Communication constraints for mobile sensors networks

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Abstract: The paper proposes the use of a mobile sensors network for the cases of large or distributed measurements requirements. An optimal approach for the computation of the trajectories for the sensors is followed. In addition, problems on the communications between mobiles are faced, producing a slight modification of the optimal control problem formulation. Simulation results are provided in order to evidence the effectiveness of the proposed solution and the effects of the constraints due to the communication requirements.

Key–Words: Sensor network, dynamic configuration, optimal motion, network communications.

1 Introduction

In the last years several problems involving very large or distributed systems monitoring and control have made the use of distributed (network) sensors systems crucial for the fulfillment of the requirements. As a consequence, the interest of many researchers for this kind of problems has been growing and growing, as proved for example by [1, 8]. The basic common feature required by sensor networks is the full coverage of the given (large) area with the union of each single field of measurement; then, the use of several sensors, suitably deployed, makes the range of measurements as wide as required. This problem has been usually faced studying optimal, suboptimal or heuristic solutions to the coverage problem in terms of collocation of sensors in the area under measurement. In this sense, the problem is to understand where the sensors have to be posed for best performances. Such a problem has been well studied in a lot of works, such as [7, 4, 14, 9, 6, 10]. In [13, 5] the problem of self-deploying mobile sensors, able to configure according to the environment, is addressed and some solutions are proposed. In these kind of approaches a common fact is the use of a lot of quasi static sensor units to cover a given area.

A different idea is to use a reduced number of sensor units moving continuously; this is the one proposed also by the authors in [2]. The advantages of such an approach are clear; the main ones are, for example, that less devices are to be used and a more quick recovery after failure of one sensor is provided. On the other hand, these advantages are to be paid: by the loose of a continuous measurement, assuring only a measurement of each point of the field within a prefixed time interval, by the necessity of providing each sensor unit with a motion capability, by the requirement of a coordinated motion, and by the consequences of a continuous change in the configuration. This last fact strongly affects the communication capabilities of the sensors in the network. For distributed sensing system the study of communication between nodes is an interesting aspect for research activities ([3, 12]). If for a static sensor network the choice of number of sensors and their deployment can take into account the range of each communication device, in a dynamic network the distances change continuously and then, in order to maintain the network connection, it is necessary to introduce some constrains on the instantaneous position of the sensors. An interesting study of the connection aspect of random deployed sensor networks is proposed in [11], but there the variability is only in the initial (and static) position.

In the present work the authors wish to show how can be easily modified the approach introduced in [2] in order to introduce also the constraints arising from these communication connections problems.

The motion problem for a set of moving sensors, under kinematic and dynamic constraints on the motion, and under distance constrains introduced to maintain a given network topology, with the objective to maximize the area covered during the movement is formulated as an optimal control problem.

As performed in [2], in order to simplify the solution of the problem, discretization of space and time are performed, so obtaining a discrete time optimal control problem equivalent to a Nonlinear Programming (NLP) one. The present solution is focused on how communication constrains caused by different network topologies can influence the coverage performances.

The paper is organized as follows. In Section 2 the mathematical model of the sensors is given, together with the constraints to be satisfied. Model and constraints are then used to propose a formulation for the optimal control problem presented in Section 3 formulated in terms of a NLP problem. Section 4 is devoted to the comparison of coverage performances for different network topologies showing simulation results. Some final comments in Section 5 end the paper.

2 The mathematical model

For sake of simplicity, the assumption of homogeneous sensor devices is assumed, that is all the sensors have the same characteristics. Clearly, the proposed approach applies also to non homogeneous sensors systems: an additional index should be added to all variables and, somewhere, sums over such an index may be required.

Under the simplifying hypothesis, each mobile sensor is modeled, from the dynamic point of view, as a material point of unitary mass, moving on a space $W \subset \mathbb{R}^2$, called the *workspace*, under the action of two independent control input forces named $u_1(t)$ and $u_2(t)$. Then, the position of the i-th sensor in W at time t is described by its Cartesian coordinates $(x_1^{(i)}(t), x_2^{(i)}(t))$. The motion is assumed to satisfy the classical simple equations

$$\begin{array}{l} \ddot{x}_1^{(i)}(t) = u_1^{(i)}(t) \\ \ddot{x}_2^{(i)}(t) = u_2^{(i)}(t) \end{array} \quad (1)$$

