Design of a Sharp Linear-Phase FIR Filter Using the α-scaled Sampling Kernel

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Abstract: In this paper, a new approach is proposed for the design of sharp linear-phase FIR filters using the α-scaled sampling kernel. The proposed filter design approach provides a closed-form expression for the filter coefficients by employing the α-scaled (here, α is real-valued) sampling kernel. The design procedure is simpler and easier compared to well-known sharp linear-phase FIR filter design methods such as interpolated FIR(IFIR) and frequency-response masking(FRM) technique. We show, by means of examples, that the proposed approach yields similar filter performance but much simpler design procedures than conventional sharp filter design methods.

Key–Words: Sharp linear-phase FIR filter, IFIR, FRM, α-scaled sampling kernel

1 Introduction

Digital FIR filters with sharp transition band find many applications in communication, multimedia, and biomedical devices. The needs for FIR filters arise from the requirement of linear phase, stability, and low coefficient sensitivity. However, a narrow transition band (or sharp) FIR filter is often associated with high computational cost due to its long filter length. To overcome this problem, many computationally efficient approaches have been proposed in the past decade, which includes the well-known prefiler-equalizer and the IFIR approaches [1, 2] for narrow-band sharp filter design, and the FRM technique for arbitrary bandwidth sharp filter design [3, 4, 5]. It is reported in [4] that a FRM filter is able to reduce the computational cost by more than 98%. However, the design procedures for these filters [1, 2, 3, 4, 5] are much more complicated than the traditional approaches due to the involvement of few subfilters. It is common that an iterative procedure is involved to design one subfilter at a time while taking into account of the contributions from other subfilter(s). In such a way, the complexity of the overall filter is gradually reduced [4, 6]. Further improvement in terms of number of multipliers is possible if nonlinear optimization techniques are employed to optimize all subfilters jointly [7, 8, 9, 10], which lead to about additional 20% savings compared to the iterative design procedure.

Although these proposed methods are effective for the design of fixed coefficient sharp filters, but it is difficult to apply these methods to programmable filters. This is because there is no closed-form expressions for filter coefficients in these filter design approaches. In this paper, a new approach is proposed for the design of sharp linear-phase FIR filters by utilizing the α-scaled sampling kernel [11]. In particular, the proposed design procedure is simpler and easier than those used for the IFIR and FRM filters. Furthermore, it provides a closed-form expression for filter coefficients. Note that the scaling factor α as in the α-scaled sampling kernel [11] can be rational as well as irrational.

This paper is organized as follows: The α-scaled sampling kernel utilized in designing sharp linear-phase FIR filters is discussed in Section 2. Section 3 provides a new method of designing sharp linear-phase FIR filters using the α-scaled sampling kernel and a window function. Section 4 presents the design examples of the proposed approach. Finally, the conclusion is drawn in Section 5.

2 α-scaled Sampling Kernel

Given a continuous-time signal $x(t)$ and its scaled version $x(\frac{t}{\alpha})$ (here, $\alpha \in \Re$, $\alpha > 0$ is a real-valued scaling factor), let's denote $x[n]$ and $x_\alpha[n]$ as their respective sampled discrete-time signals. That is,

$$x[n] = x(nT) \quad (1)$$

$$x_\alpha[n] = x(nT^{'}), \quad T^{'} = \frac{T}{\alpha} \quad (2)$$

Note that $\alpha > 1$ is for interpolation and $0 < \alpha < 1$ for decimation. Recently, it was reported that $x_\alpha[n]$ can be obtained in a single step from $x[n]$ by utilizing the following α-scaled up sampling kernel $sinc(\frac{n}{\alpha} - k)$ [11]:

$$x_\alpha[n] = \sum_{k=-\infty}^{\infty} x[k]\, sinc(\frac{n}{\alpha} - k) \quad (3)$$

