Utilization of Maximum Sensitivity in Control Quality Indication

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Abstract: For the needs of industrial practice a new controller parameter setting method has been presented recently. It is based on experimentally performed evaluation of excited frequency responses with the aim to achieve the recommended values of one or more control quality indicators known from course of the Nyquist plot. Unlike their linear origin, the indicators can be obtained in control loops involving nonlinearities even in the controller. In this sense, the method has philosophy similar to the popular Ziegler and Nichols method. For its implementation no mathematical model and theory is required. All processing is carried out by a program added to the control algorithm. No additional instrumentation is necessary. For a good software solution some investigation concerning choice of the indicator and processing of frequency responses need to be investigated. This is the main subject that the paper is reporting about.

Keywords: controller tuning, critical oscillation, maximum sensitivity, phase margin, gain margin

1 Frequency response in control performance assessment

In some previous publications [10], [11] it was discussed what is accepted by the standard industrial practice from the existing broad offer of the optimising methods helping to improve function of control loops. Conclusion was not very optimistic – except exclusive orders – no theory, no models and no expensive experts are those very often hidden preferences in the practice. Controller pre-setting and its adjustment on the place if necessary are popular steps in such intuitive working process based on empiric knowledge and experience. That is why methods such as that from Ziegler and Nichols keep their popularity up to now why and any new procedure using similar philosophy [1] is accepted much better than the others.

Realizing these circumstances a new modification of the methods utilising for controller setting excited oscillations in the control loop. Unlike Ziegler and Nichols method auto-oscillation are not invoked by reaching a critical setting of the controller, or by inserting a relay as in Aström Relay method, but by (software) adding a harmonic signal to the control error signal processed by the controller. This on one side technically simple realisation has of course several problems that must be solved within the software on the other side. For example, it is recognition of a oscillating steady state when evaluation of the magnitude and phase shift can be started. Secondly, amplitude of the inserted signal must be chosen carefully so that it does not disturb much the controlled variable, but on the other hand it must be distinguishable from the noise. To achieve desired values of indicators both changes in the frequency of exciting signal and in the controller parameter setting must be combined. If more indicators are monitored it is not easy to find out a strategy of performing these changes simultaneously. Therefore we paid our attention to the indicator based on evaluation of maximum sensitivity, because it expresses certain compromise between the optimum characterized by the phase margin and the gain margin.

A great disadvantage of the relay method is that the control function is interrupted, while the critical parameter identifying process is carried out. During this operation, the controller must be disconnected and reconnected without any bump and a steady state must been achieved before. The amplitude of the oscillation added to the controlled variable can be influenced by the parameters of the relay, but it is difficult to forecast its size in advance. Excitation of the oscillation often requires changes from the manipulated variable that are easy to simulate but difficult to execute technically.

In Fig. 1 a Nyquist plot from previous publication is depicted. It serves for recalling definitions of introduced symbols. The plot consists of the points

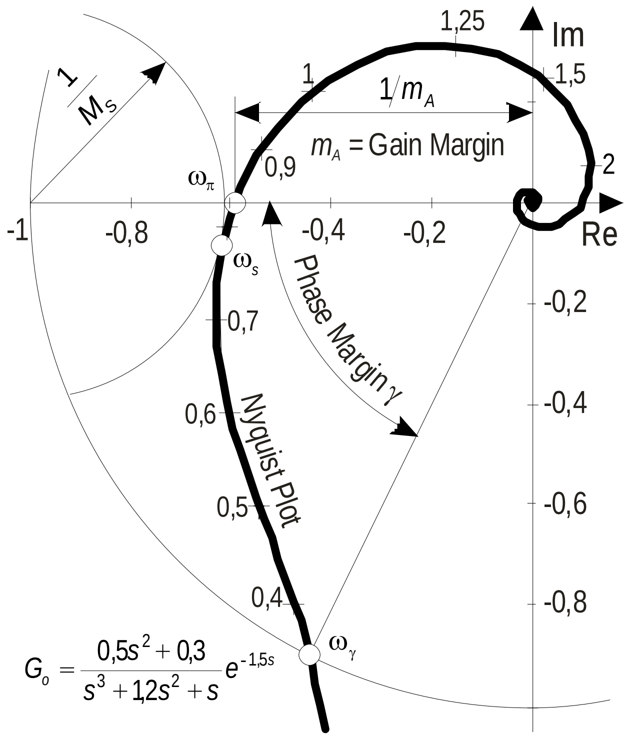

Fig. 1 Some indicators of optimal controller setting coming from the open loop frequency response (Nyquist plot)

calculated from a chosen open loop transfer function $G_o(\mathrm{j}\omega)$ linked with a linear model of the circuit.

