New ZCZ Sequence Sets Composed of Two Subsets

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Abstract: - The present paper proposes new ZCZ sequence sets which consist of two subsets. The most important property of the proposed ZCZ sequence sets is that the cross-correlation function between two arbitrary sequences which belong to different subsets has quite a large zero-cross-correlation zone. Of course, each subset is a ZCZ sequence set of which the family size is half of the original one. When the proposed ZCZ sequence sets are applied to AS-CDMA systems, one subset is assigned to users near to a base station and another subset is assigned to users far from it. By this allotment, the proposed sets can hypothetically achieve larger zero-correlation zones than conventional ZCZ sequence sets.

Key-Words: - ZCZ sequence sets, AS-CDMA, perfect sequences, unitary matrices, spreading sequences, spread spectrum communications

1 Introduction

Recently, AS-CDMA (approximately synchronized code-division multiple-access) systems have attracted a great deal of attention. In those systems, ZCZ (zero-correlation zone) sequence sets are used as spreading sequences. Generally, a ZCZ sequence set is characterized by the sequence period, the number of sequences, and the zero-correlation zone length. It is well known that the zero-correlation zone length is restricted by a mathematical upper bound. In our previous work, we have proposed a new method for constructing ZCZ sequence sets of which the zero-correlation zone length is relatively near to the upper bound [1].

In the present paper, we propose new ZCZ sequence sets which consist of two ZCZ sequence sets. They can hypothetically achieve larger zero-correlation zones than conventional ZCZ sequence sets. In addition, we introduce the concept of ZCZ ratio in order to evaluate the zero-correlation zone length properly. The proposed ZCZ sequence sets are also superior to the conventional ones from the view point of the ZCZ ratio.

2 ZCZ Sequence Sets

In this section, we explain ZCZ sequence sets and other related terms.

Let S be a sequence set with M sequences of period P. S can be represented as

$$\begin{aligned} S &= \{S_0, S_1, \cdots, S_p, \cdots, S_{M-1}\}, \\ S_p &= \left(s_0^p, s_1^p, \cdots, s_q^p, \cdots, s_{P-1}^p\right). \end{aligned} \quad (1)$$

S_p and s_q^p denote a sequence and a sequence element respectively. If all of the sequences in S satisfy the following autocorrelation and cross-correlation properties, S is referred to as a ZCZ sequence set.

$$R_{p,p'}(\tau) = \sum_{q=0}^{P-1} s_q^p s_{(q+\tau)\bmod P}^{p'*} = \begin{cases} E_p & (\tau = 0, p = p'), \\ 0 & (\tau = 0, p \neq p'), \\ 0 & (1 \leq |\tau| \leq L). \end{cases} \quad (2)$$

The symbol $*$ denotes a complex conjugate. In addition, E_p denotes the sequence energy of S_p and is defined by the following formula:

$$E_p = \sum_{q=0}^{P-1} s_q^p s_q^{p*} = \sum_{q=0}^{P-1} \left|s_q^p\right|^2 . \quad (3)$$

The range, $0 \leq |\tau| \leq L$, is referred to as a zero-correlation zone. In addition, L is referred to as the zero-correlation zone length of S. We represent the ZCZ sequence set as $Z(P, M, L)$ in order to exhibit the sequence period, the family size, and the zero-correlation zone length.

The zero-correlation zone length is restricted by the following mathematical upper bound [5], [8]:

$$L \le \frac{P}{M} - 1. \quad (4)$$

ZCZ sequence sets which satisfy this upper bound only exist under some limited conditions [4], [8]. Other known ZCZ sequence sets can achieve only small zero-correlation zones [6], [7]. So, we have proposed a new method for constructing ZCZ sequence sets [1]. The proposed method can generate various ZCZ sequence sets recursively by using perfect sequences and unitary matrices. The parameters of the ZCZ sequence sets generated by this method satisfy the following formula:

$$L = \frac{(l-2)P}{lM}, \quad (5)$$

where l is the period of the perfect sequences used to construct the ZCZ sequence sets. Although L of (5) does not satisfy the mathematical upper bound, it is relatively near to the upper bound. In the real systems, because of simplicity of implementation, binary or quadriphase sequences are preferred. In the quadriphase case in particular, the proposed method can generate ZCZ sequence sets having larger zero-correlation zones than previously known ones.

