An Integer Programming Model with Special Forms for the Optimum Provision of Needed Manufactures with an Application Example

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Abstract: - Many practical problems are concerned with the provision of manufactured components; the provision can be done using three different methods, production in normal and over times, importing/storing, and subcontracting. The decision maker can evaluate the market demand, and the total cost elements. The problem is to determine how many components should be provided using different methods in order to accomplish the required demand and to minimize the total provision cost.

A mathematical model for such a problem is formulated. The decision variables represent the number of components to be provided using different methods. The objective function is to minimize the total fixed and variable costs. A special (If-Then) form for the objective function is generated due to the fixed costs associated with some of the provision methods. Another combined (If-Then/Or) form for the objective function is generated due to the fixed cost associated with the subcontracting method. A third special form for the objective function is generated due to the stepped unit prices associated with the subcontracting method. The problem contains many different constraints: the provision of the total demand, the maximum capacity constraints, the overtime conditions, the construction of more prevision lines, the subcontracting stepping prices conditions.

The steps of the used algorithm are summarized, and a real example of application is presented, the mathematical model has 9 construction integer decision variables and 11 additional 0-1 variables. The model is solved using LINGO 9.0 software package, and the final solution is obtained. The obtained optimal solution comprises two combined different provision methods, and saves SR 580,000 or 27,750,000 compared to other solutions using only one method of provision.

***Key-Words:* -** Integer programming, Provision methods, If-then and Fixed cost conditions.

1 Introduction

Many practical problems are concerned with the provision of manufactured components; the provision of these components can be done using three different methods: production, importing/ storing, and subcontracting with a third party. The total market demand can be estimated, and also the total fixed and variable cost elements for each method can be evaluated.

In practice, it can be easy to evaluate the total fixed and variable costs associated with one of the candidate methods, but because each method has its maximum limited capacity, then it is required to establish an addition line to meet the required demand. In such cases, when the required demand is too much, the number of feasible solutions will be huge and cumbersome for evaluation.

The solution of such problems using the method of exhaustive evaluation of all possible alternatives is not practical. Even no guarantee that the selection of the shown solutions constituting the selection of one method with its full capacity and completing with another method will give an optimal solution. It is needed to build a mathematical model to represent the complete problem with the objective of minimizing the total provision cost, and to cover all possible choices for provision of components.

The second part of this research is devoted to the problem formulation, the third part to the definition of the decision variables, the fourth part to the construction of the objective function, the fifth part to the formation of the problem constraints, the sixth part to the steps of the procedure for the solution algorithm, the seventh part to presenting a practical example of application, and the last part to the conclusions and points for future researches.

2 Problem Formulation

The problem is to determine how many components should be provided using individual method with the aim to minimize the total cost of provision while to accomplish the required demand under the constraints of maximum provision capacity for each plant in every method of provision. The provision method that can be: production in normal and / or overtime, importing/ storing, and subcontracting. The total cost comprises fixed and variable cost elements of provision for all the needed components.

The problem constraints many different constraints: The provision of the total demand, the maximum capacity constraints for the production line in normal time and in overtime, importing / storing maximum capacity for different plants, and subcontracting constraints with stepped unit prices that decreases with the committing for a certain level of components. Another type of constraints is the overtime condition which constitutes that the overtime can not be performed unless the total capacity in normal time is totally exhausted. The problem contains another additional type of constraints corresponding to the construction of more provision lines, these addition lines can not be established unless the total capacity of the previous lines is totally exhausted. At last, the problem contains also the subcontracting stepping prices. The unit price for subcontracting are not constant, it has the shape of stepping prices related to the amount of components subcontracted.