It should be noted that although in the following figures, text and formulas symbols such as $|G_o(\mathrm{j}\omega)|$, $|1+G_o(\mathrm{j}\omega)|$, $\arg(G_o(\mathrm{j}\omega))$ are used, in the tuning algorithm they represent only the values obtained from measurement and assessment of excited frequency responses. Any knowledge of a frequency transfer function is not necessary and therefore any linear model replacement of the controlled plant need not to be introduced.

This note is important because in literature it is very often spoken instead of a real control circuit about its model. For the model is the correct notation control system. A graphical representation of a system can be a block scheme depicted e.g. in Fig. 2 where in the description inside both two blocks are used symbols of transfer functions ($G_R(s)$ denotes the transfer function of a controller; $G_P(s)$ represents the transfer functions of the plant). Usage of the transfer functions also says the models are linear.

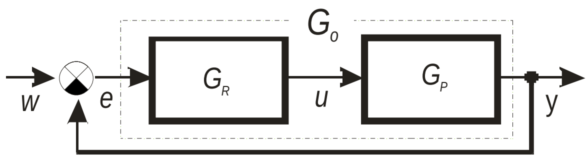

Fig. 2 Block scheme model of a control circuit

2 Control Robustness

In controller design it has been always an important objective to achieve resistance of the control circuit against different and changing conditions during its operation. In reality this situation is normal in contrast with theoretical considerations when we work with a control system instead of a real control circuit. A good designed controller is expected to be sufficiently robust to the changes which are represented e.g. by unmodelled dynamics, extraneous influences, and imperfections in parameter setting. Robustness is sometimes interpreted as reduced necessity of controller retuning. It is well known that PID controllers are preferred because of robustness of their control.

Åström et al tested the indicators for optimal control quality assessment (see Fig. 1) among them the maximum sensitivity M_s with the conclusion that robustness can usually be guaranteed when maximum values of this indicators are in the range from 1,3 to 2 [2]. The value of maximum sensitivity is connected with other indicators by the following relationships

$$m_A \geq \frac{M_S}{M_S - 1} \quad \gamma \geq 2\arcsin\left(\frac{1}{2M_S}\right) > \frac{1}{M_S} \qquad (1)$$

where the gain margin is denoted by m_A, and it is the factor which, multiplying the amplitude of the Nyquist plot characterized by the phase angle $-\pi$, causes the plot to pass the critical point -1+0.j; by γ is denoted the phase margin which expresses the amount of phase shift that can be tolerated before the control loop becomes unstable.

The recommended value of the gain margin ranges from 2 – 2,5; the optimal phase margin are quoted in the range from 30° to 60°.

3 Maximum Sensitivity

This model allows us to define the open loop transfer function $G_o(s)$ as a product of the functions $G_R(s)$ and $G_S(s)$), and also enables to express the transfer function for the load disturbance by the formula

$$G_{dy}(s) = \frac{1}{1 + G_o(s)} \qquad (2)$$

From (1) it follows that the load disturbance transfer with can be characterized by the sensitivity function $S(\mathrm{j}\omega) = G_{dy}(\mathrm{j}\omega)$. Its amplitude maximum over the range of frequencies

$$M_s = \max_{0\leq\omega\leq\infty}\left|\frac{1}{1 + G_o(j\omega)}\right| = \max_{0\leq\omega\leq\infty}\left|S(j\omega)\right| \qquad (3)$$

is defined as the maximum sensitivity M_s . In Fig. 3 shows M_s graphically by means a circle whose centre lies in the critical point -1 + 0.j, its radius is equal to the inverse of M_s and it touches the Nyquist plot in a point which distance from the critical point is evidently the shortest.

