tilde{M}] = [M] - \frac{[K] - k_0^2[M]}{4k_0^2}$$

equation (15) will becomes,

$$-j2k_0[\tilde{M}]\frac{\partial\{u\}}{\partial x} = [K]\{u\} - k_0^2[M]\{u\} \tag{16}$$

So at range step $x + \frac{\Delta x}{2}$ equation (16) can be written as,

$$-j2k_0[\tilde{M}]\frac{\partial\{u\}}{\partial x}\Big|_{x+\frac{\Delta x}{2}} = \Big\{[K] - k_0^2[M]\Big\}\{u\}^{x+\frac{\Delta x}{2}} \tag{17}$$

Using crank nicolson approximation scheme,

$$-j2k_0[\tilde{M}]\frac{\{u\}^{x+\Delta x} - \{u\}^x}{\Delta x} = \Big([K] - k_0^2[M]\Big)\Big[\frac{\{u\}^{x+\Delta x} + \{u\}^x}{2}\Big] \tag{18}$$

yields final propagation algorithm as,

$$\Big\{-j4k_0[\tilde{M}] - \Delta x\big\{[K] - k_0^2[M]\big\}\Big\}\{u\}^{x+\Delta x} = \Big\{-j4k_0[\tilde{M}] + \Delta x\big\{[K] - k_0^2[M]\big\}\Big\}\{u\}^x \tag{19}$$

4 Results and Discussions

In this section, results for wave propagation in troposphere, over irregular terrain and urban streets will be presented. Coverage diagrams at different frequencies and under different environment conditions will be presented. Initially wave propagation in troposphere over smooth terrain is described along with some ducting conditions and than results of propagation over irregular terrain will be shown. Accurate modelling of radio waves over irregular terrain and especially in urban environment is crucial for the planning of cellular communications in cities.

To demonstrate the efficacy of the present approach a number of simulations were carried out with various frequencies and media profiles. In all of the simulations the following assumptions were made until stated otherwise. A transmitting antenna of height $H_0 = 150$m above sea level with a 3dB-beam width $\theta_{\text{bw}} = 10$ and vertical polarization of the propagating wave is assume. Boundary conditions and initial field generation is described in section **??** and **??**. Simulation parameters use in this paper are summarized in table 1.

Parameter	Value(s)
Tx Antenna Height	150m
Tx Antenna Gain	43.5dB
Rx Antenna Height	150m
Antenna 3dB bandwidth	10°
Ground Conductivity	0.01mho/m
Ground Permittivity	15
PML zone	300m
Tropospheric Duct Height	300
Antenna Polarization	HPOL or VPOL
Frequency of Waves	100MHz, 1GHz

Table 1: Simulation Parameters used for wave propagation in troposphere

For all simulations finite elements of length $\lambda/10$ has been choosen. Different values of range step Δx somewhere in between 25 and 180m has taken. If a smaller Δx is use the computation takes longer and more computational resources will be needed. For large Δx, the computation is accelerated, but some significant atmosphere changes may miss and more error will incur in simulation results. Therefore, Δx should be optimally choosen accordingly for each problem. However, it is possible to choose Δx and element size having variable lengths in one simulation.

Finite element formulation of parabolic equation method using different terrain profiles is simple and straightforward. As in the case of refractivity profiles, at each range step Δx program requires a terrain profile as a function of height. However if terrain or refractivity profile is not changing along range than a single profile is sufficient. Terrain profile is entered in much the same way as refractivity profiles. All what is required is the series of data points corresponding to height versus range to describe terrain. The simplest and most effective technique models terrain as a sequence of horizontal steps, and called staircase method. In this paper we use staircase method to model terrain, however other approaches are discussed in detail in [2].

Atmospheric conditions can be entered into the simulation program as profiles of refractivity-height data where refractivity N is related with refractive index n as $N = (n - 1) \times 10^6$ [2]. In cases where horizontally inhomogeneous conditions needs to be modelled, different refractivity profiles can be entered at several ranges, and program can perform linear interpolation in range and height for use at intermediate calculation position.

Modelling ground wave propagation for upward propagated waves is a challenging signal processing task. As we discussed earlier in section **??** the upper boundary should be truncated for proper termination of grid or domain. This can be accomplished numerically by implementing an absorbing boundary condition on the upper boundary or using perfectly matched layer (PML) as described in section **??**. Effective absorption within PML block depends on number of PML layers and artificial PML medium parameters. Figure 1 and 2 presents the coverage diagram of transmitting antenna at 100MHz for standard atmospheric conditions without and with PML. As can be seen from figure 1 without the implementation of PML the waves get reflected from the upper boundary. In figure 2 when a perfectly matched layer (PML) boundary is implemented these reflections are eliminated.