3 Decision Variables

Let: x_q^r represents the number of components to be provided using the method q, in plant No. r, where $r = 1$ to n_q (the number of lines of type q), and q represents a provision method that can be: production line (p), overtime (o), importing (i), and subcontracting (s). So a problem with two production lines, two stores for imported components, and three level steps of subcontracts will contain the following decision variables:

x_p^1 = The number of components to be produced in production line No.1,

x_p^2 = The number of components to be produced in production line No.2,

x_o^1 = The number of components to be produced in overtime in production line No.1,

x_o^2 = The number of components to be produced in overtime in production line No.2,

x_i^1 = The number of components to be imported and stored in the first store,

x_i^2 = The number of components to be imported and stored in the second store,

x_s^1 = The number of components to be subcontracted with price in the first interval,

x_s^2 = The number of components to be subcontracted with price in the second interval,

x_s^3 = The number of components to be subcontracted with price in the third interval,

4 Objective Function

4.1 Objective Function Coefficients

Let f_j^n, and v_j^n respectively represent the fixed and variable cost coefficients in the objective function corresponding to the variable x_j^n.

4.2 If-Then Constraints for the Fixed Cost

To minimize the total costs of provision of all needed panels.

$$\text{Min } z = \begin{cases} \sum_j \sum_{n=1}^{n_j} v_j^n x_j^n + f_j^n, & x_j^n > 0 \\ 0, & x_j^n = 0 \end{cases}$$

This means that:

If $x_j^n > 0$, then a term f_j^n corresponding to the total fixed cost for provision mean j No. n, should be added, this term should be deleted in case when $x_j^n = 0$. So for the chosen size problem, we have:

1. If $x_p^1 > 0$, then a term corresponding to the total fixed cost for production line No. 1 in normal time f_p^1, should be added, this term should be deleted in case when $x_p^1 = 0$.
2. If $x_p^2 > 0$, then a term corresponding to the total fixed cost for production line No. 2 in normal time f_p^2, should be added, this term should be deleted in case when $x_p^2 = 0$.

3. If $x_i^1 > 0$, then a term corresponding to the total fixed cost for importing and storing the panels f_i^1, should be added, this term should be deleted in case when $x_i^1 = 0$.
4. If $x_i^2 > 0$, then a term corresponding to the total fixed cost for importing and storing the panels f_i^2, should be added, this term should be deleted in case when $x_i^2 = 0$.
5. If (x_s^1 OR x_s^2 OR x_s^3 OR any combinations of them) > 0 , then a term corresponding to the total fixed cost for subcontracting the panels f_s, should be added, this term should be deleted in case when all x_s^1, x_s^2, and $x_s^3 = 0$.

These are nonlinear in x_p^1, x_p^2, x_i^1, x_i^2, x_s^1, x_s^2, and x_s^3 because the discontinuity at the origin.

It is known that:

$0 \le x_j^n \le N_j^n$ for all j, and n.

So, we have:

$0 \le x_p^1 \le N_p^1$,

$0 \le x_p^2 \le N_p^2$,

$0 \le x_i^1 \le N_i^1$,

$0 \le x_i^2 \le N_i^2$,

$0 \le x_s^1 \le N_s^1$,

$0 \le x_s^2 \le N_s^2$,

$0 \le x_s^3 \le N_s^3$,

f_p^1, f_p^2 , f_i^1 , f_i^2 , and $f_s > 0$.

Where:

N_p^1 = The maximum production capacity of production line No. 1 in normal time,

N_p^2 = The maximum production capacity of production line No. 2 in normal time,

N_i^1 = The maximum importing and storing capacity in store No. 1,

N_i^2 = The maximum importing and storing capacity in store No. 2,

N_s^1 = The maximum subcontracting quantity with the first unit price,

N_s^2 = The maximum subcontracting quantity with the second unit price,

N_s^3 = The maximum subcontracting quantity with the third unit price

The nonlinearity can be transformed into linear minimization model as follows:

$$\text{Min. z} = \sum_j \sum_{n=1}^{n_j} v_j^n x_j^n + f_j^n y_j^n,$$

Subject to:

$$x_j^n \le N_j^n y_j^n, \ \forall \ j, n$$

$$y_j^n = 0, 1. \qquad \forall \ j, n.$$

If $0 < x_j^n \le N_j^n$, then y_j^n is forced to be 1, to eliminate the infeasibility, and

If $x_j^n = 0$, then no restrictions are imposed on y_j^n , it can be equal to 0, or 1, but according to the minimization of the objective function, it will be forced to have the value of 0.