3 ZCZ Sequence Sets Composed of Two Subsets

In this section, we define the concept of ZCZ sequence sets composed of two subsets. In addition, we propose a method for constructing such ZCZ sequence sets.

3.1 Definition

Suppose that two sequence sets, U and V, are ZCZ sequence sets. In addition, suppose that they have the same sequence period, P, the same family size, M, and the same zero-correlation zone length, L. That is, they are $Z(P,M,L)$. Now, we represent U as

$$\begin{aligned} U &= \{U_0, U_1, \cdots, U_p, \cdots, U_{M-1}\}, \\ U_p &= \left(u_0^p, u_1^p, \cdots, u_q^p, \cdots, u_{P-1}^p\right). \end{aligned} \quad (6)$$

Similarly, V is represented as

$$\begin{aligned} V &= \{V_0, V_1, \cdots, V_p, \cdots, V_{M-1}\}, \\ V_p &= \left(v_0^p, v_1^p, \cdots, v_q^p, \cdots, v_{P-1}^p\right). \end{aligned} \quad (7)$$

If all of the sequences in U and all of the sequences in V satisfy the following condition, the union of U and V, namely $T = U \cup V$, is referred to as a ZCZ sequence set composed of two subsets.

$$R_{p,p'}(\tau) = \sum_{q=0}^{P-1} u_q^p v_{(q+\tau) \bmod P}^{p'*} = 0 \quad \left(0 \le |\tau| \le \Lambda\right). \quad (8)$$

Λ is referred to as the zero-cross-correlation zone length between U and V. We describe such ZCZ sequence sets as $Z^T(P, 2M, [L, \Lambda])$ in order to exhibit all of the parameters.

3.2 Sequence Generation

Let l be an even positive integer and let $A = (a_0, a_1, \cdots, a_{l-1})$ be a perfect sequence of period l. That is, the sequence A has the following autocorrelation property:

$$R_A(\tau) = \sum_{k=0}^{l-1} a_k a_{(k+\tau) \bmod l}^{*} = \begin{cases} E_A & (\tau = 0), \\ 0 & (\text{otherwise}), \end{cases} \quad (9)$$

where E_A is the sequence energy of A. Let l_0 and l_1 be integers. The two integers are defined as

$$\begin{aligned} l &= 2 l_0 l_1, \\ 2 &\le l_1 \le \frac{l}{2}. \end{aligned} \quad (10)$$

Using A and these integers, a sequence set with l_1 perfect sequences, U^0, is defined as

$$\begin{aligned} U^0 &= \left\{U_0^0, U_1^0, \cdots, U_p^0, \cdots, U_{l_1-1}^0\right\}, \\ U_p^0 &= \left(u_0^{0,p}, u_1^{0,p}, \cdots, u_{l-1}^{0,p}\right) \\ &= \left(a_{pl_0}, a_{pl_0+1}, \cdots, a_{l-1}, a_0, \cdots, a_{pl_0-1}\right). \end{aligned} \quad (11)$$

Similarly, a sequence set, V^0, is defined as

$$\begin{aligned} V^0 &= \left\{V_0^0, V_1^0, \cdots, V_p^0, \cdots, V_{l_1-1}^0\right\}, \\ V_p^0 &= \left(v_0^{0,p}, v_1^{0,p}, \cdots, v_{l-1}^{0,p}\right) \\ &= \left(a_{(p+l_1)l_0}, a_{(p+l_1)l_0+1}, \cdots, a_{l-1}, a_0, \cdots, a_{(p+l_1)l_0-1}\right). \end{aligned} \quad (12)$$

$U_p^0 \left(0 \le p \le l_1 - 1\right)$ and $V_p^0 \left(0 \le p \le l_1 - 1\right)$ are perfect sequences derived by shifting A cyclically to the left. Therefore, they have the same autocorrelation property as A.