3 The Design of a Sharp Linear-phase FIR Filter

In this paper, (3) is further exploited and modified for the efficient design of linear phase FIR filters with sharp transition. In particular, when $h[n]$ ($n = 0, 1, \cdots, N-1$) is a linear-phase equiripple FIR filter of length N, consider the following new filter $h_{(\alpha)}[n]$:

$$h_{(\alpha)}[n] = \frac{1}{\alpha}\, h_\alpha[n] \quad (4)$$

$$= \sum_{k=0}^{N-1} h[k] \bullet \frac{1}{\alpha}\, sinc(\frac{n}{\alpha} - k) \quad (5)$$

In particular, it can be verified that the discrete-time Fourier transform (DTFT) of $h_{(\alpha)}[n]$ (i.e., $H_{(\alpha)}[e^{j\omega}]$, $\omega \in [-\pi, \pi]$) can be expressed in terms of $H[e^{j\omega}]$: i.e.,

$$H_{(\alpha)}[e^{j\omega}] = \begin{cases} H[e^{j\alpha\omega}], & \omega \in [-\frac{\pi}{\alpha}, \frac{\pi}{\alpha}] \\ o, & \omega \notin [-\frac{\pi}{\alpha}, \frac{\pi}{\alpha}] \end{cases} \quad (6)$$

For the proof of (6), let's infer the scaling properties of the discrete-time Fourier transform (DTFT)[2, 11]:

$$h[n] \xrightarrow{DTFT} H(e^{j\omega}) \quad (7)$$

$$h_\alpha[n] \xrightarrow{DTFT} \alpha\, H(e^{j\alpha\omega}) \quad (8)$$

As in [11], (4) can be also expressed by

$$h_{(\alpha)}[n] = \frac{1}{2\pi} \sum_{k=0}^{N-1} h[k] \int_{-\frac{\pi}{\alpha}}^{\frac{\pi}{\alpha}} e^{j(n-\alpha k)\omega'} d\omega' \quad (9)$$

By taking the DTFT of both sides of (9), we can complete the proof as follows:

$$H_{(\alpha)}[e^{j\omega}] = \sum_{n=-\infty}^{\infty} h_{(\alpha)}[n] e^{-j\omega n}$$

$$= \frac{1}{2\pi} \int_{-\frac{\pi}{\alpha}}^{\frac{\pi}{\alpha}} \underbrace{\sum_{k=0}^{N-1} h[k] e^{-j\alpha k\omega'}}_{H[e^{j\alpha\omega'}]}$$

$$\bullet \underbrace{\sum_{n=-\infty}^{\infty} e^{-jn(\omega-\omega')}}_{2\pi\,\delta((\omega-\omega')} d\omega'$$

$$= \begin{cases} H[e^{j\alpha\omega}], & \omega \in [-\frac{\pi}{\alpha}, \frac{\pi}{\alpha}] \\ o, & \omega \notin [-\frac{\pi}{\alpha}, \frac{\pi}{\alpha}] \end{cases}$$

From (5)-(6), we can observe that

- When $H[z]$ is a linear-phase lowpass FIR filter with bandedges at ω_p and ω_s (e.g., see Fig. 1), the widths of the passband and the transition band of $H_{(\alpha)}[z]$ are $\frac{1}{\alpha}$-th of those of $H[z]$. Accordingly, $h_{(\alpha)}[n]$ (or $H_{(\alpha)}[z]$) can be considered as a linear-phase FIR filter with a narrower passband and a sharper transition than the parent linear-phase FIR filter $h[n]$ (or $H[z]$), and thus it plays the same role as the conventional IFIR filters [2].

- The scaling factor α can be a real-valued one, and the design procedure for the α-scaled filter (4)-(5) can be completed in a single step with removing unwanted images and avoiding aliasing simultaneously [11].

- Also, the length of the new filter $h_{(\alpha)}[n]$ in (5) can be estimated by using the well-known formulas (e.g., Kaiser's formula, Bellanger's formular, Hermann's formula, etc. [2]) for establishing the minimum filter length from the given filter specification. For example, when the Kaiser's formula is utilized for the filter length estimation, the minimum value for N_α (i.e., length of $h_{(\alpha)}[n]$) is given by:

$$N_\alpha = \frac{-20\log_{10}(\sqrt{\delta_p\delta_s}) - 13}{14.6(\omega_s - \omega_p)/2\pi} + 1 \quad (10)$$

  where δ_p and δ_s denote the peak passband and stopband ripples, and ω_p and ω_s correspond to the normalized passband and stopband edge angular frequencies, respectively, given in the specification for the filter $h_{(\alpha)}[n]$. Moreover, when α is an integer, the overall filter can be efficiently implemented by using the IFIR structure [1, 2].