So for the chosen sized problem, we have:

Min. z =

$$v_p^1 x_p^1 + f_p^1 y_p^1 + v_p^2 x_p^2 + f_p^2 y_p^2 + v_i^1 x_i^1 + f_i^1 y_i^1 + v_i^2 x_i^2 + f_i^2 y_i^2 + v_s^1 x_s^1 + v_s^2 x_s^2 + v_s^3 x_s^3 + f_s y_s$$

Subject to:

$$x_p^1 - N_p^1 y_p^1 \le 0,$$
$$x_p^2 - N_p^2 y_p^2 \le 0,$$
$$x_i^1 - N_i^1 y_i^1 \le 0,$$
$$x_i^2 - N_i^2 y_i^2 \le 0,$$
$$y_p^1, y_p^2, y_i^1, y_i^2 = 0, 1.$$

4.3 Combined If-Then / Or Constraints

For the subcontracting variant:

If (x_s^1 OR x_s^2 OR x_s^3 OR any addition combinations of them) > 0 , then a term corresponding to the total fixed cost for subcontracting the panels f_s, should be added, this term should be deleted in case when all x_s^1, x_s^2, and $x_s^3 = 0$.

This can be done by adding the following constraint:

$$\sum_{j=1}^{n_s} x_s^j \le y_s \sum_{j=1}^{n_s} N_s^j ,$$

$$y_s = 0, 1.$$

4.4 Objective Function for Variable Costs

Min $z = \sum_k \sum_{m=1}^{m_k} v_k^m x_k^m$, for all k, and m having variable cost elements.

Where:
k = A method of provision having variable cost coefficients,
m_k = The number of lines of a method k.
So for the chosen size problem, we have:
Min. z = $v_o^1\ x_o^1 + v_o^2\ x_o^2 + v_s^1\ x_s^1 + v_s^2\ x_s^2 + v_s^3\ x_s^3$,
Where:

v_o^1 = The unit variable cost for over time production in line No. 1,

v_o^2 = The unit variable cost for over time production in line No. 2,

v_s^1 = The unit cost for subcontracting with normal price,

v_s^2 = The unit cost for subcontracted with the first reduced price,

v_s^3 = The unit cost for subcontracted with the second reduced price,

4.5 Objective Function for the Problem

$$\text{Min. z} = \sum_j \sum_{n=1}^{n_j} v_j^n\ x_j^n + f_j^n\ y_j^n + \sum_k \sum_{m=1}^{m_k} v_k^m x_k^m, \text{ for all } j, \text{ and } k, \quad \ldots\ldots\ldots (1)$$

So, for our problem, we will have:
Min. z

$$= v_p^1\ x_p^1 + f_p^1\ y_p^1 + v_p^2\ x_p^2 + f_p^2\ y_p^2 + v_o^1\ x_o^1 + v_o^2\ x_o^2 + v_i^1\ x_i^1 + f_i^1\ y_i^1 + v_i^2\ x_i^2 + f_i^2\ y_i^2 + v_s^1\ x_s^1 + v_s^2\ x_s^2 + v_s^3\ x_s^3 + f_s\ y_s \quad \ldots\ldots (1.1)$$

5 Constraints

5.1 Fixed Cost Coefficients Constraints

$x_j^n - N_j^n\ y_j^n \le 0$, $\forall\ j, n$ corresponding to fixed cost coefficients ……………………………… (2)

$y_j^n = 0, 1.$ $\forall\ j, n.$ ……………………….. (i.1)

So we will have the following constraints for our problem:

$x_p^1 - N_p^1\ y_p^1 \le 0,$ ……………................... …. (2.1)

$x_p^2 - N_p^2\ y_p^2 \le 0,$ ……………….................(2.2)

$x_i^1 - N_i^1\ y_i^1 \le 0,$ ……………………………… (2.3)

$x_i^2 - N_i^2\ y_i^2 \le 0,$ ……………………………… (2.4)

$y_p^1,\ y_p^2,\ y_i^1,\ y_i^2 = 0, 1.$ ……………………... (i.1.1)

