Sensitivity Analysis of Daubechies 4 Wavelet Coefficients for Reduction of Reconstructed Image Error

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Abstract: Quantization of compressed image file reduces the requirements for memory storage and the transmission bandwidth. However, the cost of quantization is loss of information. On the receiver end when image is reconstructed from lossy de-quantized file using the wavelet reconstruction coefficients, it is impossible to create a perfect image exactly like the original image. This paper records the sensitivity analysis of wavelet coefficients to determine which coefficients contribute to the image reconstruction and in what way and how they should be changed to reconstruct a perfect image. Some interesting findings have been recorded in the conclusion section.

Key Words: - Daubechies 4 Wavelet Coefficients, Image Reconstruction, Mean Squared Error, Sensitivity Analysis

1. Introduction

Raw image data requires large amount of storage space and in order to reduce the memory requirements many compression algorithms have been developed. Historically the discrete cosine transform (DCT) used in the JPEG compression standard was used. However, with the adoption of Joint photographic Experts Group's JPEG 2000 standard for still image compression, which is based on discrete wavelet transforms, wavelets have become the most popular transforms for image compression. Fig. 1 illustrates the image compression and reconstruction with wavelet coefficients.

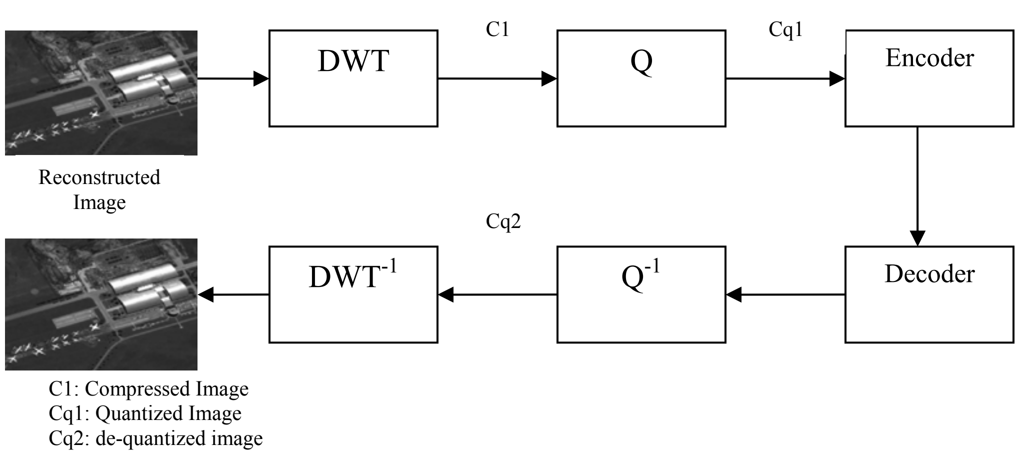


Fig. 1: Image Compression with Discrete Wavelet Transforms

Wavelets transform continuous or discrete time domain signals into frequency domain. DWT convolves the signal against specific wavelet instances at various time scales and positions resulting in a compressed representation of the original image. The compression is reversed by applying the Inverse discrete wavelet transform DWT^{-1}, which convolves the signal against an inverted order of the original wavelet instances to produce an approximation of the original signal. Wavelets conserve energy and redistribute most of the energy to the first trend sub- signal. The energy outside of the first trend signal is insignificant and can be eliminated without much significant loss of information providing a favorable compression rate at the expense of perfect reconstruction. Fig. 2 illustrates the energy distribution in a compressed signal obtained by first level of discrete wavelet transform of the AF museum.

Two main components of DWT are the scaling function $\phi(t)$, and the wavelet function $\psi(t)$, which are defined as follows:

$$\phi(t) = \sum_n h_n \phi(2t-n) \quad (1)$$

$$\psi(t) = \sum_n g_n \psi(2t-n) \quad (2)$$

Where h_n is the impulse response of the scaling filter and g_n is the impulse response of the wavelet filters. Moreover, h_n contains the set of filter coefficients corresponding to the projection of the basis functions for low pass filtering section of the DWT, and g_n contains filter coefficients corresponding to the projection of the basis functions for high filtering section of the DWT. Once transformed, the analysis of signal *x(t)* results in discrete sets of data in the wavelet domain. The inverse DWT^{-1} is used to transform coefficients from the wavelet domain back into the original signal domain. Thus the inverse transform produces the original signal *x(t)* from the wavelet and scaling coefficients.

$$x(t) = \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} d_{k,n} \psi_{k,n}(t) \quad (3)$$

