A comparison of the diffracted fields from gratings and zone plates with mixed structures

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Abstract: - We study the degree of fractality for the electromagnetic field diffracted from gratings and zone plates with internal Cantor structure. This fractality is determined by the correlation peaks which appear in the self-similarity function for the intensity distribution of the field. The results are complementary to the study of the homogeneity property, presented in a previous work [7].

Key-Words: - Numerical analysis, applied electromagnetism, signal processing, optoelectronics, Cantor structures, fractal diffractive optics.

1 Introduction

The diffracted field from gratings with different geometry is important for the electromagnetic information processing and also for the inverse problem, this is, the study of the geometry of the object starting from the geometry of the field. Diffraction gratings and zone plates are important elements in the applications of diffractive optics [1,2], antennas [3,4], etc.. In previous works we have studied the field diffracted from gratings [5,6] and zone plates [7,8] with fractal characteristics, using in both cases a construction method with periodic functions. In these cases we have shown that there is a transformation that allows to obtaining the periodic functions for both cases and then, the total structure can be transformed in the same way. We have also studied the property of homogeneity and the foci positions for Cantor zone plates with different fractal dimensions.

In this work we study the property of self-similarity for zone plates and the associated linear gratings, which are related through a quadratic transformation, as continuation of a previous work [7] referred to the analytic derivation of the homogeneous property for both cases.

2 Gratings and zone plates with fractal geometry

We have already demonstrated the possibility of obtaining Cantor functions by means of the product superposition of periodic components [9]. Each of these periodic components can be expressed through scaled Fourier series. Then, the periodic Cantor function is given by:

$$F^{(N)}(x,y)=\prod_{k=0}^{N}\left\{\sum_{n_k=-\infty}^{+\infty} C_{n_k}\,\exp\left[\frac{2\pi i}{d_0}s^k\,n_k\,x\right]\right\},\qquad(1)$$

where N is the order and s is the scaling factor of the fractal structure. Using the coordinate transformation:

$$i\pi\frac{p\,g_k}{d_0}x\rightarrow i\pi\frac{p\,g_k}{D_0^2}r^2\ ,\qquad(2)$$

where d_0 and D_0 are the corresponding periods. Then, the representation for a zone plate is finally obtained:

$$S^{(N)}(r)=\prod_{k=0}^{K}\sum_{n_k=-\infty}^{+\infty} C_{n_k}\,\exp\left[i\pi\frac{n_k\,s^{2k}}{{D_0}^2}r^2\right],\qquad(3)$$

and both expressions, Eqs. (1) and (3), are related through Eq. (2).

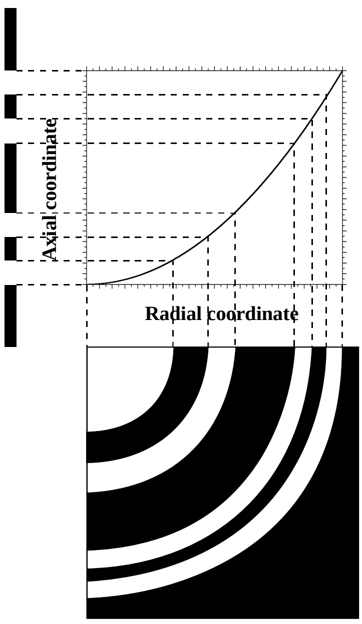
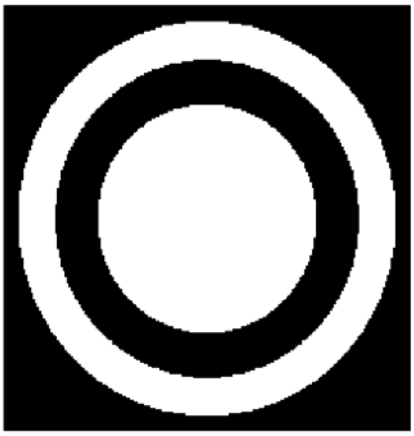


Fig. 1 – Coordinate transformation.



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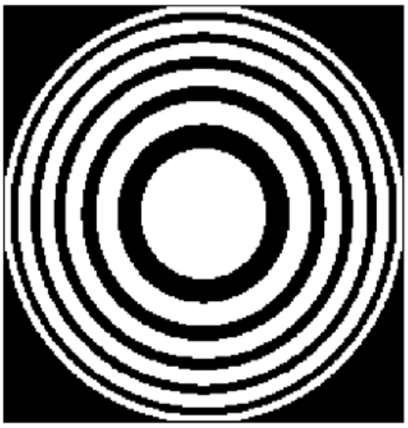