Let $B_n (n \ge 1)$ and $C_n (n \ge 1)$ be $l_1 \times l_1$ unitary matrices. In addition, let $b_{p,q}^n / \sqrt{l_1} \left(0 \le p, q \le l_1 - 1\right)$ be the (p,q) -th element of B_n and let $c_{p,q}^n / \sqrt{l_1} \left(0 \le p, q \le l_1 - 1\right)$ be the (p,q)-th element of C_n. Because they are unitary matrices, we have

$$\sum_{k=0}^{l_1-1} b_{k_0,k}^n b_{k_1,k}^{n*} = \sum_{k=0}^{l_1-1} c_{k_0,k}^n c_{k_1,k}^{n*} = \begin{cases} l_1 & (k_0 = k_1), \\ 0 & (\text{otherwise}). \end{cases} \quad (13)$$

Using U^0 and B_n, a sequence set with l_1 sequences of period ll_1^n, $U^n(n \ge 1)$, is defined as

$$\begin{aligned} U^n &= \{U_0^n, U_1^n, \cdots, U_i^n, \cdots, U_{l_1-1}^n\}, \\ U_i^n &= \left(u_0^{n,i}, u_1^{n,i}, \cdots, u_j^{n,i}, \cdots, u_{ll_1^n-1}^{n,i}\right), \\ u_j^{n,i} &= b_{i,j \bmod l_1}^n u_{\lfloor j/l_1 \rfloor}^{n-1, j \bmod l_1}, \end{aligned} \tag{14}$$

where $0 \le i \le l_1 - 1$ and $0 \le j \le ll_1^n - 1$. $\lfloor j/l_1 \rfloor$ denotes a maximum integer which does not exceed j/l_1. Similarly, using V^0 and C_n, a sequence set, $V^n(n \ge 1)$, is defined as

$$\begin{aligned} V^n &= \{V_0^n, V_1^n, \cdots, V_i^n, \cdots, V_{l_1-1}^n\}, \\ V_i^n &= \left(v_0^{n,i}, v_1^{n,i}, \cdots, v_j^{n,i}, \cdots, v_{ll_1^n-1}^{n,i}\right), \\ v_j^{n,i} &= c_{i,j \bmod l_1}^n v_{\lfloor j/l_1 \rfloor}^{n-1, j \bmod l_1}. \end{aligned} \tag{15}$$

Let T^n be the union of U^n and V^n, namely $T^n = U^n \bigcup V^n$, then we can obtain the following theorem:

Theorem 1: The sequence set T^n defined by the above formulas is a ZCZ sequence set composed of two subsets and the parameters of T^n are represented as $Z^T\left(ll_1^n, 2l_1, \left[\left(\frac{l-4}{2}\right)l_1^{n-1}, \left(\frac{l-4}{2} + l_1\right)l_1^{n-1}\right]\right)$.

The proof of this theorem is presented in the next subsection.

3.3 Proof

First, we consider T^1. T^1 consists of U^1 and V^1. The correlation function between two sequences in U^1 is calculated as

$$\begin{aligned} R_{i_0,i_1}^{U^1}(\tau) &= \sum_{k=0}^{ll_1-1} u_k^{1,i_0} u_{(k+\tau) \bmod ll_1}^{1,i_1*} \\ &= \sum_{k=0}^{ll_1-1} b_{i_0,k \bmod l_1}^1 u_{\lfloor k/l_1 \rfloor}^{0,k \bmod l_1} b_{i_1,(k+\tau) \bmod l_1}^{1*} u_{\lfloor (k+\tau)/l_1 \rfloor \bmod l}^{0,(k+\tau) \bmod l_1 *} \\ &= \sum_{k_1=0}^{l_1-1} b_{i_0,k \bmod l_1}^1 b_{i_1,(k+\tau) \bmod l_1}^{1*} \\ &\quad \sum_{k_0=0}^{l-1} a_{(\lfloor k/l_1 \rfloor + (k \bmod l_1) l_0) \bmod l} a^*_{(\lfloor (k+\tau)/l_1 \rfloor \bmod l + ((k+\tau) \bmod l_1) l_0) \bmod l} \\ &= \sum_{k_1=0}^{l_1-1} b_{i_0,k_1}^1 b_{i_1,(k_1+\tau_1) \bmod l_1}^{1*} \\ &\quad \sum_{k_0=0}^{l-1} a_{(k_0+k_1 l_0) \bmod l} a^*_{(k_0+\tau_0+\lfloor (k_1+\tau_1)/l_1 \rfloor + ((k_1+\tau_1) \bmod l_1) l_0) \bmod l} \\ &= \sum_{k_1=0}^{l_1-1} b_{i_0,k_1}^1 b_{i_1,(k_1+\tau_1) \bmod l_1}^{1*} \rho(k_1, \tau_0, \tau_1). \end{aligned} \tag{16}$$