Where

$$d_{k,n} = \int_{-\infty}^{+\infty} \psi_{k,n}(t) x(t) dt \;\; x(t) \quad (4)$$

Fig. 2a: Original Satellite AF Museum

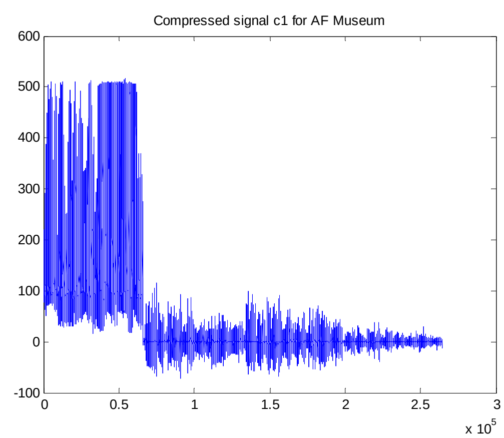


Fig. 2b: Compressed Signal of AF Museum after one level of DWT

For Daubehies 4 wavelet coefficients have the following values:
(Lo_D) h1 = {-0.1294, 0.2241, 0.8365, 0.4829}
Lo_D (1:4) in Matlab
(Hi_D) g1 = {-0.4830, 0.8365, -0.2241, -0.1294}
Hi_D(1:4) in Matlab
(Lo_R) h2 = {0.4830, 0.8365, 0.2241, -0.1294}
Lo_R(1:4) *in Matlab*
(Hi_R) g2 = {-0.1294, -0.2241, 0.8365, 0.4830}
Hi_ R(1:4) in Matlab
h1 is the set of wavelet numbers for the forward discrete wavelet transforms (DWT).
g1 is the set of scaling numbers for the DWT.
h2 is the set of wavelet numbers for the inverse DWT (DWT^{-1}).
g2 is the set of scaling numbers for the (DWT^{-1}).

A two-dimensional 2D DWT of a discrete input image **f** with M rows and N columns (M and N being even) is computed by first applying the one-dimensional (1D) transform defined by the coefficients from set h1 and g1 to the columns of f, and then applying the same transform to the rows of the resulting signal [1]. Similarly, 2D DWT^{-1} is performed by applying the 1D DWT^{-1} defined by sets h2 and g2 first to the rows and then to the columns of a previously compressed signal.
A one-level DWT decomposes **f** into M/2 by N/2 sub-images $\mathbf{h}^1$, $\mathbf{d}^1$, $\mathbf{a}^1$ and $\mathbf{v}^1$, where a^1 is the trend sub mage where most of the energy of the signal is concentrated. $\mathbf{h}^1$, $\mathbf{d}^1$, and $\mathbf{v}^1$ are its first horizontal, diagonal, and vertical fluctuation subimages, respectively.

One-level DWT may be repeated $k \leq \log_2(\min(M, N))$ times. The size of the trend signal $\mathbf{a}^i$ at level *i* of decomposition is $1/4^i$ times the size of the original image **f** (e.g., a three-level transform produces a trend sub image $\mathbf{a}^3$ that is $1/64^{th}$ the size of **f**). Nevertheless, the trend sub image will typically be much larger than any of the fluctuation sub images; for this reason, the MRA scheme computes a k-level DWT by recursively applying a one-level DWT to the rows and columns of the discrete trend signal $\mathbf{a}^{k-1}$. Similarly, a one-level DWT^{-1} is applied k times to reconstruct an approximation of the original M-by-N signal **f**. [2]

2: Impact of Reconstruction Coefficients on the Reconstructed Image

The wavelet reconstruction coefficients based on Daubechies 4 wavelets (db2 in Matlab) are used for the reconstruction of compressed images. The image reconstructed with these wavelets has certain amount of error expressed in mse (mean squared error).
However, there is no knowledge base available which tells in what way each of the eight reconstruction wavelet coefficients Lo_R (1:4) and Hi_R (1:4) impact the reconstruction of the image. Each time the coefficients are evolved they evolve to some numbers which depend on the image. For each image the evolved coefficients are different but we do not understand the relationship between the properties of the image and the evolved coefficients. This study is a step in that direction.
Approach to the Problem: In order to see the impact of each coefficient only one parameter was varied and all the other coefficients were fixed to the values obtained as the wavelet coefficients. The following "wfilters function" on the wavelet toolbox gives the coefficients:

[Lo_D,Hi_D,Lo_R,Hi_R] =filters (wave_type);

Each of the reconstruction coefficients is varied from -1.5 to 1.5 in steps of .02 and image is reconstructed using each of these values. The mean squared error of the reconstructed image was computed for each step value of the parameter. The value which gives the minimum mse is chosen for each coefficient and compared with standard wavelet coefficient. The first run of the study was done on the fruits.bmp image. Later it was extended to other images.
Fig. 3a illustrates the best value (the value for which the error is minimum) for the Lo_R (Low Frequency Wavelet Reconstruction Coefficients).
Fig. 3b illustrates the best value for Hi_R (High Frequency Wavelet Reconstruction Coefficients).

