Fault Identification Algorithms in the Presence of Intermittent Faults

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Abstract: The system diagnosis has been extensively studied in the literature in connection with fault-tolerant multiprocessor computer systems. An original graph-theoretical model for system diagnosis was introduced in a classic paper by Preparata, Metze, and Chien in 1967. Yang and Masson extended the model to the case when the system has some intermittent faults, and gave a characterization of diagnosability under their model. However, their proof is not constructive, that is present no diagnosis algorithm for systems. In this paper, we present two diagnosis algorithms under the model by Yang and Masson: one is a polynomial-time algorithm for system with high connectivity, and the other is a linear time algorithm for system with high degree.

Key–Words: Fault Diagnosis, Fault Identification, Intermittent Fault, Polynomial-Time Algorithm, t-Connected Graph,

1 Introduction

The system diagnosis has been extensively studied in the literature in connection with fault-tolerant multiprocessor computer systems. An original graph-theoretical model for system diagnosis, which is called the PMC model, was introduced in a classic paper by Preparata, Metze, and Chien [6]. In the PMC model, the testing assignment is represented by a digraph(directed graph), each vertex of which represents a processor, and each arc (u, v) of which means that processor u tests processor v. In this model, each processor is either faulty or fault-free. The fault-status of a processor does not change during the diagnosis. A testing processor evaluates a tested processor as either faulty or fault-free. The evaluation is accurate if the testing processor is fault-free, while the evaluation is unreliable if the testing processor is faulty. A syndrome is a collection of test results. The model also assumes that the number of faulty processors is bounded.

A testing assignment is said to be t-diagnosable if all faulty processors can be identified uniquely from any syndrome provided that the number of faulty processors does not exceed t. It is well-known that a testing assignment for system with n processors is t-diagnosable only if $t < n/2$ and each processor is tested by at least t distinct other processors [6]. A complete characterization of t-diagnosable system was shown by Hakimi and Amin [5].

Yang and Masson [8] extends the PMC model to the case when a multiprocessor system contains some intermittent faults. In this paper, this version of the PMC model is called the YM model. In the YM model, each processor is either of *fault-free*, *permanently faulty* (corresponding to faulty in the PMC model), or *intermittently faulty*. The fault-status of a processor does not change during the diagnosis. A testing processor evaluates a tested processor as either faulty or fault-free. When the testing processor is fault-free, the evaluation is accurate if the tested processor is fault-free or permanently faulty, while the evaluation is unreliable if the tested processor is intermittently faulty. When the testing processor is (permanently or intermittently) faulty, the test result is arbitrary. See Fig.1.

In the YM model, no diagnosis algorithm can identify all intermittently faulty processors because they might behave as good in every test. Algorithm A is called an (s, t)-diagnosis algorithm if A outputs from any syndrome a set F of processors that contains all permanently faulty processors and some intermittently faulty processors, but no fault-free processors, provided that the number of intermittently faulty processors does not exceed s and that of faulty processors does not exceed t. A testing assignment D is said to be (s, t)-diagnosable if there exists an (s, t)-diagnosis algorithm for D. Notice that if D is t-diagnosable in the PMC model then D is also $(0, t)$-diagnosable. Yang and Masson give in [8] a complete characteri-

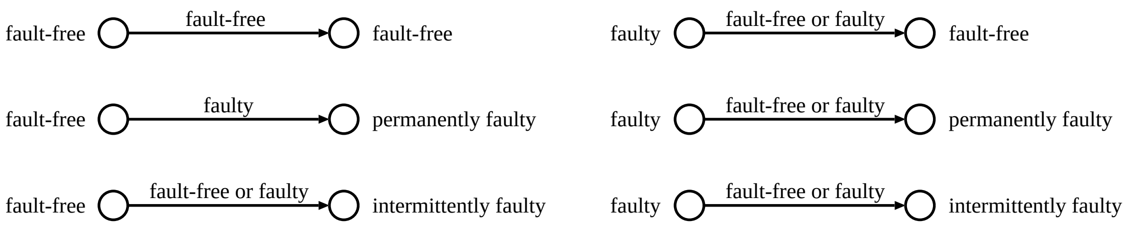

Fig. 1: Test Results

zation of an (s,t)-diagnosable testing assignment by generalizing a characterization of a $(0,t)$-diagnosable testing assignment in [5]. However, the proof by Yang and Masson is not constructive, that is they present no (s,t)-diagnosis algorithm for D.