- By applying a "lowpass-to-highpass" transformation to $h_{(\alpha)}[n]$ (or $H_{(\alpha)}[z]$), a *narrow highpass* linear-phase FIR filter with sharp transition can be also derived (e.g., $(-1)^n h[n]$ or $H_{(\alpha)}[-z]$).

- The concept of the *single-lowpass* α-scaled sampling kernel (i.e., (4) or (5)) may be further extended for the derivation of the more general "*single-lowpass* plus *multi-bandpass*" FIR filters.

- The complementary filter concept as used in the FRM approaches [2, 3] can be also utilized for the derivation of the complementary filter of $h_{(\alpha)}[n]$.

From (5), we can design sharp lowpass FIR filters with arbitrary passband width. However, since the $sinc$ function is involved in (5), the designed filter length can be long. To solve this problem, the Gaussian window function is employed to obtain desired FIR filters by minimizing the performance degradation of the filter characteristics. With the introduction of a Gaussian window function, (5) can be rewritten into:

$$h^f_{(\alpha)}[n] \;=\; h_{(\alpha)}[n]w[n],\;\; n = 0, \cdots, M-1 \quad (11)$$

where the Gaussian window $w[n]$ is given by

$$w[n] \;=\; e^{-\frac{1}{2}\beta\left(\frac{n-\frac{M-1}{2}}{\frac{M-1}{2}}\right)^2},\;\; n = 0, \cdots, M-1 \quad (12)$$

In particular, the width of $w[n]$ (here, $\beta > 2$) is inversely related to the value of β: a larger value of β produces a narrower window. In this paper, β is set to 2.5 (default). To illustrate the proposed approach, the narrow-passband sharp FIR filter example is shown in Fig. 1, where Fig. 1(a) is the frequency response of a model filter and Fig. 1(b) corresponds to the frequency response of the designed FIR filter.

To design a filter using the proposed method, the following procedure can be used:

- For a given set of filter specification, we should determine the value of α and bandedges of a model filter. The value of α depends on the filter structure being used. It can be either IFIR or FRM.

- The corresponding model filter should be designed by using any conventional filter design algorithm such as Remez. The coefficients of overall filter can be calculated by applying (5) and (11). Note that the length of Gaussian window is about the same of the effective length of IFIR filter or FRM filter according to our experience

Fig. 2 illustrates the design of a lowpass filter design using above procedure, where $\alpha = \sqrt{7}$. Fig. 2(a) shows $h_{(\alpha)}[n] = h[n]$: i.e., $\alpha = 1$; Fig. 2(b) corresponds to $h_{(\alpha)}[n]:\ \alpha = \sqrt{7}$; and Fig. 2(c) is the final filter $h^f_{(\alpha)}[n]$ obtained after windowing.

It is possible to extend the proposed method to the design of frequency-response masking filter. We demonstrate this by applying our method to the modified FRM structure presented in [5], as shown in Fig. 3. The design starts with a model filter as in Fig. 3(a). Interpolating the model filter by a factor of α results a filter $H_{s1}[z]$ with compressed frequency response as shown in Fig. 3(b). Applying IFIR technique to $H_{s1}[z]$ and selecting a proper pair of masking filters, a sharp FIR filter is produced as shown in Figs. 3(c)-(e).

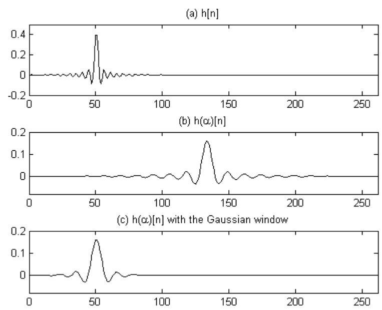

Figure 1: Design of a narrow-band sharp FIR filter.