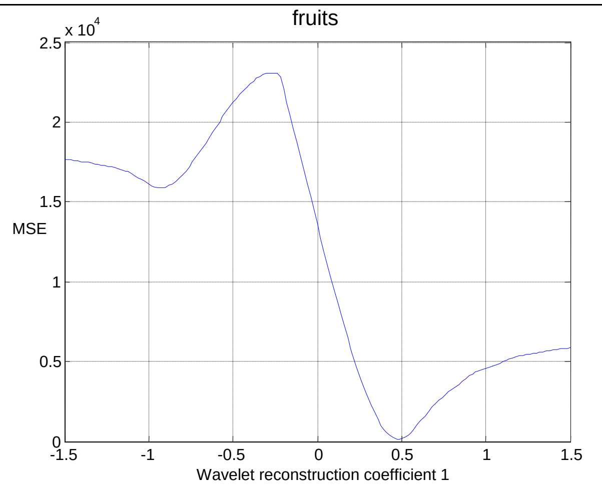


Figure: Lo_R(1) Best Value Analysis

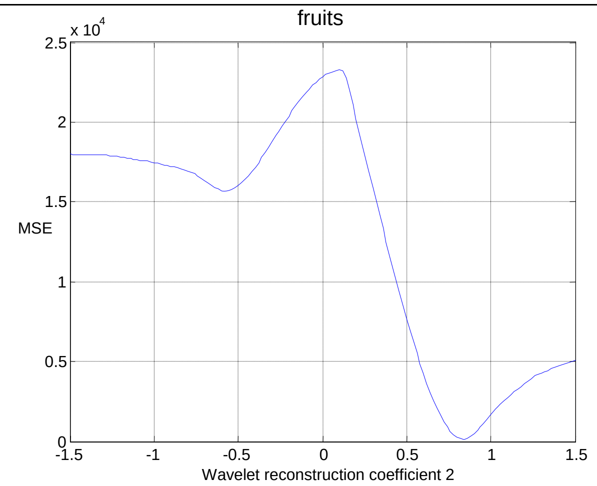


Figure: Lo_R(2) Best Value Analysis

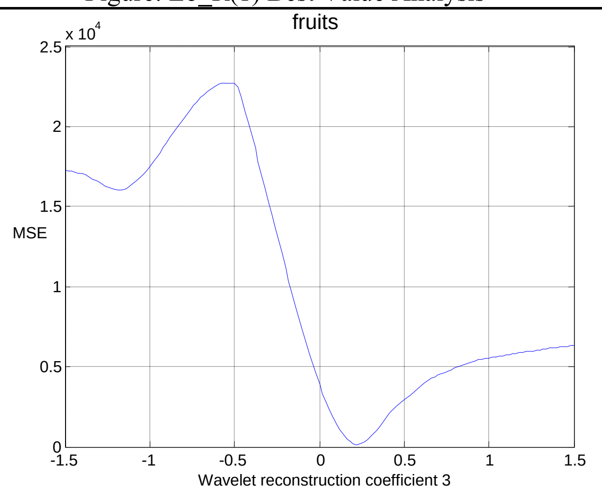


Figure: Lo_R(3) Best Value Analysis

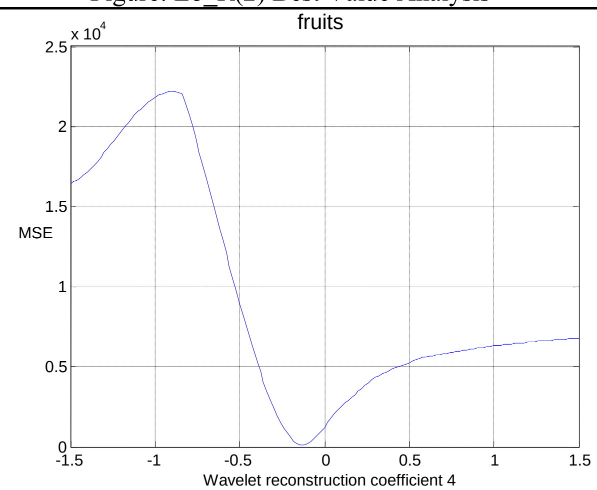


Figure: Lo_R(4) Best Value Analysis

Fig. 3a: The Best Value Analysis of Inverse Low Frequency Wavelet Coefficient

The experiments were repeated with more images to get reasonable data to arrive at some conclusion. Images in two different categories were analyzed. Some images fall in the category of satellite and others are called no-satellite.