Here, $\rho(k_1, \tau_0, \tau_1)$ is introduced for simplicity. Note that the integers, k_0, k_1, τ_0, and τ_1, are defined as

$$\begin{aligned} k &= k_0 l_1 + k_1, \\ \tau &= \tau_0 l_1 + \tau_1, \\ 0 &\le k_0, \tau_0 \le l - 1, \\ 0 &\le k_1, \tau_1 \le l_1 - 1. \end{aligned} \tag{17}$$

The following formula is obtained form (9) and (16).

$$\rho(k_1, \tau_0, \tau_1) = \begin{cases} E_A & \left(\begin{aligned} &\tau_0 + \left\lfloor \frac{k_1 + \tau_1}{l_1} \right\rfloor + ((k_1 + \tau_1) \bmod l_1) l_0 \\ &= k_1 l_0 \pmod{l} \end{aligned}\right), \\ 0 & (\text{otherwise}). \end{cases} \tag{18}$$

Note that $k_1 l_0$ and $((k_1 + \tau_1) \bmod l_1) l_0$ take only the following values:

$$0, l_0, 2l_0, \cdots, l_0 l_1 - l_0. \tag{19}$$

Suppose that $0 \le \tau_0 \le l_0 - 2$, then

$$0 \le \tau_0 + \lfloor (k_1 + \tau_1)/l_1 \rfloor \le l_0 - 1. \tag{20}$$

Therefore,

$$\rho(k_0, \tau_0, \tau_1) = \begin{cases} E_A & (\tau_0 = \tau_1 = 0), \\ 0 & (\text{otherwise}). \end{cases} \tag{21}$$

Similarly, suppose that $l - l_0 + 1 \le \tau_0 \le l - 1$, then

$$l - l_0 + 1 \le \tau_0 + \lfloor (k_1 + \tau_1)/l_1 \rfloor \le l. \tag{22}$$

Therefore,

$$\rho(k_0, \tau_0, \tau_1) = 0. \tag{23}$$

Note that $\lfloor (k_1 + \tau_1)/l_1 \rfloor = 0$ if $k_1 = 0$. Moreover, suppose that $\tau_0 = l_0 - 1$, then

$$\rho(k_0, \tau_0, \tau_1) = \begin{cases} E_A & (\tau_1 = l_1 - 1, k_1 \ge 1), \\ 0 & (\text{otherwise}). \end{cases} \tag{24}$$

Similarly, suppose that $\tau_0 = l - l_0$, then

$$\rho(k_0, \tau_0, \tau_1) = \begin{cases} E_A & (\tau_1 = 1, k_1 \le l_1 - 2), \\ 0 & (\text{otherwise}). \end{cases} \tag{25}$$

From (16), (21), (23), (24), and (25), when $|\tau| \le l/2 - 2$,

$$R_{i_0,i_1}^{U_1}(\tau) = \begin{cases} E_A l_1 & (\tau = 0, i_0 = i_1), \\ 0 & (\text{otherwise}). \end{cases} \tag{26}$$

Note that $\tau = \tau_0 l_1 + \tau_1$ and $l_0 l_1 = l/2$. Therefore, U^1 is $Z(ll_1, l_1, l/2 - 2)$. By the same manner as U^1, we can also prove that V^1 is $Z(ll_1, l_1, l/2 - 2)$.