Because $1/M_s$ is according to (3) also equal to the absolute value of its denominator $|1+G_o(\mathrm{j}\omega)|$ for the frequency when it achieves its minimum we can interpret the radius r more generally. If we use a vector interpretation of points in the complex plane like in Fig. 3 then the reciprocal value $1/M_s$ is equal the length of the vector $1+ G_o(\mathrm{j}\omega)$ which summed with the vector -1+j.0 gives the vector $G_o(\mathrm{j}\omega)$.

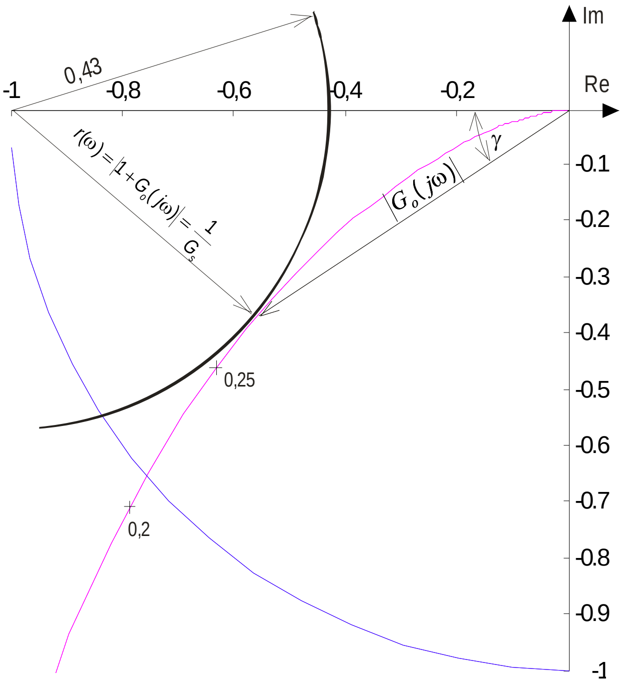

Fig. 3 Maximum sensitivity and Nyquist plot

Now, if the radius r is plotted versus the angle γ obtained from $\arg(G_o(\mathrm{j}\omega))$ for various frequencies ω, we get an interesting graph whose worth will increase if another graph is added to it.

The dependence of the radius r on the angle γ has a minimum corresponding the situation when the circle is touching the Nyquist plot, i.e. a maximum sensitivity M_s is achieved. If this extreme falls into the range of recommended values this indicates optimal setting of the controller. It is possible to check simultaneously another indicator – the phase margin where the angle of the phase margin is given by the crossing of two curves depicted in Fig. 4 .

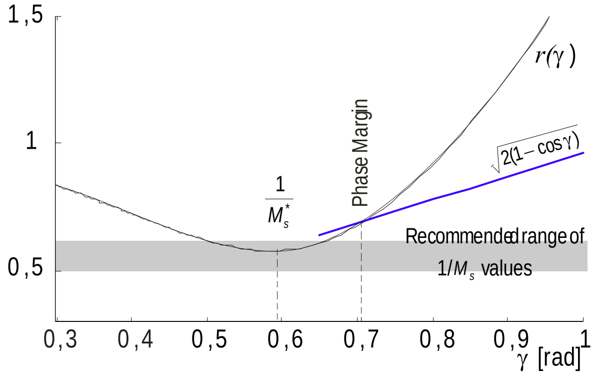

Fig. 4 Dependence $|1+G_o(\mathrm{j}\omega)|$ on the angle $\pi+\arg(G_o(\mathrm{j}\omega))$ and plot of the function $\sqrt{2(1-\cos\gamma)}$

On these two pieces of information it is much easier to organize autotuning changes of controller setting towards a globally interpreted control optimum. This should be demonstrated a little bit by an example of simulation results. Only the integral constant of the controller was changed (r_I = 0,1, 0,2, 0,3). It had a small influence on the eigen frequency of the response, but the overshoot changed with the indicated maximum sensitivity quite markedly.

4 Evaluation of Excited Frequency Responses

To evaluate the frequency response based indicators of an optimal controller setting, two principles can be used:

- phase-locked loop (PLL) identifier module
- direct frequency response assessment.