SECOND_COMPONENT

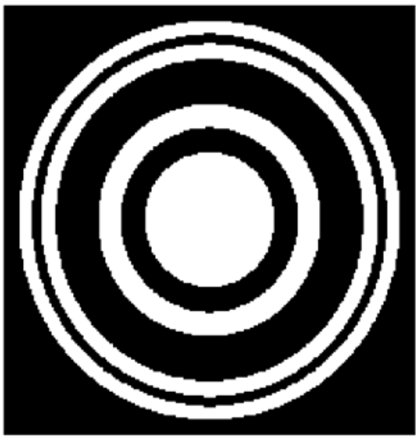

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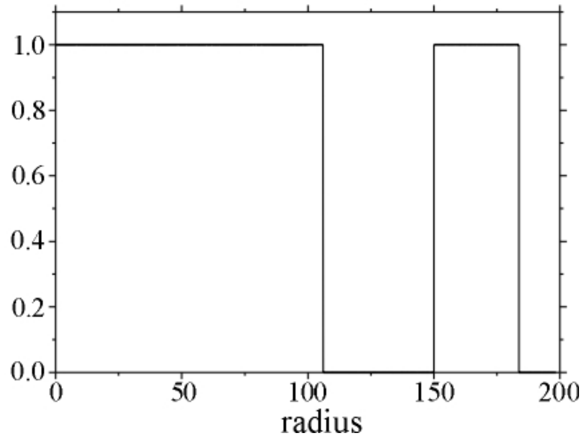
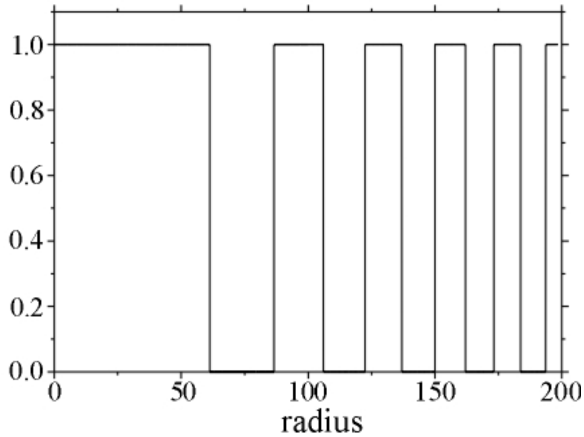
Figure 2a – Components related with linear periodic gratings and the Cantor zone plate.





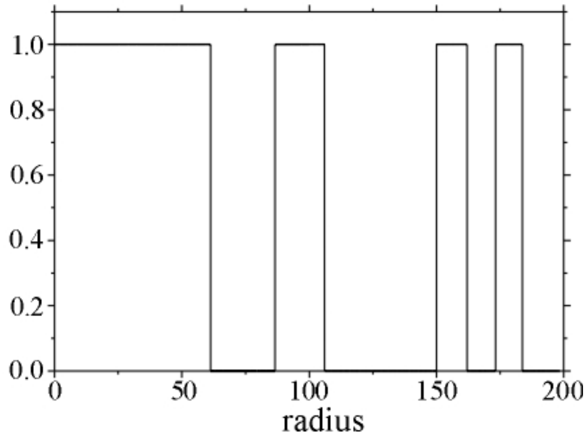


Figure 2b – Transmition function as a function of the radius of the zone plates of Fig. 2a.

3 Calculation of the diffracted field

The computation for the diffracted field, in each case, can be obtained from the Fresnel integral in Cartesian and polar coordinates:

- Linear Gratings:

$$U(x)=\frac{A(x,z)}{\sqrt{\lambda z}}\int_{-\infty}^{+\infty}F(x')\exp\left[-i\frac{\pi}{\lambda z}x'^2\right]\exp\left[-i\frac{2\pi}{\lambda z}xx'\right]dx' \tag{4}$$

- Zone plates:

$$U(r)=\frac{A(r,z)}{\lambda z}\int_{0}^{+\infty}r'F(r')\exp\left[-i\frac{\pi}{\lambda z}r'^2\right]J_0\left(\frac{2\pi}{\lambda z}rr'\right)dr' \tag{5}$$

where $A(x,z)$ and $A(r,z)$ are complex functions [10].

When the Cantor functions defined in Eqs. (1) and (3) are introduced in Eqs. (4) and (5) respectively, the diffracted field and the foci positions for such fractal structures can be obtained.

The comparison between the foci positions are shown in the intensity patterns of Fig. 3, considering that they are found along a transversal axis (for the case of linear gratings) and along the optical axis (for the case of zonal plates).

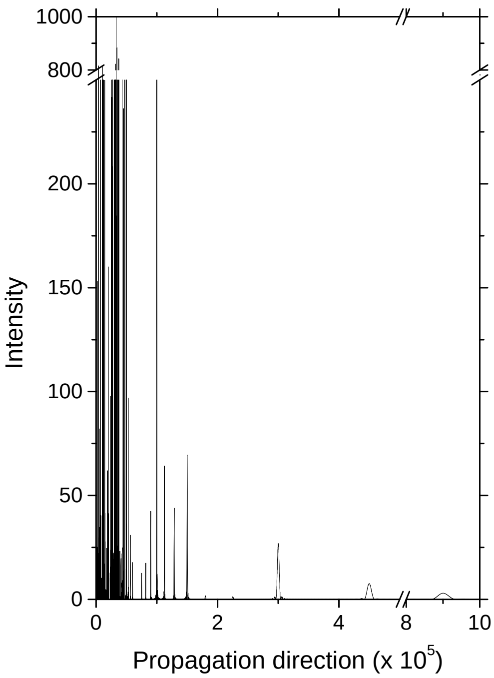


Figure 3 – Intensity distribution along the optical axis.