The correlation function between a sequence in U^1 and a sequence in V^1 is calculated as

$$\begin{aligned}R_{i_0,i_1}^{U^1,V^1}(\tau) &= \sum_{k=0}^{ll_1-1} u_k^{1,i_0} v_{(k+\tau)\bmod ll_1}^{1,i_1 *} \\ &= \sum_{k=0}^{ll_1-1} b_{i_0,k \bmod l_1}^{1} u_{\lfloor k/l_1 \rfloor}^{0,k \bmod l_1} c_{i_1,(k+\tau)\bmod l_1}^{1*} v_{\lfloor (k+\tau)/l_1 \rfloor \bmod l}^{0,(k+\tau)\bmod l_1 *} \\ &= \sum_{k_1=0}^{l_1-1} b_{i_0,k_1}^{1} c_{i_1,(k_1+\tau_1)\bmod l_1}^{1*} \\ &\quad \sum_{k_0=0}^{l-1} a_{(k_0+k_1 l_0)\bmod l}\, a^{*}_{(k_0+\tau_0+\lfloor (k_1+\tau_1)/l_1 \rfloor + ((k_1+\tau_1)\bmod l_1 + l_1) l_0)\bmod l} \\ &= \sum_{k_1=0}^{l_1-1} b_{i_0,k_1}^{1} c_{i_1,(k_1+\tau_1)\bmod l_1}^{1*} \rho'(k_1,\tau_0,\tau_1).\end{aligned} \tag{27}$$

The following formula is obtained form (9) and (27).

$$\rho'(k_1,\tau_0,\tau_1) = \begin{cases} E_A & \left(\begin{aligned} &\tau_0 + \left\lfloor \frac{k_1+\tau_1}{l_1} \right\rfloor + ((k_1+\tau_1)\bmod l_1 + l_1) l_0 \\ &= k_1 l_0 \pmod l \end{aligned} \right), \\ 0 & (\text{otherwise}). \end{cases} \tag{28}$$

Suppose that $0 \le \tau_0 \le l_0 - 1$, then

$$l_0 l_1 \le \tau_0 + \left\lfloor \frac{k_1+\tau_1}{l_1} \right\rfloor + ((k_1+\tau_1)\bmod l_1 + l_1) l_0 \le 2 l_0 l_1. \tag{29}$$

Therefore,

$$\rho'(k_0,\tau_0,\tau_1) = 0. \tag{30}$$

Note that $\lfloor (k_1+\tau_1)/l_1 \rfloor = 0$ if $k_1 = 0$. Similarly, suppose that $l - l_0 \le \tau_0 \le l - 1$, then

$$\begin{aligned} l_0 l_1 - l_0 \le \tau_0 + \left\lfloor \frac{k_1+\tau_1}{l_1} \right\rfloor + ((k_1+\tau_1)\bmod l_1 + l_1) l_0 \\ \le 2 l_0 l_1 - l_0. \end{aligned} \tag{31}$$

Therefore,

$$\rho'(k_0,\tau_0,\tau_1) = 0. \tag{32}$$

Note that $\lfloor (k_1+\tau_1)/l_1 \rfloor \ne 0$ or $(k_1+\tau_1)\bmod l_1 \ne 0$ if $k_1 = l_1 - 1$.

Moreover, suppose that $\tau_0 = l_0$, then

$$\begin{aligned} l_0 l_1 + l_0 \le \tau_0 + \left\lfloor \frac{k_1+\tau_1}{l_1} \right\rfloor + ((k_1+\tau_1)\bmod l_1 + l_1) l_0 \\ \le 2 l_0 l_1 + 1 \end{aligned} \tag{33}$$

Therefore,

$$\rho'(k_0,\tau_0,\tau_1) = \begin{cases} E_A & (\tau_1 = l_1 - 1, k_1 = 0), \\ 0 & (\text{otherwise}). \end{cases} \tag{34}$$

