Transfer Matrix Method For A Single-Chamber Mufflers

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Abstract: Mufflers are widely used for exhaust noise attenuation in vehicles, machinery and other industrial elements. Modeling procedures for accurate performance prediction had led to the development of new methods for practical muffler components in design. Plane wave based models such as the transfer matrix method (TMM) can offer fast initial prototype solutions for muffler designers. In the present paper the authors present an overview of the principles of TMM for predicting the transmission loss (TL) of a muffler. The predicted results agreed in some limits with the experimental data published in literature.

Key-Words: Muffler modelling, transfer matrix method

1 Introduction

Mufflers are commonly used in a wide variety of applications. Industrial flow ducts as well as internal combustion engines frequently make use of silencing elements to attenuate the noise levels carried by the fluids and radiated to the outside atmosphere by the exhausts. Design of a complete muffler system is, usually, a very complex task because each of its elements is selected by considering its particular acoustic performance and its interaction effects on the entire acoustic system performance. For the frequency analysis of the muffler, as can be seen from the references [1,3,5], it is very convenient to use the transfer matrix method.
The present paper deals with the fundamentals of the Transfer Matrix Method (TMM) and the method is applied to a specific muffler configuration for the prediction of Transmission Loss.

2 Transfer Matrix Method Theory

2.1 Plane wave propagation [1,3,5]

For plane wave propagation in a rigid straight pipe of length L, constant cross section S, and transporting a turbulent incompressible mean flow of velocity V (see Fig. 1), the sound pressure p and the volume velocity ν anywhere in the pipe element can be represented as the sum of left and right traveling waves. The plane wave propagation model is valid when the influence of higher order modes can be neglected. Using the impedance analogy, the sound pressure p and volume velocity ν at positions 1 (upstream end) and 2 (downstream end) in Fig. 1 ($x = 0$ and $x = L$, respectively) can be related by

$$p_1 = Ap_2 + B\nu_2 , \tag{1}$$

and

$$\nu_1 = Cp_2 + D\nu_2 , \tag{2}$$

where A, B, C, and D are called the four-pole constants.

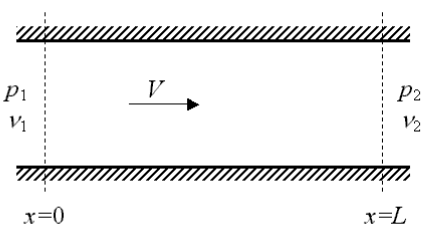

Fig. 1. Plane wave propagation in a rigid straight pipe transporting a turbulent incompressible mean flow [3].

Munjal shows that the four-pole constants for non-viscous medium are

$$A = \exp(-jMk_c L)\cos k_c L , \tag{3}$$

$$B = j(\rho c / S)\exp(-jMk_c L)\sin k_c L , \tag{4}$$

$$D = \exp(-jMk_c L)\cos k_c L ,(5), \tag{5}$$

$$C = j(S / \rho c)\exp(-jMk_c L)\sin k_c L , \tag{6}$$

where $M=V/c$ is the mean flow Mach number ($M<0.2$), c is the speed of sound (m/s), $k_c=k/(1-\tilde{}M^2)$ is the convective wave-number (rad/m), $k = \omega/c$ is the acoustic wave-number (rad/m), ω is the angular frequency (rad/s), ρ is the fluid density (kg/m^3), and j is the square root of - 1. Substitution of M=0 in equations (3) to (6) yields the corresponding relationships for stationary medium.

2.2 Transfer matrix method computations[3]

It can be seen that equations (1) and (2) can be put into the matrix form

$$\begin{bmatrix} p_1 \\ \upsilon_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} p_2 \\ \upsilon_2 \end{bmatrix}, \quad (7)$$

where the second matrix is the matrix that relates the total sound pressure and volume velocity at two points in a muffler element, such as the straight pipe discussed in the previous section. Thus, a transfer matrix is a frequency-dependent property of the system. Reciprocity principle requires that the transfer matrix determinant be 1. In addition, for a symmetrical muffler A and D must be identical. In practice, there are different elements connected together in a real muffler, as shown in figure 2, such as perforated tube. However, the four-pole constants values of each element are not affected by connections to elements upstream or downstream as long as the system elements can be assumed to be linear and passive. So, each element is characterized by one transfer matrix, that depends on its geometry and flow conditions. Therefore, it is necessary to model each element and then to relate all of them to obtain the overall acoustic property of the muffler.