A number of $(0,t)$-diagnosis algorithms have been developed in the literature [1–3, 7]. The result of parallel diagnosis by Fu and Beigel [4] leads to an (s,t)-diagnosis algorithm for a testing assignment represented by a complete graph. However, no (s,t)-diagnosis algorithm for a large class of testing assignments have been developed.

In this paper, we present a polynomial-time (s,t)-diagnosis algorithm for a testing assignment represented by a $(s+t)$-connected digraph, and a linear-time (t,t)-diagnosis algorithm for any (t,t)-diagnosable testing assignment.

2 Preliminaries

Let D be a digraph(directed graph), and let $V(D)$ and $E(D)$ be the vertex set and arc(directed edge) set of D, respectively. Let

$$\Gamma^-_D(v) = \{u : (u,v) \in E(D)\}, \tag{1}$$

$$\delta^-_D(v) = |\Gamma^-_D(v)|, \tag{2}$$

$$\Gamma^+_D(v) = \{u : (v,u) \in E(D)\}, \tag{3}$$

$$\delta^+_D(v) = |\Gamma^+_D(v)| \tag{4}$$

for any vertex $v \in V(D)$, and

$$\Gamma^-_D(X) = \bigcup_{v\in X} \Gamma^-_D(v) - X, \tag{5}$$

$$\delta^-_D(X) = |\Gamma^-_D(X)|, \tag{6}$$

$$\Gamma^+_D(X) = \bigcup_{v\in X} \Gamma^+_D(v) - X, \tag{7}$$

$$\delta^+_D(X) = |\Gamma^+_D(X)| \tag{8}$$

for any $X \subseteq V(D)$. Define that

$$\delta^-(D) = \min_{v\in V(D)} \delta^-_D(v), \tag{9}$$

$$\delta^+(D) = \min_{v\in V(D)} \delta^+_D(v) \tag{10}$$

Let σ be a syndrome on D, that is a mapping of $E(D)$ onto $\{0,1\}$. A partition (H,I,P) of $V(D)$ is said to be *consistent* with σ if the following two conditions are satisfied:

- $u,v \in H$, $(u,v) \in E(D) \Rightarrow \sigma(u,v) = 0$;
- $u \in H$, $v \in P$, $(u,v) \in E(D) \Rightarrow \sigma(u,v) = 1$.

Then, σ is also said to be *consistent* with (H,I,P).

An *(s,t)-diagnosis algorithm* for D is one that, for any partition (H,I,P) of $V(D)$ with $|I| \le s$ and $|I|+|P| \le t$, finds F with $P \subseteq F \subseteq P \cup I$ from any syndrome σ consistent with (H,I,P). D is *(s,t)-diagnosable* if there exists an (s,t)-diagnosis algorithm for D. The following theorem is proved in [8].

Theorem I [8] *D is (s,t)-diagnosable if and only if the following three conditions are satisfied:*

- $n = |V(D)| \ge 2t+1$*;*
- $\delta^-(D) \ge t+s$*;*
- $\delta^+_D(X) > p+2s$ *for any* $X \subset V(D)$ *with* $|X| = n-2t+p$ $(0 \le p < t-s)$. □

Notice that the proof of Theorem I is not constructive. Therefore, it is very important to design a (s,t)-diagnosis algorithm.

Let $\mathcal{D}_D(\sigma,s,t)$ denote the set of partitions (H,I,P) of $V(D)$ consistent with σ such that $|I| \le s$ and $|P|+|I| \le t$.