The linearity of 1 allows one to write the dynamics in the form

$$\begin{array}{l} \dot{z}^{(i)}(t) = Az^{(i)}(t) + Bu^{(i)}(t) \\ y^{(i)}(t) = Cz^{(i)}(t) \end{array} \quad (2)$$

where

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

once the state vector $z^{(i)}(t) = \left(\dot{x}_1^{(i)}(t), x_1^{(i)}(t), \dot{x}_2^{(i)}(t), x_2^{(i)}(t) \right)^T$ and the output $y^{(i)}(t) = \left(x_1^{(i)}(t), x_2^{(i)}(t) \right)^T$ are defined. Clearly, $y^{(i)}(t)$ denotes the trajectory followed by the i–th mobile sensor. If N sensor units are considered, the vector

$$y(t) = (y^{(1)}(t) \quad y^{(2)}(t) \quad \cdots \quad y^{(N)}(t))^T$$

can be defined to denote the generalized trajectory of the whole system such that, for each t, N points in the workspace are given representing the instantaneous positions (configuration) of the N sensors.

If the workspace W is supposed to be a rectangular subset of $\mathbb{R}^2$, the trajectory must satisfy the constraints

$$\begin{array}{c} x_{1,min} \leq x_1^{(i)}(t) \leq x_{1,max} \\ x_{2,min} \leq x_2^{(i)}(t) \leq x_{2,max} \end{array}$$

Moreover, physical limits on the actuators (for the motion) and/or on the sensors (in terms of velocity in the measure acquisition) suggest the introduction of the following additional constraints

$$\begin{array}{l} |\dot{x}_1^{(i)}(t)| \leq v_{max} \\ |\dot{x}_2^{(i)}(t)| \leq v_{max} \\ |u_1^{(i)}(t)| \leq u_{max} \\ |u_2^{(i)}(t)| \leq u_{max} \end{array}$$

Each mobile sensor at time t is assumed to take measures within a circular area of radius ρ_S around its current position $y^{(i)}(t)$.

Such an area under sensor *visibility* will be denoted as

$$M^{(i)}(t) = \sigma(y^{(i)}(t), \rho_S^{(i)}) \quad (3)$$

In other words, $M^{(i)}(t)$ denotes the area over which the i-th sensor can take measures at time t. The introduction of directional sensors can be modeled, within the present framework, by the simple change of 3 into

$$M^{(i)}(t) = \sigma(y^{(i)}(t), \rho_S^{(i)}, \theta_0^{(i)}, \Delta\theta^{(i)})$$

where $\theta_0^{(i)}$ denotes the main direction and $\Delta\theta^{(i)}$ the amplitude of the directional cone.

In addition, it is assumed that two sensors can communicate if the distance between them is smaller than a given communication radius ρ_C.

The function used to evaluate how sensors trajectories *cover* the space is based on the distance

$d(y(t), P)$ between the points $\{P|P \in W\}$ of the workspace and the generalized trajectory $y(t)$.

Defining the distance between a point P of the workspace and a generalized trajectory $y(t)$, within a time interval $\Theta = [0, t_f]$, as

$$d(y(t), P) = \min_{t\in\Theta, j\in\{1,2,\dots N\}} ||P - y_j(t)|| \quad (4)$$

and making use of the function

$$\text{pos}(\xi) = \begin{cases} \xi & \text{if} \quad \xi > 0 \\ 0 & \text{if} \quad \xi \leq 0 \end{cases} \quad (5)$$

that fixes to zero any nonpositive value, the function

$$\hat{d}(y(t), P, \rho_S) = \text{pos}\left(d(y(t), P) - \rho_S\right) \geq 0$$

can be defined. Then, a measure of how the generalized trajectory $y(t)$ produces a good coverage of the workspace can be given by

$$J(y(t)) = \int_{P\in W} \hat{d}(y(t), P, \rho_S) \quad (6)$$

Smaller is $J(y(t)$, better is the coverage. If $J(y(t)) = 0$ than $y(t)$ covers completely the workspace.

2.1 Communication Constrains

Communication between mobile sensors is very important, since the mobile units constitute the communication network used for data exchange and transmission, but also for sensor localization, coordination and commands communication.

In order to assure communication between sensors, a full connection of the sensors network is required. This can be imposed introducing some motion constrains.

