The Study of Fuzzy Performance Evaluation

Ching-Cheng Shen
Graduate Program in Travel Industrial Management
Kaohsiung Hospitality College
1, Sung-Ho Rd., Hsiao-Kang, Kaohsiung, Taiwan, R.O.C.

Kun-Lin Hsieh*(Corresponding author)
Department of Information Management
National Taitung University
684, Sec.1, Chung-Hua Rd., Taitung 950, Taiwan, R.O.C.

Abstract: - As for the performance comparison, we can compare the difference between the actual performance and ideal performance with simultaneously taking all criteria into consideration. Hence, the practitionerw will meet a problem of MCDM. Besides, the importance of each criterion may have different priority and it should be taken into analysis. In this study, we will apply the concept of fuzzy weight aggregation into performance aggregation. An illustrative example will be also applied to demonstrate the effectiveness of proposed procedure.

Key-Words: - multiple criteria decision-making (MDCM), fuzzy weight aggregation (FWA), performance evaluation

1 Introduction

The choice of weight value, the summation effect of the weight values for several criterions, and the difference comparison among several performances with weighted consideration frequently bother the practitioner for the issue of multiple criteria decision-making (MDCM) (Zeleny, 1982; Zeleny, 1992, Shen et al., 2006). As for the choice of weight value, it will be not easy to be determined during the multiple criteria consideration for those practitioners. Hence, analytic hierarchy process (AHP) (Saaty, 1980; Lai, et al., 1999; Ramanathan & Ganesh, 1995; Ghodsypour & O'Brien, 1998; Tam & Tummala, 2001; Koe, et al., 1999; Ngai, 2003) was developed to overcome such issue by dividing them into the primary criteria and sub-criteria with hierarchy consideration. The feasible weight value can be computed and the test of consistency can also be made via AHP methodology. However, the test of consistency during AHP frequently limited the real applications. Restated, how to resolve the test of consistency will be a worthy issue to be studies. As for the summation effect of the weight value, the practitioners will face the problem of how to summarize the weight effect of each criterion with the case of several experts. The mean operation was the method frequently been used to compute the weight value of each criterion in such case. However, the variation between different experts will be omitted by using the concept of mean to compute the weight value. Restated, the problem of common consensus during those experts will be met. Generally, the larger importance degree of criterion will denote that the corresponding weight value must set a larger value.

As for the difference comparison among criteria' performance, we frequently apply the standardization or fuzzy membership function into the base of performance evaluation. Restated, it will analyze separately each criterion since making the necessary comparison. Due to overcome such problems mentioned above, therefore, we will initially propose a MDCM based on the fuzzy aggregation operator, one is for weight value (it is called as FWA) and another is for performance (it is called as FPA), to address the problem of weight value. The weight value of each criterion will be obtained with the consideration of the average intension for each criterion and the common consensus for experts at the same time (Shen et al., 2006). Then, we will incorporate FWA and FPA to analyze the MCDM problem. Finally, we also take an example, owing to the competence analysis for the citied Motels at Taiwan, to demonstrate the rationality and feasibility of our proposed model.

2 Literature Review

Generally, the Delphi technique (Noorderhaver, 1995; Schermerhorn, Hunt & Osborn, 1985; Robbins, 1991; Hwang & Lin, 1987) is a known way to obtain the weight value of each criterion based on the questionnaire investigation. The viewpoint of weight value for several experts can be obtained by using Delphi technique. However, the procedures of performing the Delphi need enough time even though that it will be done via e-mail and Internets.

The analytic hierarchy process (AHP), which was introduced by Saaty (1971), is measurement method to determine the relative importance or preference of a set of activities, in a multiple criteria decision-making (MCDM) problem. It can incorporate judgments on intangible qualitative criteria, as well as tangible quantitative criteria. This method uses pairwise comparisons of multiple criteria be collect data for decision-making. Using an AHP, an evaluation team is able to systematically evaluate and determine the priorities of the criteria and sub-criteria. Based on this information, the team can then effectively evaluate several potential projects and select the best project. The AHP has been successfully applied to widespread problems, including a multimedia authoring system selection (Lai, et al., 1999), resource allocation problems (Ramanathan & Ganesh, 1995), suppliers selection (Ghodsypour & O'Brien, 1998; Tam & Tummala, 2001), convenience stores location (Koe, et al., 1999), online advertising selection (Ngai, 2003) and so on. The detailed AHP evaluation procedure can be referred to Satty (1980).