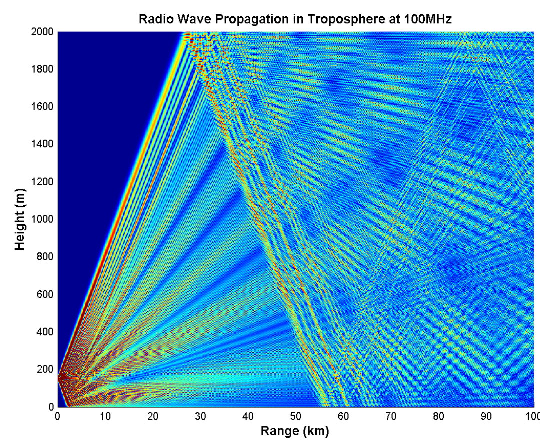

Figure 1: Coverage diagram with out PML at 100MHz

It can be seen easily that waves propagates undisturbed through the tropospheric medium under normal atmospheric conditions. In figures 3, a bilinear surface ducting profile is included at 100MHz, starting from the sea level to an altitude of $300m$. Standard atmospheric conditions over this altitude were also assumed. Figures 4 and 5 shows trapping mechanism at 1GHz and 3GHz respectively. From the results, it is clear that a duct of sufficient intensity is capable of capturing whole energy at 3GHz. At 100MHz duct is unable to divert waves but at 3GHz waves are almost completely trapped between the sea level and duct height.