Similarly, suppose that $\tau_0 = l - l_0 - 1$, then

$$\begin{aligned} l_0 l_1 - l_0 - 1 \le \tau_0 + \left\lfloor \frac{k_1+\tau_1}{l_1} \right\rfloor + ((k_1+\tau_1)\bmod l_1 + l_1) l_0 \\ \le 2 l_0 l_1 - 2 l_0. \end{aligned} \tag{35}$$

Therefore,

$$\rho'(k_0,\tau_0,\tau_1) = \begin{cases} E_A & (\tau_1 = 1, k_1 = l_1 - 1), \\ 0 & (\text{otherwise}). \end{cases} \tag{36}$$

From (27), (30), (32), (34), and (36), when $|\tau| \le l/2 + l_1 - 2$,

$$R_{i_0,i_1}^{U_1,V_1}(\tau) = 0. \tag{37}$$

Note that $\tau = \tau_0 l_1 + \tau_1$ and $l_0 l_1 = l/2$. Therefore, T^1 is $Z^T\left(ll_1, 2l_1, \left[\frac{l-4}{2}, \frac{l-4}{2} + l_1 \right] \right)$.

Next, we give Lemma2 in order to prove Theorem 1.

Lemma 2: If $T^{n-1} = U^{n-1} \cup V^{n-1} (n \ge 2)$ is $Z^T(P_{n-1}, 2M_{n-1}, [L_{n-1}, \Lambda_{n-1}])$, $T^n = U^n \cup V^n$ obtained from T^{n-1} by the formulas (14) and (15) is $Z^T(P_{n-1} l_1, 2M_{n-1}, [L_{n-1} l_1, \Lambda_{n-1} l_1])$.

Now, we prove Lemma2. Because U^{n-1} is $Z(P_{n-1}, M_{n-1}, L_{n-1})$, U^n is $Z(P_{n-1} l_1, M_{n-1}, L_{n-1} l_1)$. The proof is presented in [1]. Therefore, we omit it. Similarly, V^n is $Z(P_{n-1} l_1, M_{n-1}, L_{n-1} l_1)$. The correlation function between a sequence in U^n and a sequence in V^n is calculated as

$$\begin{aligned}R_{i_0,i_1}^{U^n,V^n}(\tau) &= \sum_{k=0}^{P_{n-1} l_1 - 1} u_k^{n,i_0} v_{(k+\tau)\bmod P_{n-1} l_1}^{n,i_1 *} \\ &= \sum_{k=0}^{P_{n-1} l_1 - 1} b_{i_0,k \bmod l_1}^{n} u_{\lfloor k/l_1 \rfloor}^{n-1,k \bmod l_1} c_{i_1,(k+\tau)\bmod l_1}^{n*} v_{\lfloor (k+\tau)/l_1 \rfloor \bmod P_{n-1}}^{n-1,(k+\tau)\bmod l_1 *} \\ &= \sum_{k_1=0}^{l_1-1} b_{i_0,k_1}^{n} c_{i_1,(k_1+\tau_1)\bmod l_1}^{n*} \\ &\quad \sum_{k_0=0}^{P_{n-1}-1} u_{k_0}^{n-1,k_1} v_{(k_0+\tau_0+\lfloor (k_1+\tau_1)/l_1 \rfloor)\bmod P_{n-1}}^{n-1,(k_1+\tau_1)\bmod l_1 *} \\ &= \sum_{k_1=0}^{l_1-1} b_{i_0,k_1}^{n} c_{i_1,(k_1+\tau_1)\bmod l_1}^{n*} R_{k_1,(k_1+\tau_1)\bmod l_1}^{U^{n-1},V^{n-1}}(\tau_0 + \lfloor (k_1+\tau_1)/l_1 \rfloor).\end{aligned} \tag{38}$$

The integers, k_0, k_1, τ_0, and τ_1, are defined as

$$\begin{aligned} &k = k_0 l_1 + k_1, \\ &\tau = \tau_0 l_1 + \tau_1, \\ &0 \le k_0, \tau_0 \le P_{n-1} - 1, \\ &0 \le k_1, \tau_1 \le l_1 - 1. \end{aligned} \tag{39}$$