Phase-locked method

The PPL identifier module, whose block scheme is depicted in Fig. 5 , is based on an assessment of the product of two harmonic signals. By means an oscillator, whose frequency is controlled by an external signal, two signals are generated:

$$u_1(t) = a\cos\omega t \qquad u_2(t) = b\sin\omega t \tag{4}$$

The first signal $u_1(t)$ is used to excite the dynamic system (e.g. the control loop) and then as an output of the system (after amplitude and phase changes have come out), the output is brought to the multiplier whose second input is the directly brought

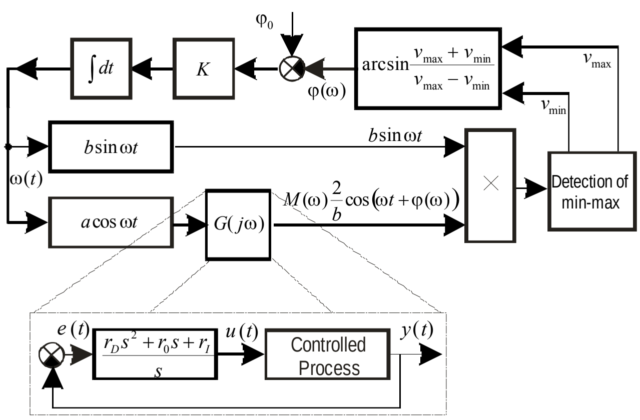

Fig. 5 Block scheme of phase angle identification via PPL method

signal $u_2(t)$. Finding two extreme values $y_{\min}$, $y_{\max}$ from a steady state record of $y(t)$ we can determine the magnitude $M(\omega)$ in a very simply way by subtracting the two output extreme values, especially if a convenient choice for the amplitude $a = 2/b$ is made

$$M(\omega) = \frac{y_{\max} - y_{\min}}{2} \tag{5}$$

The phase angle results from adding $y_{\min}$, $y_{\max}$ (again when $a = 2/b$)

$$\varphi(\omega) = \arcsin \frac{y_{\max} + y_{\min}}{y_{\max} - y_{\min}} \tag{6}$$

The velocity of finding this frequency depends on constant K.

Direct frequency response assessment

Direct assessment is a part of the block which is in Fig. 6 denoted by the label Exciting and Evaluation. In fact this block represents the complete algorithm performing autotuning of the controller building practically one unit with the controller. Because the function of the autotuner is not discussed here in details we do not pay attention to another information that should be somehow presented in the figure such as initial setting, limits for parameters and exciting frequency, accuracy for determining steady state, considered periods for data processing, choice of the exciting signal amplitude, etc. We focus on a short description of steady state fixation, and then on the most important information for optimal control quality indicators – the amplitude gain and the phase shift in harmonic signal transfer.

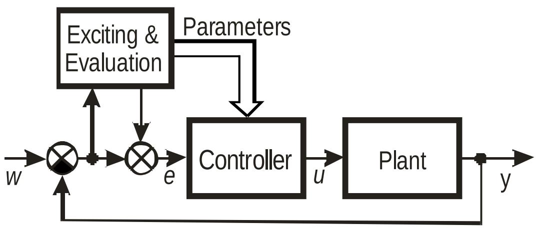

Fig. 6 Block scheme of a control circuit with added block for frequency response exciting and evaluation

Evaluation of the gain and phase shift can be carried on in a steady state which starts to be tested after a time interval of the length T_p to override the control circuit dynamics. After this time a harmonic signal $e_i(t)$ of the suitable amplitude and frequency is added to the control error. Because its period T_k is known for a triple of that period T_k it is waited and feedback signal $e_o(t)$ is saved before the evaluation is started. At the steady state evaluation start instant t, and in any time after that, it is tested whether average of three mean values of recorded values e_o (i.e. in the