The network can be represented by a connected graph $\mathcal{G}$, where the nodes represent the sensors, and each arch the bidirectional communication link between two sensors. The network topology is given and can be represented by an *adjacency matrix* $A_\mathcal{G}$ that, as well known, is composed only by 0 or 1:

$$A_\mathcal{G}(i, j) = 1$$

if there is a link between node i and node j

$$A_\mathcal{G}(i, j) = 0$$

if there is not a link between node i and node j

The existence of a link between nodes depends on the distance between them according to the rule

$$||y^{(i)}(t) - y^{(j)}(t)|| \leq \rho_C \; \forall t \in \Theta \; \Rightarrow \; A_\mathcal{G}(i, j) = 1$$

3 The Optimal Control Problem formulation

Making use of the element introduced in previous subsections, the Optimal Control Problem can be formulated in order to find the best trajectory $y^*(t)$ that maximizes the area covered by measurement of the N moving sensors during the time interval Θ, and satisfies the constraints. Then a constrained optimal control problem is obtained, whose form is ([2])

$$\min J(\Lambda(u(t)))$$

$$f(u(t)) = 0$$

$$g(u(t)) \leq 0$$

$$d(\Lambda_i(u^{(i)}(t), \Lambda_j(u^{(j)}(t)) \leq \rho_C \text{ if } A_\mathcal{G}(i, j) = 1 \; \forall t \in \Theta \quad (7)$$

In (7), the cost functional $J(\cdot)$ is given by (from (6))

$$J(\Lambda(u(t))) = \int_{p\in W} \hat{d}(\Lambda(u(t)), p, \rho_S) \quad (8)$$

where

$$u(t) = (\; u^{(i)}(t) \quad \dots \quad ^{(N)}(t) \;)^T$$

and

$$\Lambda(u(t)) = (\; \Lambda_1(u^{(1)}(t)) \quad \dots \quad \Lambda_N(u^{(N)}(t)) \;)$$

The optimal solution $u^*(t)$ is given by the control inputs that produce the optimal trajectory $y^*(t) = \Lambda\left(u^*(t)\right)$, $(t \in \Theta)$.

In general is not possible to solve analytically the optimal control problem defined in the precedent section, due the form of the functional $J(\cdot)$ in (7). In next section a solvable discrete problem is defined and solved.

In order to overcome the difficulty of solving a problem as (7) due to the complexity of the cost function $J(\cdot)$, a discretization is performed, both with respect to space W, and with respect to time in all the time dependent expressions.

The workspace is divided into square cells $c_{i,j}$ with resolution (size) l_{res}, and the trajectories are discretized with sample time T_s. The equations of the discrete time dynamics for a single moving sensor are:

$$z^{(i)}((k+1)T_s) = A_d z^{(i)}(kT_s) + B_d u^{(i)}(kT_s)$$

$$y^{(i)}(kT_s) = C z^{(i)}(kT_s) \quad (9)$$

where $A_d = e^{AT_s}$ and $B_d = \int_0^{T_s} e^{A\tau} B d\tau$

The state $z^{(i)}(t)$ at the generic time instant $t = kT_s$ depends on the initial state $z_0^{(i)}$ and on the discrete control $u^{(i)}(t)$ from time $t = 0$ to time $t = (k-1)T_s$

$$z^{(i)}(kT_s) = A_d^k z_0^{(i)} + \sum_{i=0}^{N-1} A_d^i B_d u^{(i)}((N-1)T_s - iT_s) \tag{10}$$

The following vectors and matrices are now defined

$$Z_k^{(i)} = \begin{bmatrix} z^{(i)}(T_s) \\ \vdots \\ z^{(i)}(kT_s) \end{bmatrix}$$

$$Y_k^{(i)} = \begin{bmatrix} y^{(i)}(0) \\ \vdots \\ y^{(i)}(kT_s) \end{bmatrix}$$

$$H_k = \begin{bmatrix} A_d & B_d & ... & 0 & 0 \\ A_d^2 & A_d B_d & ... & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A_d^k & A_d^{k-1} B_d & ... & A_d B_d & B_d \end{bmatrix}$$

$$U_k^{(i)} = \begin{bmatrix} z^{(i)}(0) \\ u^{(i)}(0) \\ \vdots \\ u^{(i)}((k-1)T_s)) \end{bmatrix} \qquad U_{max} = \begin{bmatrix} u_{max} \\ \vdots \\ u_{max} \end{bmatrix}$$