For the stepped price subcontracting, we have:

$\sum_{j=1}^{n_s} x_s^j \le y_s \sum_{j=1}^{n_s} N_s^j$, …………………………… (3)

$y_s = 0, 1.$ ………………………………………….. (i.2)

So, we will have for our problem:

$x_s^1 + x_s^2 + x_s^3 - y_s(N_s^1 + N_s^2 + N_s^3) \le 0,$ … (3.1)

$y_s = 0, 1.$ ………………………………………….. (i.2.1)

5.2 Provision of Total Demand

$$\sum_q \sum_{r=1}^{n_q} x_q^r = \text{N}, \quad \ldots\ldots\ldots\ldots (4)$$

Where:
N = The total No. of required components.
So for the chosen problem, we have:

$x_p^1 + x_o^1 + x_p^2 + x_o^2 + x_i^1 + x_i^2 + x_s^1 + x_s^2 + x_s^3 = \text{N}$ ……………………………………………(4.1)

5.3 Maximum Capacity Constraints

$x_q^r \le N_q^r \quad \forall q, r,$ …………………………………(5)

Where:
N_q^r = maximum capacity of method of provision q No. r.

5.3.1 Production Line Normal Capacity

$x_p^1 \le N_p^1,$ ……………………………………… (5.1)

$x_p^2 \le N_p^2.$ ……………………………………… (5.2)

Where: N_p^1 = The maximum production capacity of production line No. 1 in normal time,
N_p^2 = The maximum production capacity of production line No. 2 in normal time.

5.3.2 Production Line Capacity (in Over Time)

$x_o^1 \le N_o^1,$ ……………………………………… (5.3)

$x_o^2 \le N_o^2.$ ……………………………………… (5.4)

Where: N_o^1 = The maximum production capacity of production line No. 1 in over time,

N_o^2 = The maximum production capacity of production line No. 1 in over time.

5.3.3 Importing/ Storing Maximum Capacity

$x_i^1 \le N_i^1$, ………………….……………… (5.5)

$x_i^2 \le N_i^2$ ……………………...………… (5.6)

Where: N_i^1 = The maximum production capacity of production line No. 1 in over time,

N_i^2 = The maximum production capacity of production line No. 1 in over time.

5.3.4 Subcontracting Maximum Capacity

$x_s^1 \le N_s^1$, ……………………………… (5.7)

$x_s^2 \le N_s^2$, ………….…………………… (5.8)

$x_s^3 \le N_s^3$. ……………………………… (5.9)

Where: N_s^1 = Maximum amount of subcontracted components with the first unit price,

N_s^2 = Maximum amount of subcontracted components with the second unit price,

N_s^3 = Maximum amount of subcontracted components with the third unit price.

5.4 IF-Then Constraints

5.4.1 If-Then Overtime Constraints

The overtime can not be performed unless the total capacity in normal time is totally exhausted. The If-Then constraints are as follows:

If $x_p^r < N_p^r$ then $x_o^r = 0$, $r = 1, 2, n_p$,

Where: n_p = number of production lines considered.

In a two production lines problem, we have:

1- If $x_p^1 < N_p^1$ then $x_o^1 = 0$,

2- If $x_p^2 < N_p^2$, then $x_o^2 = 0$.

To transform the conditional constraints into an equivalent set of constraints without the conditional relation, and knowing that:

$x_p^r \le N_p^r$,

$x_o^r \le N_o^r$,

The following formulation can be used, for each production line $r = 1, 2, ..., n_p$:

$x_p^r \ge N_p^r \lambda_o^r$, ………………..……………… (6)

$x_o^r \le N_o^r \lambda_o^r$, …………………..……..……… (7)

$\lambda_o^r = 0 , 1.$ ………………...………..……… (i.3)

If $x_p^r < N_p^r$, then the constraint (5) will force λ_o^r to be equal to 0, and then from the constraint (6), x_o^r will be equal to 0.

If $x_p^r = N_p^r$, then from (5) λ_o^r can take the value 0 or 1, and according to (6), λ_o^r will be adjusted to 1 if x_p^r is to be positive in the optimal solution.