Suppose that $0 \le \tau_0 \le \Lambda_{n-1} - 1$, then

$$0 \le \tau_0 + \lfloor (k_1+\tau_1)/l_1 \rfloor \le \Lambda_{n-1}. \tag{40}$$

Therefore,

$$R_{k_1,(k_1+\tau_1)\bmod l_1}^{U^{n-1},V^{n-1}}\left(\tau_0+\lfloor (k_1+\tau_1)/l_1 \rfloor\right)=0. \quad (41)$$

Suppose that $P_{n-1}-\Lambda_{n-1}\le \tau_0 \le P_{n-1}-1$, then

$$P_{n-1}-\Lambda_{n-1}\le \tau_0+\lfloor (k_1+\tau_1)/l_1 \rfloor \le P_{n-1}. \quad (42)$$

Therefore,

$$R_{k_1,(k_1+\tau_1)\bmod l_1}^{U^{n-1},V^{n-1}}\left(\tau_0+\lfloor (k_1+\tau_1)/l_1 \rfloor\right)=0. \quad (43)$$

Moreover, suppose that $\tau_0=\Lambda_{n-1}$ and $\tau_1=0$, then $\tau_0+\lfloor (k_1+\tau_1)/l_1 \rfloor=\Lambda_{n-1}$. Therefore,

$$R_{k_1,(k_1+\tau_1)\bmod l_1}^{U^{n-1},V^{n-1}}\left(\tau_0+\lfloor (k_1+\tau_1)/l_1 \rfloor\right)=0. \quad (44)$$

From (38), (41), (43), and (44), we have

$$R_{i_0,i_1}^{U^n,V^n}(\tau)=0 \quad (|\tau|\le \Lambda_{n-1}l_1). \quad (45)$$

Note that $\tau=\tau_0 l_1+\tau_1$. This formula shows that T^n is $Z^T(P_{n-1}l_1, 2M_{n-1}, [L_{n-1}l_1, \Lambda_{n-1}l_1])$. Thus, Lemma 2 has been proved.

Starting from T^1, it is clear that T^n becomes $Z^T\left(ll_1^n, 2l_1, \left[\left(\frac{l-4}{2}\right)l_1^{n-1}, \left(\frac{l-4}{2}+l_1\right)l_1^{n-1}\right]\right)$ by repeating Lemma 2. Thus, Theorem 1 has been proved by mathematical induction.

4 Application to AS-CDMA Systems

ZCZ sequence sets are studied mainly for the application to the uplink of AS-CDMA systems. Now, we will explain a signal design method for AS-CDMA. Suppose that the sequence set S defined in (1) is $Z(P,M,L)$. A sequence $\hat{S}_p$ of length $P+2L$ is obtained by adding guard chips of length L to both sides of the sequence S_p.

$$\hat{S}_p=\left(s_{P-L}^p,\cdots,s_{P-1}^p,s_0^p,\cdots,s_{P-1}^p,s_0^p,\cdots,s_{L-1}^p\right). \quad (46)$$

The left side guard chips are equivalent to the last L chips of S_p. Similarly, the right side guard chips are equivalent to the first L chips of S_p. Information bits are spread by the sequence $\hat{S}_p\ (0\le p\le M-1)$. On the other hand, received signals are despread by the sequence $S_p\ (0\le p\le M-1)$. The aperiodic cross-correlation function between $\hat{S}_p$ and S_p is calculated as

$$\left(x_{-P-L+1},\cdots,x_{-L-1},0,\cdots,0,E_p,0,\cdots,0,x_{L+1},\cdots,x_{P+L-1}\right), \quad (47)$$

where each x_i denotes a complex number. The zero-correlation zone of the autocorrelation function of S_p appears in the central part. On the other hand, the aperiodic cross-correlation function between $\hat{S}_p$ and $S_{p'}\ (p\ne p')$ is calculated as

$$\left(x_{-P-L+1},\cdots,x_{-L-1},0,\cdots,0,0,0,\cdots,0,x_{L+1},\cdots,x_{P+L-1}\right), \quad (48)$$

Similarly, the zero-correlation zone of the cross-correlation function between S_p and $S_{p'}$ appears in the central part. For detail, see [4].