Theorem 1 *D is (s,t)-diagnosable if and only if the following condition is satisfied:*

Let σ be any syndrome such that $\mathcal{D}_D(\sigma,s,t) \ne \emptyset$, and let (H_1,I_1,P_1) and (H_2,I_2,P_2) be any two partitions in $\mathcal{D}_D(\sigma,s,t)$. Then, $H_1 \cap P_2 = \emptyset$ and $H_2 \cap P_1 = \emptyset$.

Proof: It is easy to see the necessity of the theorem.

Let σ be any syndrome such that $\mathcal{D}_D(\sigma, s, t) \neq \emptyset$. Then, define F as the set of vertices v such that $v \in P$ for some $(H, I, P) \in \mathcal{D}_D(\sigma, s, t)$. Then, it is easy to see that $P' \subseteq F$ for any $(H', I', P') \in \mathcal{D}_D(\sigma, s, t)$. Moreover, $H' \cap F = \emptyset$ for any $(H', I', P') \in \mathcal{D}_D(\sigma, s, t)$ because of the assumption. Hence, D is (s, t)-diagnosable. □

By Theorem 1, we obtain a simple (s, t)-diagnosis algorithm in Fig. 2. However, the time complexity of the algorithm is $O(3^{|V(D)|} \cdot |E(D)|)$, that is the algorithm is an exponential-time algorithm. So, we present two polynomial-time diagnosis algorithms in the next section. The one is a polynomial-time (s, t)-diagnosis algorithm for an $(s+t)$-connected digraph, and the other is a linear-time (t, t)-diagnosis algorithm.

algorithm Simple-(s, t)-Diagnosis
  Input: Digraph D, Syndrome σ;
  Output: Faulty Set F;
begin
  $H_{\text{temp}} \leftarrow \emptyset$;
  for $\forall$ partition (H, I, P) of $V(D)$ **do**
    if $|I| \leq s$ **and** $|I| + |P| \leq t$
        and (H, I, P) is consistent with σ
    then $H_{\text{temp}} \leftarrow H_{\text{temp}} \cup H$;
  $F \leftarrow V(D) - H_{temp}$;
end

Fig. 2: (s, t)-Diagnosis Algorithm for $(s + t)$-Connected Graph

3 Diagnosis Algorithms

3.1 (s, t)-Diagnosis Algorithms for $(s + t)$-Connected Digraphs

A dipath $(w_0, w_1, \ldots, w_l)$ from w_0 to w_l is called a *disagreed dipath* on σ if $\sigma(w_i, w_{i+1}) = 0$ for each $i \in \{0, 1, \ldots, l-2\}$ and $\sigma(w_{l-1}, w_l) = 1$. Fig. 3 shows a disagreed dipath from u to v. In the PMC model, if u is fault-free then v is faulty while if v is fault-free then u is faulty. On the other hand, in the FB model, we cannot determine the status of v even if u is fault-free, because w can be intermittently faulty.

v is called a *disagreed vertex* with a vertex u if there exist at least $(s+1)$ internally vertex-disjoint disagreed dipath from v to u. Fig. 4 shows that u is a disagreed vertex with v