$$Z_{max} = \begin{bmatrix} v_{max}^{(i)} \\ x_{1,max}^{(i)} \\ v_{max}^{(i)} \\ x_{2,max}^{(i)} \\ -- \\ \vdots \\ -- \\ v_{max}^{(i)} \\ x_{1,max}^{(i)} \\ v_{max}^{(i)} \\ x_{2,max}^{(i)} \end{bmatrix} \qquad Z_{min} = \begin{bmatrix} -v_{max}^{(i)} \\ x_{1,min}^{(i)} \\ -v_{max}^{(i)} \\ x_{2,min}^{(i)} \\ -- \\ \vdots \\ -- \\ -v_{max}^{(i)} \\ x_{1,min}^{(i)} \\ -v_{max}^{(i)} \\ x_{2,min}^{(i)} \end{bmatrix}$$

Making use of such matrices, the sequence of values for the sampled state vector $z^{(i)}(kT_s)$ can be expressed by the simple compact form

$$Z_k^{(i)} = H_k^{(i)} U_k^{(i)} \tag{11}$$

For every mobile sensor is possible to define the matrices $A_{model}^{(i)} = \begin{bmatrix} H_k^{(i)} \\ -H_k^{(i)} \end{bmatrix}$ and $B_{model}^{(i)} = \begin{bmatrix} Z_{max}^{(i)} \\ -Z_{min}^{(i)} \end{bmatrix}$

For the set of N moving sensors, one has

$$U_k^N = \begin{bmatrix} U_k^{(1)} \\ \vdots \\ U_k^{(N)} \end{bmatrix}$$

$$Y_k^N = \begin{bmatrix} Y_k^{(1)} \\ \vdots \\ Y_k^{(N)} \end{bmatrix}$$

$$A_{model}^N = \begin{pmatrix} A_{model}^{(1)} & 0 & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & ... & A_{model}^{(N)} \end{pmatrix}$$

$$B_{model}^N = \begin{pmatrix} B_{model}^{(1)} \\ \vdots \\ B_{model}^{(N)} \end{pmatrix}$$

The cost function can then be written as:

$$J(Y_k^N) = \sum_{i=1}^{\nu_x} \sum_{j=1}^{\nu_y} \hat{d}(\Lambda(U_k^N), c_{i,j}, \rho_S)$$

where $\nu_x = \frac{(x_{max} - x_{min})}{l_{res}}$, $\nu_y = \frac{(y_{max} - y_{min})}{l_{res}}$ and $\Lambda(U_k^N) = Y_k^N$.

3.1 The Nonlinear Programming Problem formulation

The problem of finding the maximum area coverage trajectory, under the constraint of connectivity maintenance in the communications connection, can now be written as a discrete optimization problem with linear inequality, box and nonlinear constraints

$$\min_{U_k^N} \sum_{i=1}^{\nu_x} \sum_{j=1}^{\nu_y} \hat{d}(\Lambda(U_k^N), c_{i,j}, \rho_S)$$

$$A_{model}^N U_k^N \leq B_{model}^N$$

$$-U_{max}^N \leq U_k^N \leq U_{max}^N \tag{12}$$

$$d(\Lambda U_k, i, \Lambda U_k, j) \leq \rho_C \text{ if } A_{\mathcal{G}}(i,j) = 1$$

Suboptimal solutions can be computed using numerical methods. In the simulations performed, the SQP (Sequential Quadratic Programming) method has been applied. The obtained model can be customized according to the specific task, as shown in the following section.

4 Simulation Results

In this section simulations results are reported in order to put in evidence the capabilities and he effectiveness of the proposed solution, and to show how different topology constrains influence the coverage performances. The values of parameters used in all the simulations are:

$$u_{max} = 0.5N,$$
$$v_{max} = 1.5\frac{m}{sec},$$
$$T_s = 0.5sec$$

Ring Network

In the *ring topology* (figure 1) each node is directly connected with two other nodes; With this structure the network maintain connection even with the fault of one sensor node. The solutions for ring network of three moving nodes are showed in figure 2.

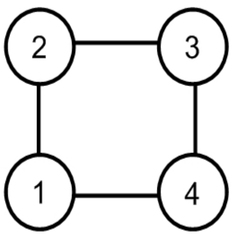

Figure 1: Ring Topology with four nodes

The area covered with measures is the 75% of the total. The same results are showed in figure 3 for a ring network with four moving nodes on a larger workspace

The area covered with measures is the 66% of the total.