It is difficult to synchronize all signals from all terminals at a base station. However, AS-CDMA systems do not require precise synchronization among spreading sequences by using the zero-correlation property. Now, we suppose that all terminals send their signals at the same time. In this case, there are different time delays depending on the distance between the base station and the terminals. We have to select a ZCZ sequence set so that the maximum difference of the time delays can be held within the zero-correlation zone. ZCZ sequence sets with large zero-correlation zones are useful because they can be applied to AS-CDMA systems of which the cell sizes are large. They can also increase the information transmission rates of AS-CDMA systems by increasing their chip rates.

We propose the application of ZCZ sequence sets composed of two subsets to AS-CDMA. Suppose that $T=U\cup V$ is $Z^T(P,2M,[L,\Lambda])$ and $\Lambda>L$. The concept is shown in Fig.1. The maximum difference of the time delays is the difference of the time delays between the nearest terminal and the farthest terminal. On the other hand, the difference of the time delays between terminals which are near to each other is not large. Therefore, one subset, U, is assigned to terminals near to the base station and another subset, V, is assigned to terminals far from it. By this allotment, terminals which are near to each other utilize the zero-correlation zone of length L and terminals which are far from each other utilize the zero-cross-correlation zone of length Λ or that of length $2L$. Thus, the ZCZ sequence sets composed of two subsets can have the virtual zero-correlation zone of length $\min\{2L,\Lambda\}$. The parameters of the proposed ZCZ sequence sets satisfy the following formula:

$$\Lambda=\frac{(l-4+2l_1)P}{lM}, \qquad 2L=\frac{2(l-4)P}{lM}. \quad (49)$$

From (5) and (19), Λ is always larger than the zero-correlation zone length of the conventional ZCZ sequence sets. On the other hand, $2L$ is larger than the zero-correlation zone length of the conventional ones if $l \geq 6$.

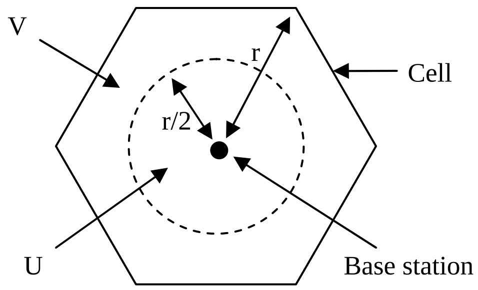

Fig.1 Allotment of Subsets

Because the guard chips are added in order to retain the zero-correlation property in AS-CDMA systems, we have to take into account the length of them in order to evaluate the zero-correlation zone length properly. Therefore, we introduce the following "ZCZ ratio":

$$\gamma = \frac{L'}{P + 2L'}$$
$$L' = \begin{cases} L & (\text{Conventional ZCZ sets}) \\ \min\{2L, \Lambda\} & (\text{Proposed ZCZ sets}) \end{cases} \quad (50)$$

We compare γ of the proposed ZCZ sequence sets and that of the conventional ZCZ sequence sets. Table 1 shows the result of some comparisons in the quadriphase case. It is clear that the proposed ZCZ sequence sets have larger γ than the conventional ones under the equivalent sequence period and family size.

Table 1 Comparison of ZCZ ratio

$Z(P,M,L)$	γ	$Z^T(P,2M,[L,\Lambda])$	γ
$Z(64,4,14)$	0.152	$Z^T(64,4,[12,16])$	0.167
$Z(256,8,28)$	0.090	$Z^T(256,8,[24,40])$	0.119
$Z(1024,16,56)$	0.049	$Z^T(1024,16,[48,112])$	0.079

5 Conclusion

The ZCZ sequence sets composed of two subsets have been proposed in the present paper. They can hypothetically achieve larger zero-correlation zones than the conventional ones. In addition, we have confirmed that the proposed ZCZ sequence sets are superior to the conventional ones from the view point of the ZCZ ratio using some examples. Therefore, it can be expected that the proposed ZCZ sequence sets are useful for designing spreading sequences for AS-CDMA systems.

6 Acknowledgment

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