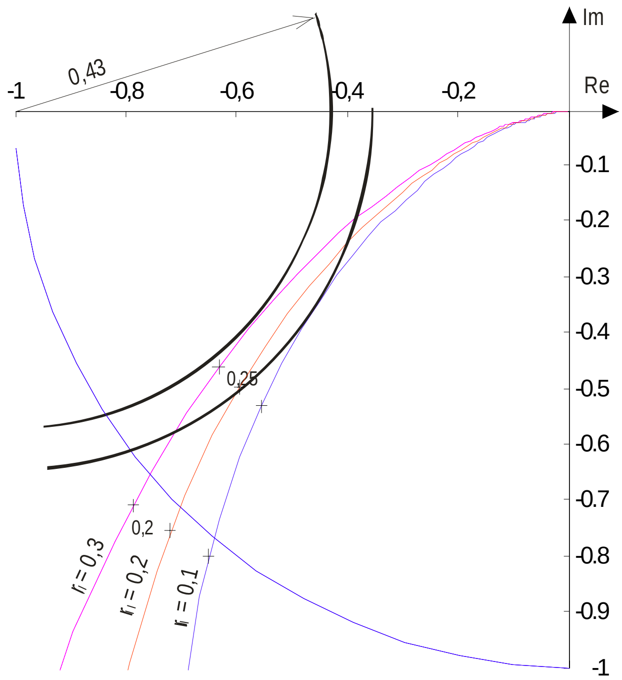

Fig. 7 Maximum sensitivity dependence on r_I

intervals $\langle t - T_k, t\rangle$, $\langle t - 2T_k, t\rangle$, $\langle t - 1{,}5T_k, t - 0{,}5\, T_k\rangle$) is less than a prescribed accuracy. If the condition is fulfilled, a signal of readiness for the magnitude and shift evaluation is given out.

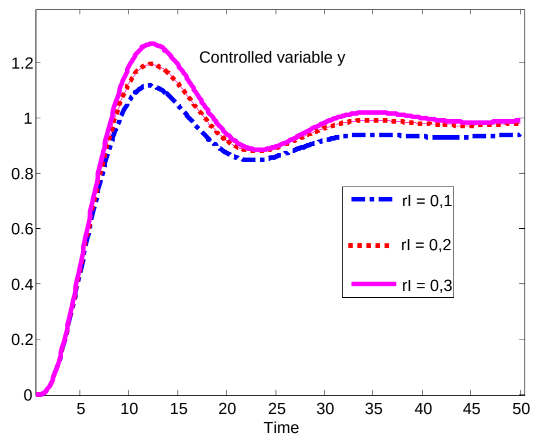

Fig. 8 Step responses for the setting above

These frequency response parameters are determined from the records t_{hist}, e_{ihist}, e_{ohist} containing data for last time interval of the length T_k (eventually from one discrete time step more). It means that there are always two zero point crossing both in the record of the input signal e_{ihist} and also in the record of the output signal e_{ihist}. Until now searching these points and extreme values as well has been done in Matlab which provides special support for such operation. It may be interesting that the recorded data are represented by vectors in this program making possible array operations, e.g. multiplication vectors element by element. Time difference between the minimum found from the left in index sequence in such a way defined product $e_{ihist}.*e_{ihist}$ and the minimum in the product $e_{ohist}.*e_{ohist}$ from the right expresse the time shift between both signals. If it is zero then the output signal $e_o(t)$ shows delay equal to the phase angle -180° and some conclusions on the gain margin can be directly made. If it is positive, then, after the conversion into degrees, this time shift corresponds to the phase margin. In finding both extremes, i.e. maxima and minima a similar way improved by interpolation can be applied.

Experience made from application of the evaluating procedures to the simulation model of a two tank cascade characterized by a large changeability of dynamics will be presented together with the first autotuning results in a separate paper. It is also prepared a comparison of results obtained from a nonlinear case with those coming from a linear PI control of the linear model of the same cascade under same conditions. Linearization can be performed in any optional operating point in the framework of one simulation program. In such a way it can be demonstrated limits in possible use of linear models and imperfectness of conclusions done on basis of linear models with artificially introduced changeability of their parameters.

5 Conclusion

In the paper, a new mechanism for optimal controller parameter tuning has been presented by means of which it is possible to make use of the indicator based on maximum sensitivity evaluation. The methods using responses to a periodic signal additionally exciting real control circuits for optimal control quality assessment do not need any mathematical models. They can be used for PID controller autotuning in which achieved optimum is considered from the global viewpoint and not only from the course of a response. Advantage of the maximum sensitivity indicator in optimal controller setting search is that it can serve as an all-in-one criterion combining more aspects bound to the other indicators. In such a way more indicators can be more easily simultaneously reflected in strategy of controller parameter changes.

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