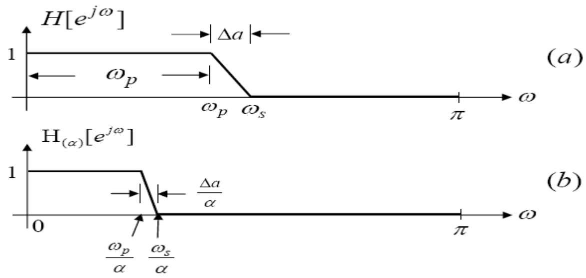

Figure 2: The sharp FIR filter design procedures: ((a) $h_{(\alpha)}[n] = h[n]:\ i.e.,\ \alpha = 1$, (b) $h_{(\alpha)}[n]:\ \alpha = \sqrt{7}$, and (c) the final filter $h^f_{(\alpha)}[n]$ obtained after windowing).

4 Design Examples

To illustrate the proposed filter design approach, the following lowpass FIR filter is considered: $\omega_p = 0.25\pi$, $\omega_s = 0.3\pi$, $\delta_p = 0.002$, and $\delta_s = 0.001$. If this filter is designed using the IFIR technique as in [2], the interpolation factor is 2 and the filter lengths for model filter and masking filter are 68 and 15,

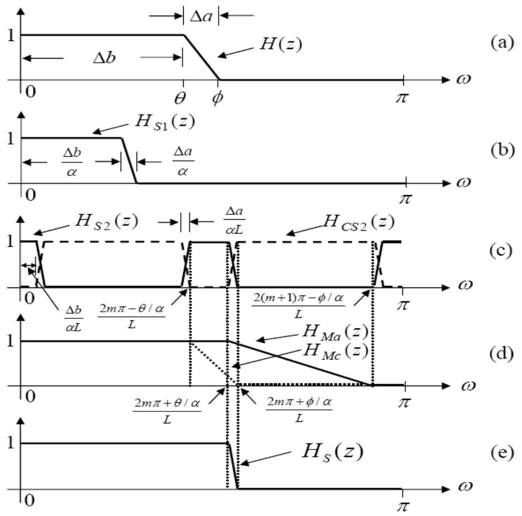

Figure 3: Design of a wide-band sharp FIR filter using masking filters with real number scaling factor.

respectively. The frequency response is shown in Fig. 4 in solid lines. If the same filter is designed using the proposed approach, the filter length of the overall designed type-I FIR filter $h^{f}_{(\alpha)}[n]$ is 149 as shown in Table 1. It is clear from Fig. 4 that both FIR filters, obtained by the IFIR technique and the proposed approach, satisfy the given filter specification. Note that the filter length of the overall cascade in the IFIR filter case is almost same as that in the proposed approach.

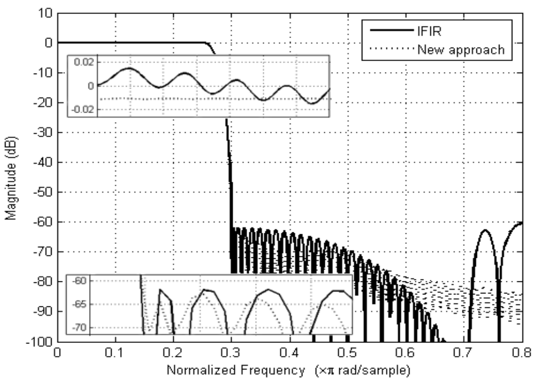

Figure 4: A sharp FIR filter with narrow lowpass band (i.e., Fig. 3(b)).