It was observed from these experiments that the low frequency coefficients were very close to the standard wavelet coefficients which have the fixed value of:

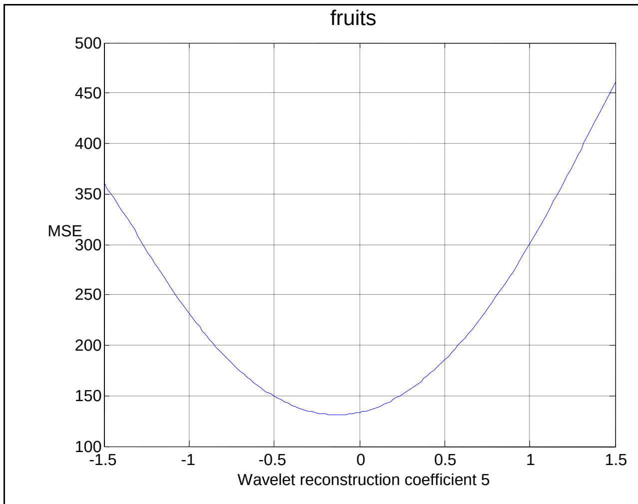
Lo_R(1:4):0.4830 0.8365 0.2241 -0.1294

The last two digits are always 00 as in .xx00. These discrepancies are because the step size was chosen to be .02. If the step size chosen was .0001 then we can get an exact value up to the four decimal places, however, the computation time would increase 220 times.

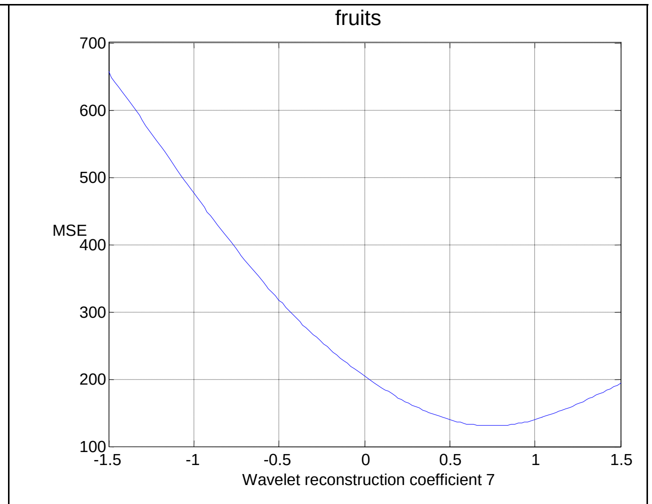
The best values obtained for the Hi_R coefficients were different from the fixed one level wavelet coefficients. The standard Hi_R coefficients are:

Hi_R(1:4) -0.1294 -0.2241 0.8365 -0.4830

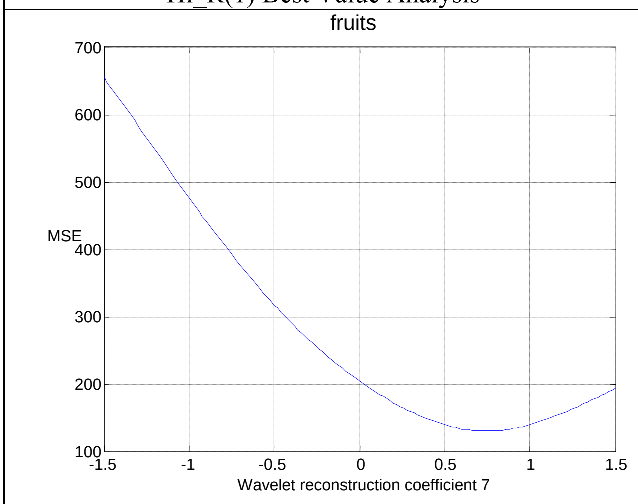
Since these reconstruction coefficients work on compressed and de-quantized image which has lost information because of quantization, these Hi_R wavelet coefficients have to adapt in order to reconstruct image which is as close to the original image.