4 Self-similarity function

In both cases it is possible to use the self-similarity function defined in Ref. [11], to see the effect of the coordinate transformation defined by Eq. (2) on the scaling property of each diffractive structure. Some properties of this self-similarity function were studied in Refs. [12,13]. It is defined by means of the correlation:

$$S(m)=\frac{\int_R\left(I(\xi)-\langle I(\xi)\rangle\right)\left(\left|I\left(\frac{\xi}{m}\right)\right|-\left\langle\left|I\left(\frac{\xi}{m}\right)\right|\right\rangle\right)d\xi}{\sqrt{\int_R\left(I(\xi)-\langle I(\xi)\rangle\right)^2d\xi\int_R\left(\left|I\left(\frac{\xi}{m}\right)\right|-\left\langle\left|I\left(\frac{\xi}{m}\right)\right|\right\rangle\right)^2d\xi}} \tag{6}$$

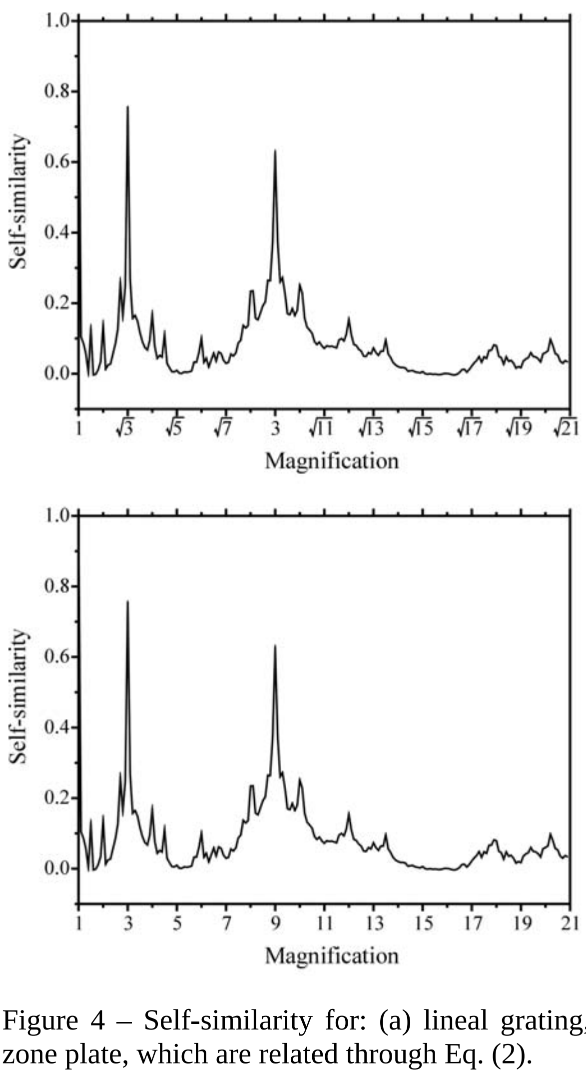


Figure 4 – Self-similarity for: (a) lineal grating, (b) zone plate, which are related through Eq. (2).

Fig. 4 shows the fractality in relation to the scaling for the linear grating (above) associated to the Cantor zone plate (below). This function is calculated, into a region R, on the intensity distribution of the field given by $I(\xi)$, where ξ is a generalized coordinate.

The scaling factor for each case is s and $s^{0.5}$ respectively. In the plot of self-similarity the magnification is defined as a variable scaling factor (m), which presents correlation peaks in correspondence with the scaling factor of the Cantor structures, this is for $m=s^{\alpha/2}$ (for the linear gratings) and $m=s^{\alpha}$ (for zone plates).

5 Conclusions

In this work the characteristics of self-similarity have been compared for linear gratings and zone plates, with internal Cantor structure. Furthermore, the foci positions for both geometries are shown. For the linear grating a direct relationship between the scaling of the structure and the position of the correlation peaks is obtained. For the case of the zone plate a quadratic relationship is obtained. This is very important for the possible applications of such types of structures. The results are related with the expressed by the homogeneity relation from Ref. [6], in the sense that if the linear grating has scaling factor $s^{0.5}$ ($=3^{0.5}$, for the triadic Cantor set) then the self-similarity of the field has correlation peaks for powers of $s^{0.5}$. For the zone plate obtained by means of the transformation defined in Eq. (2), the field has correlation peaks for powers of s.

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