The proposed approach can be applied to the design of FRM filter as well. Let us consider the design of a lowpass filter. The passband and stopband edges are at $\omega_p = 0.537\pi$ and $\omega_s = 0.538\pi$, respectively, as in [5], where the passband ripple is at most 0.001 and the minimum stopband attenuation is 80 dB. Applying the modified FRM design technique in [5], it is easy to find that the bandedges of the bandedge shaping filter, as shown in Fig. 3(a), are: $\theta = 0.6\pi$ and $\phi = 0.62\pi$, respectively. In the proposed approach, the optimum interpolation factor L is set to 10 for the FRM filter and the IFIR filter pair uses an interpolation factor of 2. The lengths of $H_{Ma}[z]$ and $H_{Mc}[z]$ are 55 and 121, respectively. The length of the Gaussian window is chosen to be 483, which is equal to the length of the overall filter obtained by the IFIR technique for the comparison purpose. Also, it is verified in Fig. 5 that the designed FIR filter obtained by the proposed approach satisfies the filter design specification.

Table 1: Filter Coefficients $h^{f}_{(\alpha)}[n]\ (n = 0, \cdots, 148)$

g[0]=g[148]	0.000195170	g[38]=g[110]	-0.001503600
g[1]=g[147]	0.000073628	g[39]=g[109]	-0.004758800
g[2]=g[146]	-0.000137060	g[40]=g[108]	-0.004874700
g[3]=g[145]	-0.000261390	g[41]=g[107]	-0.001347900
g[4]=g[144]	-0.000189210	g[42]=g[106]	0.003655600
g[5]=g[143]	0.000041020	g[43]=g[105]	0.006577900
g[6]=g[142]	0.000284830	g[44]=g[104]	0.004954200
g[7]=g[141]	0.000372550	g[45]=g[103]	-0.000598170
g[8]=g[140]	0.000197090	g[46]=g[102]	-0.006435400
g[9]=g[139]	-0.000184130	g[47]=g[101]	-0.008218600
g[10]=g[138]	-0.000520480	g[48]=g[100]	-0.004073600
g[11]=g[137]	-0.000519420	g[49]=g[99]	0.003682000
g[12]=g[136]	-0.000100290	g[50]=g[98]	0.009718900
g[13]=g[135]	0.000486230	g[51]=g[97]	0.009279200
g[14]=g[134]	0.000808690	g[52]=g[96]	0.001804000
g[15]=g[133]	0.000567830	g[53]=g[95]	-0.008078900
g[16]=g[132]	-0.000156840	g[54]=g[94]	-0.013291000
g[17]=g[131]	-0.000901670	g[55]=g[93]	-0.009238100
g[18]=g[130]	-0.001094700	g[56]=g[92]	0.002407200
g[19]=g[129]	-0.000477360	g[57]=g[91]	0.014032000
g[20]=g[128]	0.000623300	g[58]=g[90]	0.016865000
g[21]=g[127]	0.001438100	g[59]=g[89]	0.007327200
g[22]=g[126]	0.001286800	g[60]=g[88]	-0.009470600
g[23]=g[125]	0.000125780	g[61]=g[87]	-0.022081000
g[24]=g[124]	-0.001320200	g[62]=g[86]	-0.020114000
g[25]=g[123]	-0.001994800	g[63]=g[85]	-0.002134600
g[26]=g[122]	-0.001261100	g[64]=g[84]	0.021539000
g[27]=g[121]	0.000547000	g[65]=g[83]	0.034008000
g[28]=g[120]	0.002221100	g[66]=g[82]	0.022717000
g[29]=g[119]	0.002464400	g[67]=g[81]	-0.010404000
g[30]=g[118]	0.000875180	g[68]=g[80]	-0.046527000
g[31]=g[117]	-0.001609200	g[69]=g[79]	-0.058159000
g[32]=g[116]	-0.003235700	g[70]=g[78]	-0.024400000
g[33]=g[115]	-0.002653200	g[71]=g[77]	0.055232000
g[34]=g[114]	0.000010102	g[72]=g[76]	0.156920000
g[35]=g[113]	0.003038200	g[73]=g[75]	0.241930000
g[36]=g[112]	0.004199800	g[74]	0.274980000
g[37]=g[111]	0.002361100		

5 Conclusion

In this paper, we have presented a new filter design approach for sharp linear-phase FIR filters. A