3 Proposed Procedure

In this section, we will clearly describe the logistic thinking which incorporates the fuzzy aggregation operator and the grey relational analysis (Deng, 1989) into and integrated procedure. The primary concept can be explained as following: "Due to the problem we addressed is a MCDM issue, how to choose the weight value for each criterion will be met initially. In order to consider the mean intension of each criterion and the common consensus among several experts at the same time, we design a fuzzy aggregation operator to compute the rational and feasible weight value. After the weight value being computed, we can take the GRC value derived from the grey relationship analysis to evaluate those projects." An integrated procedure including nine steps will be described clearly as follows:

Step1.Compute the membership degree μij according to Xij in a evaluation matrix.

Assume that there are n criteria, m experts, an important evaluation value Xij of i-th criterion for j-th expert, then we can construct an evaluated matrix by n*m. Then, we can obtain the ideal point of each criterion by finding the maximum value Xij among those criteria. Next, the membership degree μij for each point with respect to the ideal point can be computed via the Equ (1):

$$\mu_{ij} = \frac{X_{ij}}{\max_i \{X_{ij}\}} \tag{1}$$

Step2.The harmonizing mean of each criterion can be computed as Equ (2). Where α will denote the degree of importance, and the larger α will represent the enlarger effect of importance.

$$h_i = \frac{1}{m}\sum_{j=1}^{m}\frac{1}{(\mu_{ij})^{\alpha}} \tag{2}$$

Step3.The average degree of weight value (ei) of i-th criterion can be computed as Equ (3) by using the Equ (2).

$$e_i = \frac{1}{h_i} \tag{3}$$

Step4.The average weight of i-th criterion (wi) can be computed as Equ (4) by using ei.

$$w_i = \frac{e_i}{\sum_{j=1}^{n} e_i} \tag{4}$$

Step5.Construct the ideal performance Y_{oi} for criteria i and compute the close degree for Yki in evaluation matrix with respect to the ideal performance.

Assuming there are n criterions and the actual performance for the k-th project for the i-th criterion, we can construct the evaluation matrix. During such matrix, the maximum ideal point for each criterion can be determined as Y_{oi} =max Yki. Then, the close degree for each project with respect to the ideal point Pki can be computed as Equ (5):

$$P_{ki} = \frac{Y_{ki}}{\max_k \{Y_{ki}\}} \tag{5}$$

Step6. The summarization performance value APk for the k-th project can be computed as Equ (6).

Where w_i will denote the weight value for the i-th criterion.

$$AP_k = \frac{1}{\sum_{i=1}^{n} w_i \frac{1}{P_{ki}}} \tag{6}$$

In order to verify the rationality and feasibility of the proposed procedure, we will assume several scenarios to make the necessary deduction. The characteristics of wi obtained from the fuzzy aggregation operator (FWA) and the performance value APk obtained from the fuzzy aggregation operator (FPA) can be given as follows:

From Figure 1 and Figure 2, we can find out the fact that the degree of sensitivity gradually increase along with the larger α value. The margin of the change for e_i will have a significantly effect when the α exceeds 1; and the margin of the change for e_i will have a non-significantly effect when the α does not exceed 1. Besides, reviewing Figure 3, the sensitivity analysis of the weight value derived from FWA for the different α value under the different combination of two experts will be graphically depicted.