Line Network

The *line topology* (figure 4), is the less constraining topology, and the one who allows the best coverage performances. The problem of this network structure is that it is not directly fault tolerant, because the fault of one of the internal nodes cause the loss of network connection if no recover manoeuvre is performed. Solutions for ring network of 3 moving nodes are showed in 5

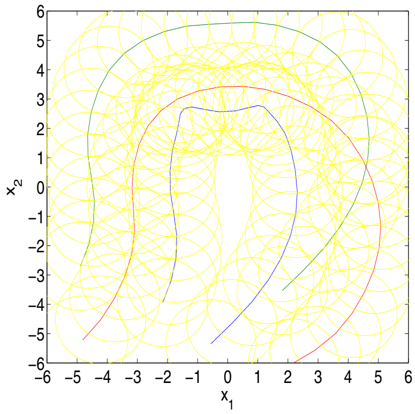

Figure 2: Suboptimal trajectory for a moving sensor network with three nodes and ring topology ($x_{max} = y_{max} = 6m$, $x_{min} = y_{min} = -6m$).

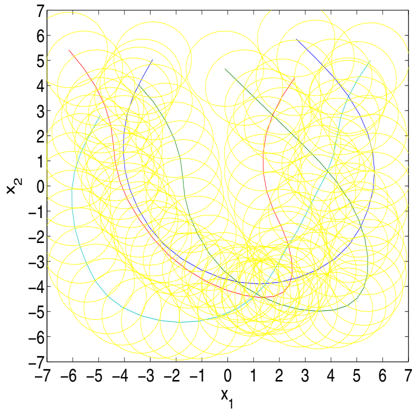

Figure 3: Suboptimal trajectory for a moving sensor network with four nodes and ring topology ($x_{max} = y_{max} = 7m$, $x_{min} = y_{min} = -7m$).

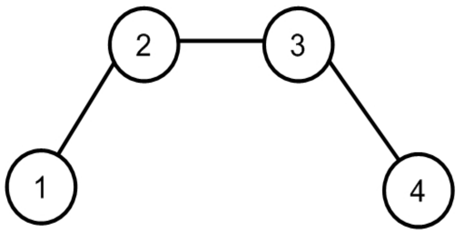

Figure 4: Line Topology with four nodes

The area covered with measures is the 83% of the total. The same results are showed in figure 6 for a ring network with four moving nodes on a larger workspace

The area covered with measures is the 82% of the total. The growth of coverage performance with respect to the ring topology is evident.

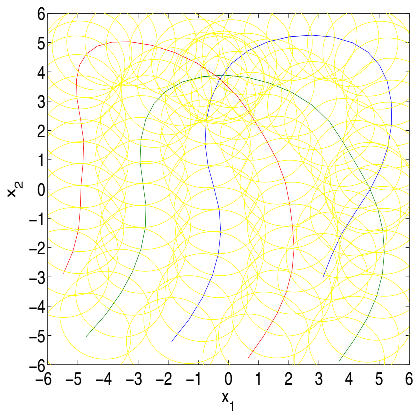

Figure 5: Suboptimal trajectory for a moving sensor network with tree nodes and ring topology ($x_{max} = y_{max} = -6m$, $x_{min} = y_{min} = -6m$).

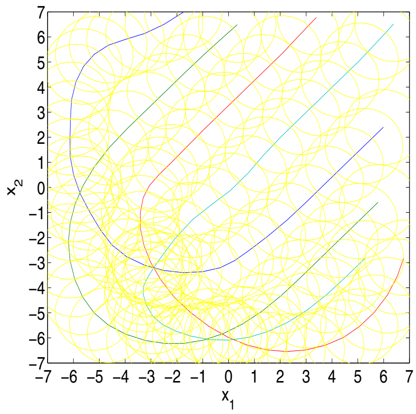

Figure 6: Suboptimal trajectory for a moving sensor network with four nodes and ring topology ($x_{max} = y_{max} = -7m$, $x_{min} = y_{min} = -7m$).

5 Conclusion

In the present paper a measurement system composed by a network of several sensors moving within the area under measure has been considered. This system has been called *dynamic sensor network*. For this kind of system the formulation for an optimal solution to the area coverage problem has been provided. The constraints on the maximal distance able to maintain a communication connection between sensors have been considered and their influence on the coverage capability has been evidenced.

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