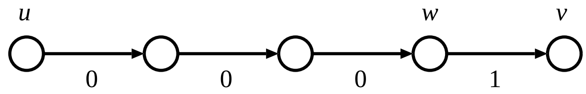

Fig. 3: Disagreed path

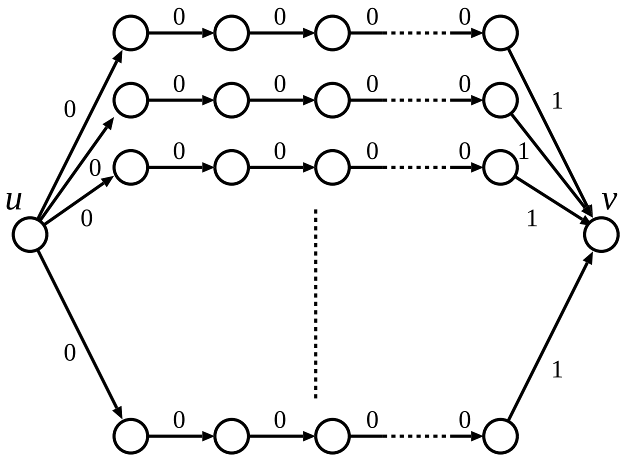

Fig. 4: Disagreed vertex

Lemma 2 *Assume that u is a disagreed vertex with v. If u is fault-free then v is (intermittently or parmanently) faulty. Moreover, If v is fault-free then u is (intermittently or parmanently) faulty.*

Proof: Assume that neither u nor v is intermittently faulty. Since u is a disagreed vertex with v, there exist at least $(s+1)$ internally vertex-disjoint disagreed dipath from v to u. Since D has at most s intermittently faulty vertices, there there exists a disagreed dipath from v to u containing no intermittently faulty vertex. Hence, provided that neither u nor v is intermittently faulty, we conclude that u is fault-free if and only if v is parmanently faulty, which completes the proof of the lemma. □

Let $\Phi_\sigma(v)$ denote the set of disagreed vertices with v.

Lemma 3 *If v is permanently faulty then $|\Phi_\sigma(v)| \geq t+1$.*

Proof: Let F denote the set of faulty vertices on D, and let $F^- = F - \{v\}$. Since D is $(s+t)$-connected and $|F^-| \leq t-1$, we obtain that $D - F^-$ is $(s+1)$-connected. Hence, for any $u \in V(D) - F$, there exist at least $(s+1)$ internally vertex-disjoint dipaths from u to v on $D - F^{-}$. All of these dipaths are disagreed with v since every vertex of $D-F$ is fault-free. Hence $V(D) - F \subseteq \Phi_\sigma(v)$. Since $|V(D)| \geq (2t+1)$ and $|F| \leq t$, we conclude that $|\Phi_\sigma(v)| \geq |V(D) - F| \geq (2t+1) - t = t+1$. □

Lemma 4 *If v is fault-free then $|\Phi_\sigma(v)| \leq t$.*

Proof: Let H denote the set of fault-free vertices on D, and consider any vertex $u \in H$. If P is a disagreed dipath from u to v, P must contain an intermittent faulty vertex because of the definition of a consistent syndrome. Since D has at most s intermittent faulty vertices, there exist at most s internally vertex-disjoint disagreed dipaths from u to v, and hence $u \notin \Phi_\sigma(v)$. Since $\Phi_\sigma(v) \subseteq V(D) - H$, we conclude that $|\Phi_\sigma(v)| \leq |V(D) - H| \leq t$. □

From lemmas 3 and 4, we obtain an (s,t)-diagnosis algorithm shown in Fig. 5.

algorithm (s,t)-Diagnosis
  Input: Digraph D, Syndrome σ;
  Output: Faulty Set F;
begin
  $F \leftarrow \emptyset$;
  for $\forall v \in V(D)$ **do**
    if $|\Phi_\sigma(v)| \geq t+1$ **then** $F \leftarrow F \cup \{v\}$;
end

Fig. 5: (s,t)-Diagnosis Algorithm for $(s+t)$-Connected Graph

For example, consider a digraph and a syndrome in Fig. 6 as an input to algorithm $(1,2)$-Diagnosis. In this example, digraph D has 6 disagreed vertices with v (See Fig. 7), and so algorithm $(1,2)$-Diagnosis diagnoses v as faulty. Moreover, algorithm $(1,2)$-Diagnosis diagnoses any other vertex as fault-free because each of these vertices has at most one directed edge labeled with 1 as an end-vertex.