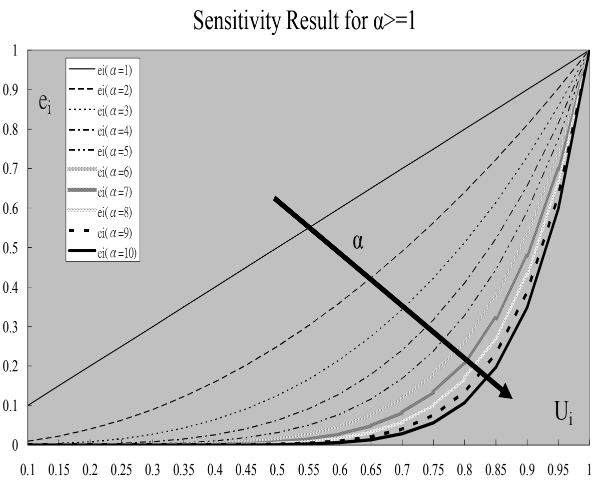

Figure 1. The change diagram of sensitivity analysis for α >= 1.

Characteristic 1: The larger Pki value will lead to a larger APk.

We can make the proof by taking the first order deviation of APk with respect to Pki. That is, we must prove $\frac{\partial AP_k}{\partial P_{ki}} \succ 0$ and the proof procedure will be listed in Equ (7). Although the larger Pki will lead to a larger APk, the added magnitude of APk will gradually decrease along with adding Pki. The larger Pki will lead to a larger APk, and the added magnitude of APk will gradually increase along with adding w_i.

$$\frac{\partial AP_k}{\partial P_{ki}} = \frac{\frac{w_i}{p_{ki}^2}}{\left(\sum_{i=1}^{n} w_i \cdot \frac{1}{P_{ki}}\right)^2} = AP_{ki}^2 \cdot \frac{w_i}{p_{ki}} \succ 0 \tag{7}$$

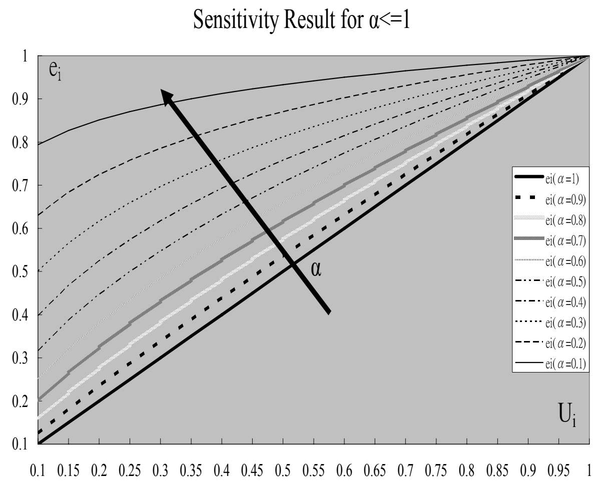

Figure 2. The change diagram of sensitivity analysis for α <= 1.

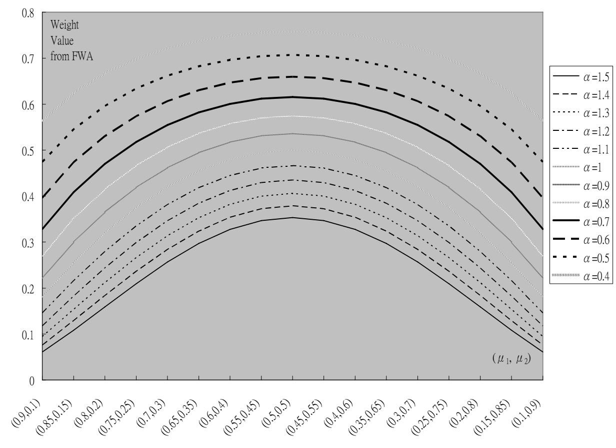

Figure 3. The sensitivity analysis of the weight value derived from FWA for the different α value under the different combination of two experts.

Characteristic 2: If the weight value of one criterion increase, the other criterion will decrease due to that the summation of all weight values equal to 1.