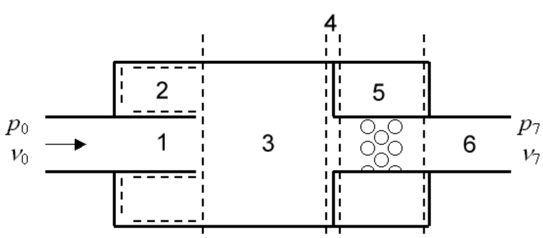

Fig. 2. Real muffler [3].

If several muffler elements, such as sudden expansions, sudden contractions, extended-tubes and/or perforated tubes are connected together in series, then the overall transfer matrix of the entire system is given by the product of the individual system matrices. For example, the muffler shown in Fig. 2 includes a straight extended tube, sudden expansion and extended inlet, uniform tube, sudden contraction, concentric resonator with perforated tube and a straight tail pipe. Then, for this particular muffler we can write the total transfer matrix of the system

$$T = T_1 T_2 T_3 T_4 T_5 T_6 . \quad (8)$$

For the continuities sections, such as extended tubes, the transfer matrix has the form

$$T_i = \begin{bmatrix} \cos(kl_i) & jY_i \sin(kl_i) \\ \frac{j}{Y_i}\sin(kl_i) & \cos(kl_i) \end{bmatrix}, \quad (9)$$

with

$$Y_i = \frac{c}{\pi R_i^2}, \quad (10)$$

while for the discontinuities sections, such as sudden expansions or sudden contractions the transfer matrix has the form

$$T_i = \begin{bmatrix} 1 & 0 \\ \frac{j}{Y_i ctg(kl_i)} & 1 \end{bmatrix}, \quad (11)$$

with

$$Y_i = \frac{c}{\pi(R_i^2 - R_{i-1}^2)}, \quad (12)$$

where R_i is the radius of the muffler section into consideration.

3 Transmission Loss By TMM For A Single-Chamber Mufflers

Considering a muffler as shown in figure 3, the transfer matrices are

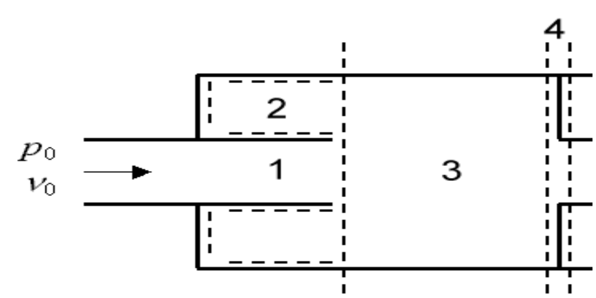

Fig. 3. Partial muffler symmetric in the mirror for right hand side

$$T_1 = \begin{bmatrix} \cos(kl_1) & jY_1 \sin(kl_1) \\ \frac{j}{Y_1}\sin(kl_1) & \cos(kl_1) \end{bmatrix}, \tag{13}$$

$$T_2 = \begin{bmatrix} 1 & 0 \\ \frac{j}{Y_2 ctg(kl_2)} & 1 \end{bmatrix}, \tag{14}$$

$$T_3 = \begin{bmatrix} \cos(kl_3) & jY_3 \sin(kl_3) \\ \frac{j}{Y_3}\sin(kl_1) & \cos(kl_3) \end{bmatrix}, \tag{15}$$

$$T_4 = \begin{bmatrix} 1 & 0 \\ \frac{j}{Y_4 ctg(kl_4)} & 1 \end{bmatrix}, \tag{16}$$