We can prove the correctness of algorithm (s,t)-Diagnosis as follows:

Theorem 5 *Algorithm (s,t)-Diagnosis in Fig. 5 outputs F that contains all permanently faulty vertices and no fault-free vertex.*

Proof: By Lemmas 3 and 4, we conclude that

- if $|\Phi_\sigma(v)| \leq t$ then v is not permanently faulty, and

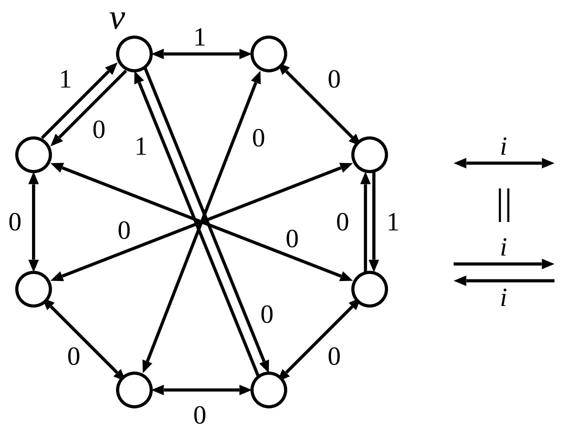

Fig. 6: Digraph D and Syndrome σ on D

- if $|\Phi_\sigma(v)| \geq t+1$ then v is not fault-free.

Hence, F contains all permanently faulty vertices and no fault-free vertex. □

Next, we present how to decide whether there exist at least $(s+1)$ internally vertex-disjoint disagreed dipaths from u to v on D. For any digraph D and syndrome σ on D, $D^v_{\sigma=0}$ is the graph obtained from D as follows (See Fig. 8):

$$V(D^v_{\sigma=0}) = V(D); \quad (11)$$

$$E(D^v_{\sigma=0}) = \{(x,v) : \sigma(x,v) = 1\} \cup \{(x,y), x, y \neq v, \sigma(x,y) = 0\}. \quad (12)$$

By definition, the following lemma can be proved

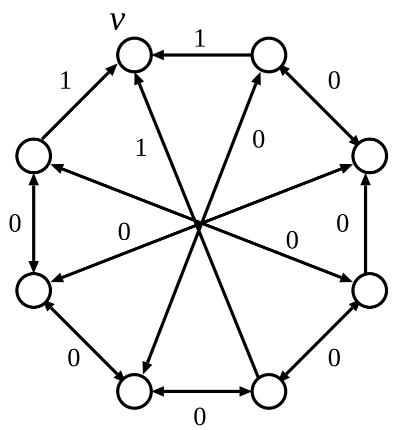

Fig. 8: Digraph $D^v_{\sigma=0}$ for digraph D and syndrome σ in Fig. 6

easily:

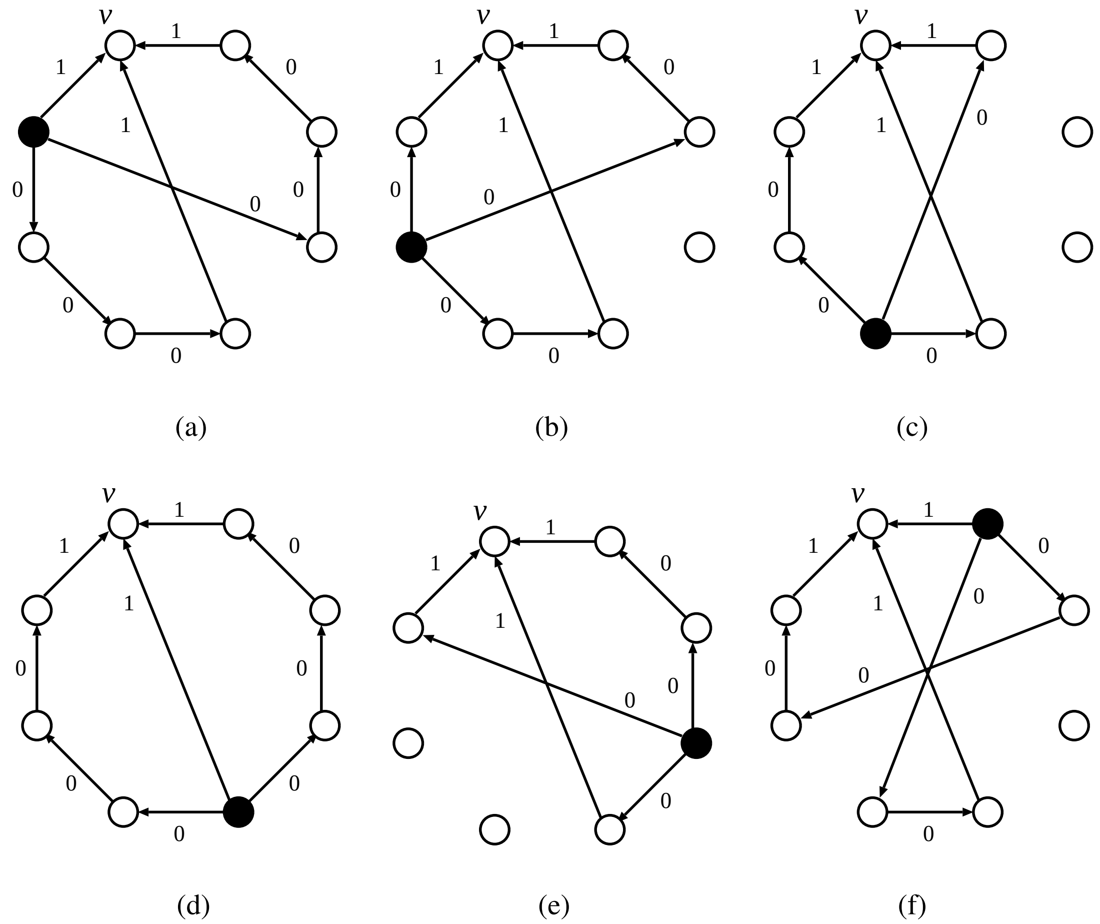

Fig. 7: 6 Disagreed Vertices with v

Lemma 6 *A dipath P from u to v on D is a disagreed dipath with v if and only if P is a dipath on $D^v_{\sigma=0}$.* □

In order to determine whether u is a disagreed vertex with v or not, it suffices to check that $D^v_{\sigma=0}$ has at least $(s+1)$ internally vertex-disjoint dipath from u to v by Lemma 6. Given a digraph $\mathcal{D}$ and a pair of vertices (x, y) on $\mathcal{D}$, it is well known how to count the number of internally vertex-disjoint paths from x to y on $\mathcal{D}$. In fact, the following lemma can be proved.

Lemma 7 *Given a digraph $\mathcal{D}$, a pair of vertices (x, y) on $\mathcal{D}$, and positive integer s, we can determine in $O(s \cdot (|V(\mathcal{D})| + |E(\mathcal{D})|))$ time whether $\mathcal{D}$ has at least $(s+1)$ internally vertex-disjoint paths from x to y.*

Proof: Let $\mathcal{D}$ be a digraph. $\mathcal{D}'$ is the graph defined as follows (See Fig. 9):

$$V(\mathcal{D}') = \{w^-, w^+ : w \in V(\mathcal{D})\}; \quad (13)$$

$$E(\mathcal{D}') = \{(w^-, w^+) : w \in V(\mathcal{D})\} \cup \{(x^+, y^-) : (x, y) \in E(\mathcal{D})\}. \quad (14)$$