The conditional constraint for the first production line can be formulated as follows:

$x_p^1 \ge N_p^1 \lambda_o^1$,

$x_o^1 \le N_o^1 \lambda_o^1$,

$\lambda_o^1 = 0 , 1.$

These relations can be written in the form:

$x_p^1 - N_p^1 \lambda_o^1 \ge 0$, ………………..………… (6.1)

$x_o^1 - N_o^1 \lambda_o^1 \le 0$, …………..……...………… (7.1)

$\lambda_o^1 = 0 , 1$. …………..….……………… (i.3.1)

In the same manner the conditional constraint for the second production line can be formulated as follows:

$x_p^2 - N_p^2 \lambda_o^2 \ge 0$, ……………………………
(6.2)

$x_o^2 - N_o^2 \lambda_o^2 \le 0$, ……….…..……………… (7.2)

$\lambda_o^2 = 0 , 1$. ….…………..…………… (i.3.2)

5.4.2 If-Then Constraints for Additional Production Lines

An additional production line can not be established unless the total capacity of the previous production line in normal and over times is totally exhausted. This constraint can be formulated as follows, for each production line $r = 1, 2, ..., n_p - 1$:

If $(x_p^r + x_o^r) < (N_p^r + N_o^r)$ then $x_p^t = 0$, $t = r+1, r+2, ..., n_p$.

Then, for a problem with two production lines, we have:

If $(x_p^1 + x_o^1) < (N_p^1 + N_o^1)$ then $x_p^2 = 0$.

This If-Then conditional constraint can be transformed into an equivalent set of constraints without the conditional relation as in the overtime restriction constraint.

Knowing that:

$x_p^r \le N_p^r$, and

$x_o^r \le N_o^r$.

The following formulation can be used:

$(x_p^r + x_o^r) \ge (N_p^r + N_o^r)\ \lambda_p^r$, $r = 1, 2, n_p - 1$, … (8)

$x_p^t \le N_p^t\ \lambda_p^r$, $r = 1, 2, n_p - 1$, $t = r+1, r+2, ..., n_p$, …………………..…….. (9)

$\lambda_p^r = 0, 1$, $r = 1, 2, n_p - 1$. ……...………… (i.4)

In a problem with two production lines, we have:

$(x_p^1 + x_o^1) \ge (N_p^1 + N_o^1)\ \lambda_p^1$,

$x_p^2 \le N_p^2\ \lambda_p^1$,

$\lambda_p^1 = 0, 1.$

These relations can be written in the following form:

$x_p^1 + x_o^1 - (N_p^1 + N_o^1)\lambda_p^1 \ge 0$, ……….….. (8.1)

$x_p^2 - N_p^2\ \lambda_p^1 \le 0$, ……………….………. (9.1)

$\lambda_p^1 = 0, 1$. ………………….……….…. (i.4.1)

5.4.3 If-Then Next Importing/ Storing Constraints

The following formulation can be used, for each:

$x_i^r \ge N_i^r\ \lambda_i^r$, $r = 1, 2, n_i - 1$, ……...…...…… (10)

$x_i^t \le N_i^r\ \lambda_i^t$, $r = 1, 2, n_i - 1$, $t = r+1, r+2, ..., n_i$, ……………………....…….……….. (11)

$\lambda_i^r = 0, 1$, $r = 1, 2, n_i - 1$. ……….………… (i.5)

In a problem with two importing/ storing lines, we have:

$x_i^1 \ge N_i^1\ \lambda_i^1$,

$x_i^2 \le N_i^2\ \lambda_i^1$,

$\lambda_i^1 = 0, 1.$

These relations can be written in the following form:

$x_i^1 - N_i^1\ \lambda_i^1 \ge 0$, …..……...……..…..…… (10.1)

$x_i^2 - N_i^2\ \lambda_i^1 \le 0$, ……….……..……….…….. (11.1)

$\lambda_i^1 = 0, 1.$ ……………………....…………. (i.5.1)

5.5 Subcontracting Stepping Prices Constraints

The unit price for the subcontracting variant are not constant, it has the shape of stepping prices related to the amount of components subcontracted x_s, for example for three steps prices the subcontracting prices are:

v_s^1 for subcontracting quantities $0 \le x_s \le N_s^1$, v_s^2 for subcontracting quantities $N_s^1 < x_s \le N_s^2$, v_s^3 for subcontracting quantities $N_s^2 < x_s \le N_s^3$.