$$T_5 = \begin{bmatrix} \cos(kl_5) & jY_5 \sin(kl_5) \\ \frac{j}{Y_5}\sin(kl_5) & \cos(kl_5) \end{bmatrix}, \tag{17}$$

$$T_6 = \begin{bmatrix} 1 & 0 \\ \frac{j}{Y_6 ctg(kl_6)} & 1 \end{bmatrix}, \tag{18}$$

$$T_7 = \begin{bmatrix} \cos(kl_7) & jY_7 \sin(kl_7) \\ \frac{j}{Y_7}\sin(kl_7) & \cos(kl_7) \end{bmatrix}, \tag{19}$$

$$T_8 = \begin{bmatrix} 1 & 0 \\ \frac{j}{Y_8 ctg(kl_8)} & 1 \end{bmatrix}, \tag{20}$$

$$T_9 = \begin{bmatrix} \cos(kl_9) & jY_9 \sin(kl_9) \\ \frac{j}{Y_9}\sin(kl_9) & \cos(kl_9) \end{bmatrix}, \tag{21}$$

where

$$Y_i = \frac{c}{\pi R_i^2}\text{, for } i\text{=1,3,5,7,9} \tag{22}$$

and

$$Y_i = \frac{c}{\pi (R_i^2 - R_{i-1}^2)}\text{, for } i\text{=2,4,6,8.} \tag{23}$$

The characterization of a muffling device used for noise control applications can be given in terms of the attenuation, insertion loss, transmission loss, and the noise reduction.

Transmission loss, *TL*, is independent of the source and requires an anechoic termination at the downstream end. It describes the performance of what has been called “the muffler proper”. It is defined as 10 times the logarithm of the ratio between the power incident on the muffler proper (W_i) and that transmitted downstream (W_t) into an anechoic termination. Transmission loss does not involve neither the source nor the radiation impedance. It is thus an invariant property of the element. Being made independent of the terminations, *TL* finds favor with researchers who are sometimes interested in finding the acoustic transmission behavior of an element or set of elements in isolation of the terminations (Munjal, 1987). Transmission Loss can be calculated in terms of the four-pole constants as

$$TL = 20\log\left[\left(\frac{Y_9}{Y_1}\right)^{1/2}\left|\frac{T_{11} + T_{12}/Y_9 + T_{21}Y_1 + T_{22}Y_1/Y_9}{2}\right|\right], \tag{24}$$

where the terms T_{ij} are the elements of the global transfer matrix of the muffler.

4 Results And Conclusions

In figures 5-8 are presented the dependence of the *TL* of the muffler as function of the frequency in the domain 10-4000 Hz, for different constructive configurations. As can be seen from the figures the *TL* manifests itself in the low frequency range. It can be concluded that the TMM is an effective tool in order to estimate the *TL* of a

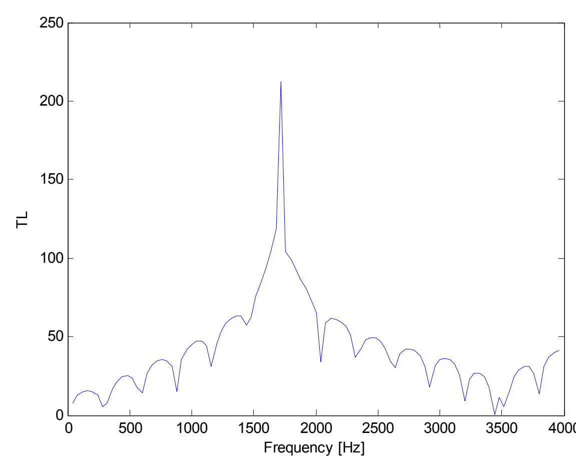

Fig. 5. Transmission loss of the muffler for R_1=R_5=R_9, R_2=R_3=R_4=R_6=R_7=R_8, R_2/ R_1= 1.5

muffler in the design early stage taking into account the specific constructive elements of the muffler. By this way the paper high-lights the possibilities of optimizing the constructive elements of a muffler with two sudden expansions and two sudden contractions.