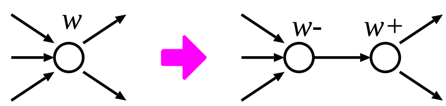

Fig. 9: Construction of $\mathcal{D}'$

Then, by corresponding a dipath $(x_1, x_2, \ldots, x_l)$ on $\mathcal{D}$ with a dipath $(x_1^+, x_2^-, x_2^+, \ldots, x_l^-)$ on $\mathcal{D}'$, it is easy to see that $\mathcal{D}$ has τ internally vertex-disjoint dipaths from x to y if and only if $\mathcal{D}'$ has τ edge-disjoint dipaths from x^+ to y^-. We can count the maximum number of edge-disjoint dipaths from x^+ to y^- on $\mathcal{D}'$ by computing the maximum flow from x^+ to y^- on $\mathcal{D}'$ with the capacity of each directed edge as 1. Since it suffices to determine whether $\mathcal{D}'$ has a flow from x^+ to y^- with value of $(s+1)$ by Ford-Fulkerson algorithm in $O(s \cdot (|V(\mathcal{D}')| + |E(\mathcal{D}')|))$ time, we can check in $O(s \cdot (|V(\mathcal{D}')| + |E(\mathcal{D}')|))$ time whether $\mathcal{D}'$ has at least $(s+1)$ edge-disjoint dipaths from x^+ to y^-, that is in $O(s \cdot (|V(\mathcal{D})| + |E(\mathcal{D})|))$ time whether

$\mathcal{D}$ has at least $(s+1)$ internally vertex-disjoint dipaths from x to y. □

Since $|V(D^v_{\sigma=0})| = |V(D)|$, $|E(D^v_{\sigma=0}| \leq |E(D)|$, and $|E(D)| \geq (s+t)|V(D)| \geq |V(D)|$, we can determine in $O(s \cdot |E(D)|)$ time whether u is a disagreed vertex with v or not by Lemma 7, and so compute $|\Phi_\sigma(v)|$ in $O(s \cdot |E(D)| \cdot |V(D)|)$ time for any $v \in V(D)$. Hence, we obtain the following theorem:

Theorem 8 *Let s and t be positive integers with $s \leq t$. Then, algorithm (s,t)-Diagnosis in Fig. 5 works in $O(smn^2)$ time, where $n = |V(D)|$ and $m = |E(D)|$.* □

3.2 (t,t)-Diagnosis Algorithms

Let $\Psi^i_\sigma(v) = \{u \in \Gamma^-_D(v) : \sigma(u,v) = i\}$ for any $i \in \{0,1\}$.

Lemma 9 *If v is fault-free then $|\Psi^1_\sigma(v)| \leq t$. If v is permanently faulty then $|\Psi^1_\sigma(v)| \geq t+1$.*

Proof: If v is fault-free then every vertex of $\Psi^1_\sigma(v)$ is permanently or intermittently faulty, and so $|\Psi^1_\sigma(v)| \leq t$.

If v is permanently faulty then at least $t+1$ vertices in $\Gamma^-_D(v)$ is fault-free since $\delta^-_D(v) \geq 2t$. Hence, $|\Psi^1_\sigma(v)| \geq t+1$. □

algorithm (t,t)-Diagnosis
  Input: Digraph D, Syndrome σ;
  Output: Faulty Set F;
begin
  $F \leftarrow \emptyset$;
  for $\forall v \in V(D)$ **do**
    if $|\Psi^1_\sigma(v)| \geq t+1$ **then** $F \leftarrow F \cup \{v\}$;
end

Fig. 10: (t,t)-Diagnosis Algorithm for (t,t)-Diagnosable Graph

From Lemma 9, we obtain the following theorem:

Theorem 10 *Algorithm (t,t)-Diagnosis in Fig. 10 outputs F that contains all permanently faulty vertices and no fault-free vertex. Moreover, the algorithm computes F in a linear time.*

4 Conclusions

This paper presented two diagnosis algorithms: a polynomial-time (s,t)-disgnosis algorithm for $(s+t)$-connected digraph and a linear-time (t,t)-diagnosis algorithm for (t,t)-diagnosable digraph.

Here are two open problems. The first one is to design a polynomial-time (s,t)-diagnosis algorithm for an (s,t)-diagnosable digraph. The second one is to develop more faster (s,t)-diagnosis algorithm for an $(s+t)$-connected digraph.

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