Consider the following decision variables x_s^r representing the subcontracting amount for subcontracting unit price = c_s^r, r = 1, n_s= number of steps of unit prices.

The constraints are as follows:

If $x_s^r < N_s^r$, then $x_s^t = 0$, $t = r+1$, $r+2$, …., n_s, r = 1, 2, ……, n_s - 1.

Then, for a problem with 3 price steps:

1. If $x_s^1 < N_s^1$, then both x_s^2, $x_s^3 = 0$,
2. If $x_s^2 < N_s^2$, then $x_s^3 = 0$,

Knowing that:

$x_s^r \le N_s^r$, r = 1, 2, ……, n_s,

The conditional constraints can be transformed as follows:

$x_s^r \ge N_s^r\ \lambda_s^r$, r = 1, 2, ……, n_s-1, ………..…. (12)

$x_s^t \le N_s^t\ \lambda_s^r$, r = 1, 2, ……, n_s-1, $t = r+1$, $r+2$, …., n_s, ……………………..…….….…. (13)

$\lambda_s^r = 0, 1.$ …………………….…………….…… (i.6)

So, for a problem with 3 stepped subcontracted prices, we will have for the first constraint:

$x_s^1 \ge N_s^1\ \lambda_s^1$,

$x_s^2 \le N_s^2\ \lambda_s^1$,

$x_s^3 \le N_s^3\ \lambda_s^1$,

$\lambda_s^1 = 0, 1.$

These relations can be written as:

$x_s^1 - N_s^1\ \lambda_s^1 \ge 0$, ……...…...……...………. (12.1)

$x_s^2 - N_s^2\ \lambda_s^1 \le 0$, ……………...….…..………. (13.1)

$x_s^3 - N_s^3\ \lambda_s^1 \le 0$, ……….……….....………. (13.2)

$\lambda_s^1 = 0, 1$. ……........….……...….…..………. (i.6.1)

In the same way, the conditional constraint No. 2 can be transformed as follows:

$x_s^2 - N_s^2\ \lambda_s^2 \ge 0$, …………...……………. (12.2)

$x_s^3 - N_s^3\ \lambda_s^2 \le 0$, ………………...………. (13.3)

$\lambda_s^2 = 0, 1$. ……........………….....………. (i.6.2)

5.6 Integral Constraints

$x_q^r\ \forall\ q, r$ Integers. …………….…….….…. (14)

So, for the considered problem, we have:

x_p^1 , x_p^2 , x_o^1 , x_o^2 , x_i^1 , x_i^2 , x_s^1 , x_s^2, x_s^3 Integers ……………….……….……….……. (14.1)

5.7 0-1 Constraints

y_q^r, λ_q^r = 0, 1 $\forall$ q, r . ………………….……. (15)

So, for the considered problem, we have:

y_p^1, y_p^2 , y_i^1 , y_i^2, y_s, λ_o^1 , λ_o^2 , λ_p^1, λ_i^1 , λ_s^1, λ_s^2 = 0 , 1 …………………..……….……..……. (15.1)

6 Steps of the algorithm

The steps of the used algorithm can be summarized as follows: the decision maker should make estimation or forecasting of the demand quantity for the needed manufactured component (N), calculate the needed number of production lines, and import/storing facilities, calculate the maximum capacity for each provision method, calculate the corresponding fixed and variable cost elements, design the corresponding mathematical model, then solve the model and obtain the optimal solution.