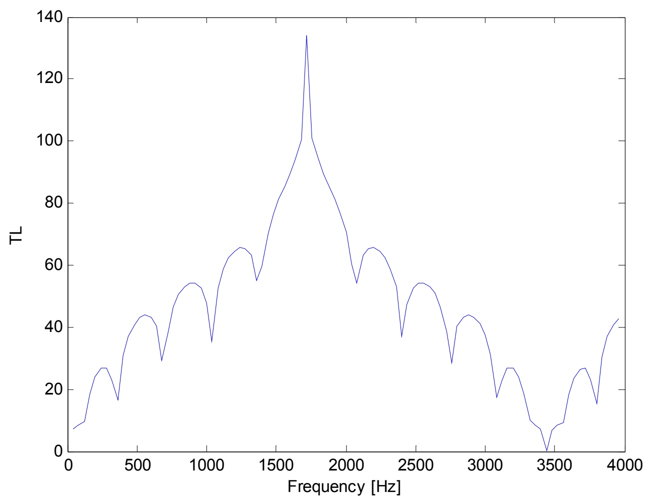

Fig. 6. Transmission loss of the muffler for $R_1=R_5=R_9$, $R_2=R_3=R_4=R_6=R_7=R_8$, $R_2/R_1= 2$.

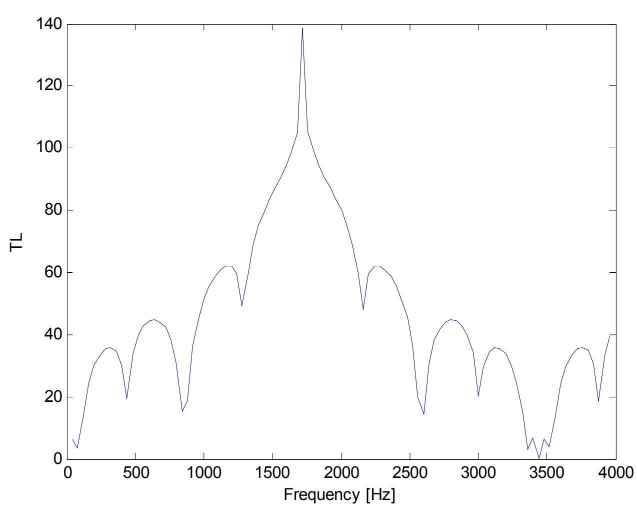

Fig. 7. Transmission loss of the muffler for $R_1=R_5=R_9$, $R_2=R_3=R_4=R_6=R_7=R_8$, $R_2/R_1= 2.5$

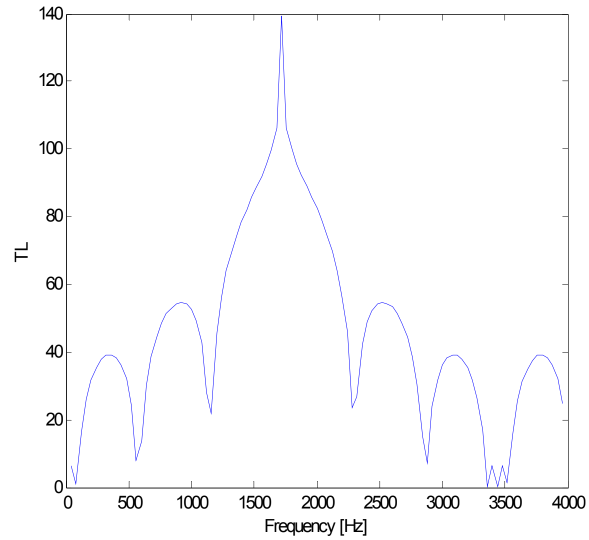

Fig. 8. Transmission loss of the muffler for $R_1=R_5=R_9$, $R_2=R_3=R_4=R_6=R_7=R_8$, $R_2/R_1= 3.0$

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