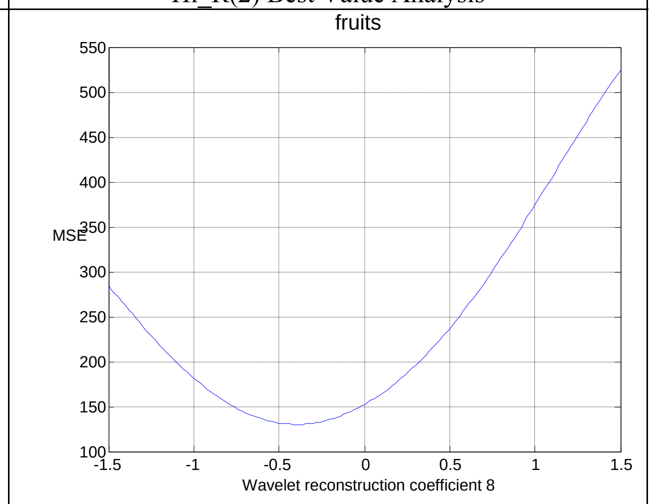
Hi_R(1) Best Value Analysis



Hi_R(2) Best Value Analysis



Hi_R(3) Best Value Analysis



Hi_R(4) Best Value Analysis

Fig. 3b: The Best Value Analysis of Inverse High Frequency Wavelet Coefficients

If the best value for each reconstruction wavelet coefficient Lo_R (1:4) and Hi_R (1:4) is picked for reconstructing the image, the overall quality of the image worsens as is shown in Fig. 4a of Fruits where the mse of the best value coefficient became 140.4532 in comparison to the mse of 131.1139 for the image constructed with wavelet coefficients.

However, if only those coefficients for the best value are picked for which the error was less than the wavelet coefficients then there is small improvement in the image over the image constructed with the wavelet coefficients. The following Fig. 4b shows that the mse of the reconstructed image reduced to 128.1626 when the best values for Hi_R (1:4) were picked and the rest were left with the standard wavelet coefficients. The above experiments show that the reconstruction wavelet coefficients are not independent.

The best value for each reconstruction parameter is best only when the other seven were fixed. When the other seven parameters are changed best value for a particular parameter does not remain best. This shows that all the parameters should be taken in unison for finding the best value which will minimize the error of the reconstructed image.

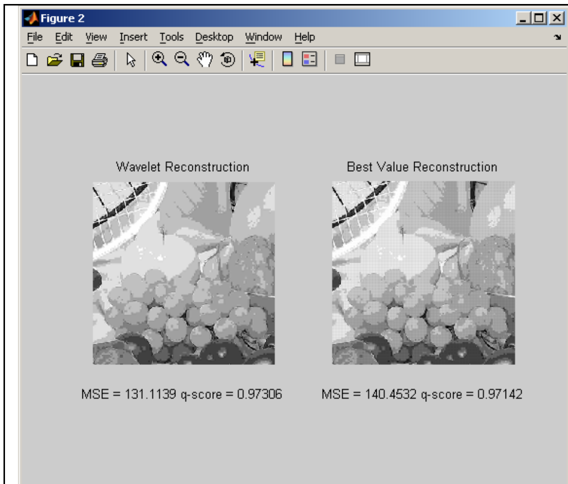


Fig. 4a: Image Reconstruction with best value for Lo_R(1:4) and Hi_R (1:4) Parameters

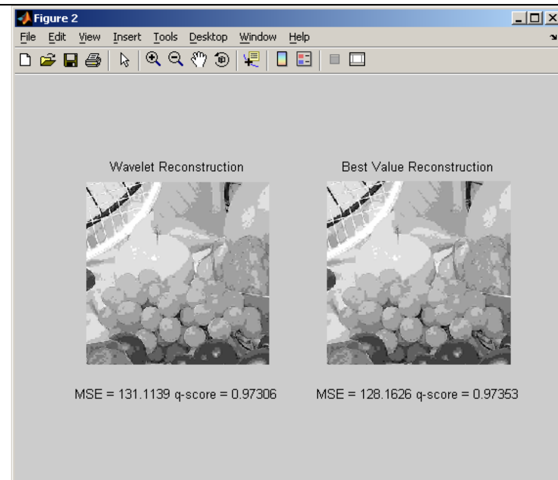


Fig. 4b: Image Reconstruction with best values from Hi_R(1:4)

3. Conclusion

The experiments in this study conclude that wavelet coefficients are not independent of each other. The best value for Reconstruction Coefficients Lo_R and Hi_R deviate from the original discrete wavelet coefficients substantially. This is because the reconstruction coefficients work on the quantized compressed image to reconstruct the image. There is loss of information in the quantized compressed image. Therefore in order to minimize the error in the reconstructed image the best value reconstruction parameters have to deviate from the standard constant values of discrete wavelet coefficients. Hi_R coefficients deviate the most and that explains why the error in the reconstructed image is mostly confined around the edges.

The techniques to reduce the error in the reconstructed image should focus on finding the best value of all the reconstruction coefficients taken together and not taken individually.

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