Hence, we can apply the full deviation of APk with respect to w_i into analyzing the affection of APk derived from the weight values and it is denoted as Equ(8). From Equ (8), when w_i increase Δw and

w_j decrease $-\Delta w$ under the condition Pki > Pkj, APk will increase and the added magnitude of APk will gradually increase along with adding Δw.

$$dAP_k = \sum_{i=1}^{n}\left(\frac{\partial AP_k}{\partial w_i}\cdot dw_i\right) = \sum_{i=1}^{n}\left(\frac{\frac{-1}{P_{ki}}}{(\sum_{i=1}^{n} w_i \cdot \frac{1}{P_{ki}})^2}\cdot dw_i\right) = \sum_{i=1}^{n}\left(AP_k^2 \cdot \frac{-1}{P_{ki}} \cdot dw_i\right) \quad (8)$$

4 Illustrative Example

In this section, we apply an illustrative example, owing to the issue of competence analysis for ten citied Motels at Taiwan, to demonstrate the rationality and feasibility of the proposed procedure. After the interview to those dealers of ten citied Motels, we take several evaluation indices depending on the recommendation from dealers or the related theories, which including attraction of marketing, specialty of attendant, uniqueness of equipments, difference of price and the index of finance to analyze the competence. Herein, the attraction of marketing also includes three sub-criteria: the website construction, advertisement kanban and print media. As for the specialty of attendant, three sub-criteria are included, i.e. the attitude of service, the knowledge of specialty and the skills of service. Besides, three sub-criteria are also included for the difference of price: the pricing of room, the discount between workday and holiday, and the additional value. However, only two sub-criteria will consist of the uniqueness of equipments including the design of scenarios and the additional equipment. Finally, the index of finance will be consisted of five sub-criteria including the cost of investment, the turnover rate of room, the capability of gain and capital recovery factor. Next, the weight values of the primary criteria and the sub-criteria can be computed according to the fuzzy aggregation operator. Then, we will obtain the evaluation score of the real performance for those ten motels. Herein, we will take Likert scale of 5. That is, score 5 will denote the best case and score 1 will denote the worst case. Next, we will compute the close degree for each criterion with respect to their ideal point. Finally, we can compute the final performance value by using our FWA and FPA, and the result will be given in Table 1.

From Table 1, we can find out that the average performance value of Firm5、Firm6、 Firm7 are the largest and the standard deviations will denote as Firm5 > Firm6 > Firm7. And the result obtained from our proposed approach will also denote as the same sequence Firm5 > Firm6 > Firm7. However, the result obtained from the weighted average will denote a different sequence result as Firm7 > Firm6 > Firm5. As for Firm1, Firm2, Firm3 and Firm10 with the same performance value and the standard deviation will denoted as Firm2 > Firm1=Firm10 > Firm3, the result obtained from our proposed approach will represent as Firm3> Firm1> Firm10> Firm2. The result obtained from the weighted average will denote a wrong sequence as Firm10 > Firm 3> Firm 2> Firm1. From the above discussion, we can demonstrate the rationality and feasibility of our proposed approach.

Table 1.Comparison table for the performance values obtained from the weighted average and the proposed approach.

	Average	Std. dev	Weighted Average	Rank	FPA	Rank
Firm 1	0.866667	0.037374	0.882351	7	0.862105	6
Firm 2	0.866667	0.042164	0.882596	6	0.852126	8
Firm 3	0.866667	0.031873	0.883796	5	0.868898	4
Firm 4	0.853333	0.030654	0.865238	8	0.862954	5
Firm 5	0.906667	0.026667	0.917603	2	0.90662	1
Firm 6	0.906667	0.033046	0.917222	3	0.899825	2
Firm 7	0.906667	0.047275	0.924436	1	0.886819	3
Firm 8	0.800000	0.039036	0.806806	10	0.783272	10
Firm 9	0.813333	0.04125	0.832176	9	0.803224	9
Firm 10	0.866667	0.037374	0.892625	4	0.860304	7

6 Concluding Remarks

From the deduction procedure and demonstration about the proposed MDCM based on FWA and FPA, no only the weight value of each criterion can be obtained and the average intension of criterion's importance and common consensus of experts also can be incorporate into the considerations of weight summation effect. The weight value derived from the proposed MDCM will be more subjective and it also be possibly identified by most practitioners. Besides, for resolving the problem of MCDM, we also incorporate the concept of fuzzy analysis into our proposed procedure. Finally, we also provide an illustrative example, owing to the issue of competence analysis for the citied Motels at Tawian, to demonstrate the rationality and feasibility of our proposed model.

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