7 An example of Application

The Power Transmission & Distribution (PTD) sector in the kingdom of Saudi Arabia continues to show strong and consistent growth as a result of high power demands arising from private, commercial and industrial consumers. PTD in Saudi Arabia has been and continues to play a substantial role in providing products, system and solutions for the power transmission sector. PTD manufactures and markets power transmission and distribution equipment and energy service solutions for electric utilities, independent power producers, co-generators and other energy-intensive industries.

One of the famous companies in the field of PTD offers complete solutions ranging from products, to systems, to turn key solutions, all related after sales support, training, and consulting services. In order to locally support the ever-growing demand of the market, the company has also started its manufacturing. The company has, and continues to be a preferred source to the customers.

7.1 Problem Data

The problem data consists of: The fixed and variable cost elements for the production system lines, labor cost, the variable cost elements for the production system lines in over-time, the fixed and variable cost elements for the importing and storing lines, the fixed and variable cost elements for the importing and storing lines, and the maximum capacity for each source of provision.

7.2 The Complete LINGO Program

```
MIN = 18500*x1 + 18500*x2 + 22500*x3 +
22500*x4 + 40250*x5 + 40250*x6 + 45000*x7 +
40000*x8 + 37000*x9 + 7440000*x10 +
7240000*x11 + 1035000*x12 + 835000*x13 +
300000*x14 ;
x1 - 880*x10 <= 0; x2 - 880*x11 <= 0;
x5 - 1500*x12 <= 0; x6 - 1500*x13 <= 0;
x7 + x8 + x9 - 3500*x14 <=0;
x1 + x2 +x3 + x4 + x5 + x6 + x7 + x8 + x9 = 2000;
x1 <= 880; x2 <= 880; x3 <= 880; x4 <= 880;
x5 <= 1500; x6 <= 1500; x7 <= 500; x8 <= 1000;
x9 <= 2000;
x1 - 880*x15 >=0; x3 - 880* x15 <=0;
x2 - 880*x16 >= 0; x4 - 880*x16 <= 0;
x1 + x3 - 1760*x17 >=0; x2 - 880*x17 <= 0;
x5 - 1500*x18 >= 0; x6 - 1500*x18 <=0;
x7 - 500*x19 >=0; x8 - 1000*x19 <=0;
x9 - 2000*x19 <=0; x8 -1000*x20 >=0;
x9 - 2000*x20 <=0;
@GIN(x1); @GIN(x2); @GIN(x3); @GIN(x4);
@GIN(x5); @GIN(x6); @GIN(x7); @GIN(x8);
@GIN(x9); @BIN(x10); @BIN(x11); @BIN(x12);
@BIN(x13); @BIN(x14); @BIN(x15); @BIN(x16);
@BIN(x17);  @BIN(x18); @BIN(x19); @BIN(x20);
```

7.3 Model Solution

Global optimal solution found at iteration: 59

Objective value = 0.5462000E+08

Non-zero variables:

Variable	Value	Meaning
X1	880.0000	x_p^1
X3	880.0000	x_o^1
X7	240.0000	x_s^1

This means that the optimum provision pattern is a mix between more than one method, this is very hard to consider by a decision maker laying only on intuition or try and error decision behavior.

7.4 Solution Using Only One Method

The total cost for using only one method of provision is calculated in order to compare the obtained cost with that of the optimal solution. The cost is calculated for the method of production, importing and storing, and for the method of subcontracting. All these methods give higher cost than that of the optimal one.

8 Conclusions and Points for Future Researches

8.1 Conclusions

This research has the following conclusions:

1- The problem of provision of components using different methods like: production, importing, and subcontracting with multiple lines has a huge number of alternative solutions that are very difficult to search and to evaluate.

2- It is possible to establish a mathematical model to represent such problems. The formulation has some special forms such as: If-Then, combined If-Then/Or, addition of fixed costs to the objective function, and stepped variable unit costs.

3- The optimal solution may comprise different combined methods of provisions that are very difficult to obtain by try and error or by exhaustive search methods. This optimal solution is less costing than any method using only one type of provision that can be evaluated in practical applications.

8.2 Points for Future Researches

1- To develop similar mathematical model for multi-component problems.

2- To develop a computer program for complete modeling and solving such problems using a modeling and/or general purpose languages.

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