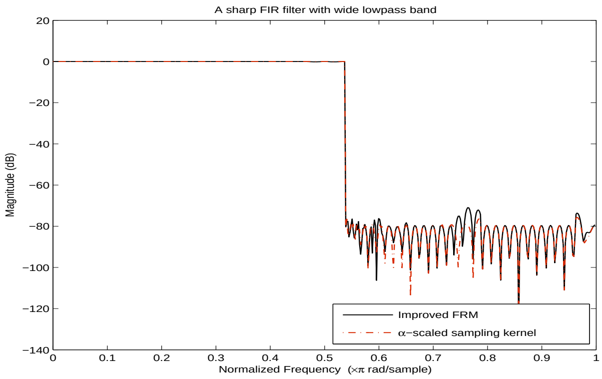

Figure 5: A sharp FIR filter with wide lowpass band (i.e., Fig. 3(e)).

closed-form expression for filter coefficients is given by employing the α-scaled sampling kernel (here, α is real-valued) and a window function. The approach is simple and easy for the design of sharp FIR filters compared with the conventional IFIR and FRM technique. The proposed approach may be extended further to design various types of linear-phase FIR filters with sharp transition (e.g., wide-lowpass, narrow-bandpass, wide-bandpass, wide-highpass, multi-bandpass, etc.) as required in digital communications, multirate signal processing, biometric signal processing, speech and audio signal processing.

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References:

[1] P.S.R. Diniz, E.A.B. da Silva, and S.L. Netto, *Digital Signal Processing - System Analysis and Design*, Cambridge University Press, 2002.

[2] S.K. Mitra, *Digital Signal Processing: A Computer-Based Approach*, McGraw-Hill, 3rd Ed., 2006.

[3] Y.C. Lim, Frequency-response masking approach for the synthesis of sharp linear phase digital filters, *IEEE Trans. on Circuits and Systems*, vol. CAS-33, no. 4, Apr. 1986, pp. 357-364.

[4] Y.C. Lim, and Y. Lian, “The optimum design of one- and two-dimensional FIR filters using the frequency response masking technique,” *IEEE Trans. on Circuits and Systems Part 2*, vol. 40, no. 2, 1993, pp. 88-95.

[5] Y. Lian, L. Zhang, and C.C. Ko, An improved frequency response masking approach for designing sharp FIR filters, *Signal processing*, vol. 81, no. 12, Apr. 2001, pp. 2573-2581.

[6] T. Saramäki, and Y. C. Lim, Use of the Remez algorithm for designing FRM based FIR filters, *Circuits, Systems, Signal Processing*, vol. 22, no. 2, Mar. 2003, pp. 77-97.

[7] T. Saramäki, and H. Johansson, Optimization of FIR filters using the frequency-response masking approach, *The Proceeding of IEEE Int. Symp. on Circuits Syst.*, vol. 2, May 2001, pp. 177-180.

[8] Y. J. Yu, and Y. C. Lim, FRM based FIR filter design - the WLS approach, *The Proceeding of IEEE Int. Symp. on Circuits Syst.*, vol. 3, May 2002, pp. 221-224.

[9] W. S. Lu, and T. Hinamoto, Optimal design of frequency-response-masking filters using semidefinite programming, *IEEE Trans. Circuits Syst. I*, vol. 50, Apr. 2003, pp. 557-568.

[10] W. S. Lu, and T. Hinamoto, Optimal design of IIR frequency-response-masking filters using second-order cone programming, *IEEE Trans. Circuits Syst. I*, vol. 50, Nov. 2003, pp. 1401-1412.

[11] S.C. Pei and M.P. Kao, A two-channel nonuniform perfect reconstruction filter bank with irrational down-sampling factors, *IEEE Signal Processing Letters*, vol. 12, no. 2, 2005, pp. 116-119.

[12] A.V. Oppenheim, R.W. Schafer, and J.R. Buck, *Discrete-Time Signal Processing*, 2nd Ed., Prentice Hall, 1999.

[13] A. Antoniou, *Digital Signal Processing: Signals, Systems, and Filters*, McGraw-Hill, 2006.

[14] A. Ambardar, *Digital Signal Processing: A Modern Introduction*, Nelson, Thomson, 2007.