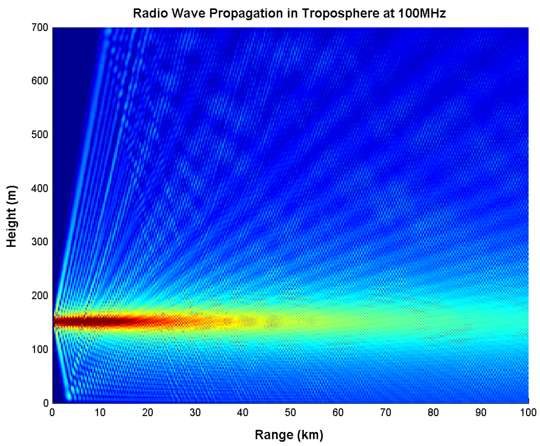

Figure 2: Coverage diagram with PML at 100MHz

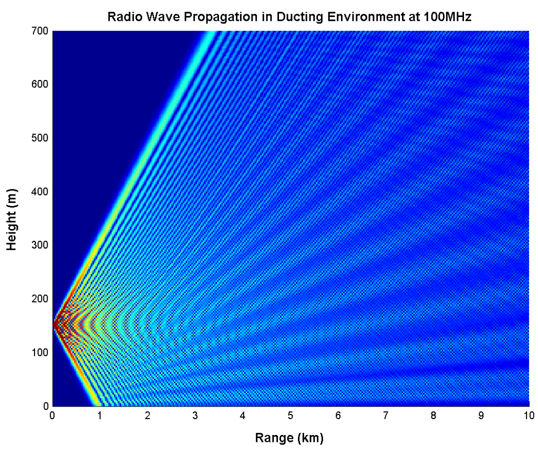

Figure 3: Coverage Diagrams at different duct intensity profiles at 100MHz

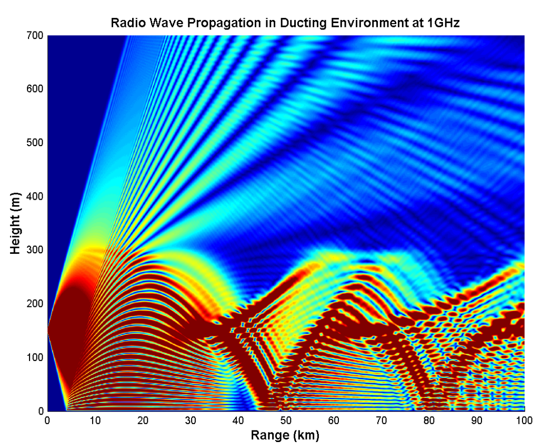

Figure 4: Coverage Diagrams at low duct intensity profiles at 1GHz

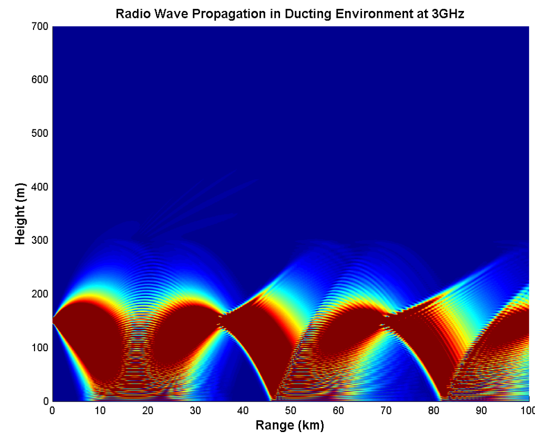

Figure 5: Coverage Diagrams at low duct intensity profiles at 3GHz

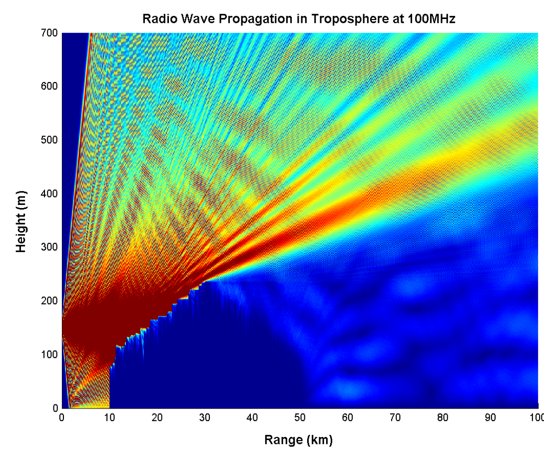

Figure 6: Coverage Diagrams at 100MHz on an irregular terrain

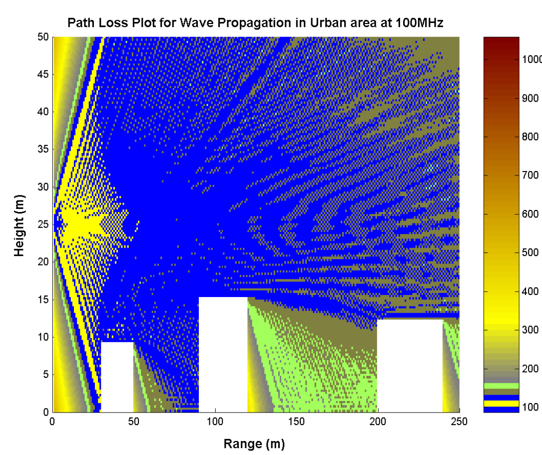

Figure 7: Path Loss in an urban street at 100MHz

In generating results for terrain modelling, two different scenarios are choosen. In first scenario (figure 6), an irregular terrain is assumed while in second case (7), street in an urban area is considered. Three buildings of arbitrarily heights are placed on the street and span of street is assumed to be 50m. Figure 6 shows coverage diagram in the presence of irregular terrain generated by using *rand* function in matlab.

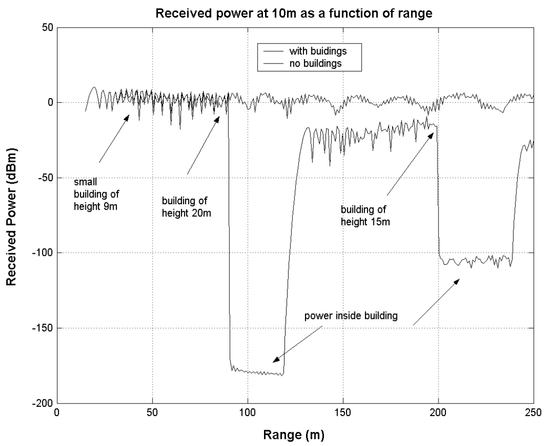

Figure 8: Received Power in an urban street at 100MHz

Figure 7 shows paath loss contour in an urban street. For urban environment radiating antenna is assumed to radiate at 25m while receiving antenna is at 10m. Three buildings of different heights and widths are assumed along the street. For the sake of simplicity it has been assumed that the building surfaces are perfectly conducting.

Figure 8 shows the received power at receiver height in dBm. Diffraction of waves along the edges of building is quite clear in figure 7. Buildings of variable heights are choosen to show the effect of shadowing as seen in figure 7. Behind the large building strength of signal is very low and obviously inside buildings received power is very low (ideally zero) as shown in figure 8. These results clearly shows that the proposed method works well for propagation modelling over irregular terrain as well as for an urban environment.

5 Conclusion

In this paper, the wide angle form of parabolic equation for radio wave propagation modelling under different profiles was introduced using finite element approach and simulation examples were presented. Finite element method formulation can easily process complex refractivity profiles of any kind and can handle complex geometries without much computational burden. Moreover, the refractive index and terrain variations being independent between consecutive range steps, giving the ability to include inhomogeneous tropospheric profiles and different terrain variations.

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