An Approximated Detection Method for Sequential Detection in Wireless Sensor Networks

Dmitry Kramarev
Gwangju Institute of Science and Technology (GIST)
Department of Information and Communications
1 Oryong-dong, Buk-gu, Gwangju, 500712
Republic of Korea

Insoo Koo
University of Ulsan
Department of Electrical Engineering
Ulsan
Republic of Korea

Kiseon Kim
Gwangju Institute of Science and Technology (GIST)
Department of Information and Communications
1 Oryong-dong, Buk-gu, Gwangju, 500712
Republic of Korea

Abstract: In this paper we study a sequential type-based detection scheme with the suboptimal detection rule for wireless sensor networks. First, we propose the approximation of the optimal detection rule and derive the suboptimal detection rule. Then we compare the performances of the optimal and the suboptimal detection rules and show that the approximated detection rule provides the similar results as the optimal detection rule in terms of both average number of observations and total energy consumption. At the same time the suboptimal detection rule allows the significant reduction of the computational complexity such that the complexity and detection delay per one iteration does not depend on the number of sensor nodes in the network. That makes the proposed approximated detection rule useful for real-time application.

Key–Words: Sequential detection, Type-Based Multiple Access, Gaussian approximation

1 Introduction

Recent significant advances in sensor devices manufacturing and low-power wireless networking have generated the development of wireless sensor networks (WSNs), which can be successfully used for distributed signal processing. The typical tasks for WSNs include acquisition of information, further transmission of the information, and data fusion [1]. Particularly, distributed detection of certain events or targets is an important application of WSNs (the summary of results obtained in the last couple of decades is provided in [2], [3]).

A traditional way of studying of the distributed detection is to assume that every sensor node has full information regarding the signal source statistics [2]-[4]. Thus the smart sensors can process the data and produce local decisions, which are used by the fusion center to make a global decision. However, for practical implementation, the storage of statistical information and the capability to make local decisions require more complex, expensive and powerful sensors.

Another common problem in different applications of distributed detection is related to communication between the nodes and the fusion center; for example, real sensors typically have constrained resources, such as battery power, making it is impossible to achieve perfect quality in data transmission. As a result, information received by the fusion center is often corrupted by channel uncertainties, markedly decaying the detection accuracy. Also, for simultaneous data transmission in a large-scale network a large bandwidth is required to avoid detection delays.

Recently, Type-Based Multiple Access (TBMA) has been proposed by Liu and Sayeed [5] and independently by Mergen and Tong [6]. This method utilizes the multiple access channel (MAC) and allows to avoid the problems mentioned above. Also, the scheme employs simple sensors as counters of histogram statistics; since no local decisions are made in the nodes, sensors do not require an event probabilistic model, such that their construction becomes relatively simple and the network becomes universal for performing different tasks. At the same time, with a large number of sensors, the accuracy of a type-based detector tends to be similar to the accuracy of centralized detection. Additionally, the problem of detection of a deterministic signal in correlated Gaussian noise through MAC has been studied in [7].

In a class of applications, sequential detection can be actually more preferable because the gathering of observations is made in sequential manner; for the distributed detection problems the sequential approach has been studied in [8] - [10]. Moreover, it was shown [9], that to perform a sequential test with pre-given detection accuracy a smaller number of observations is required, comparing to non-sequential tests.

In the previous work [11] we studied the optimal algorithm for sequential type-based detection. In this paper with the motivation of reducing the computational complexity of the optimal detection rule, we propose a suboptimal detection method which provides the similar performance in terms of two performance metrics: the average number of observations, and the average energy consumption; additionally, there is no losses in desired accuracy. At the same time, the proposed method essentially reduces the computational complexity of the proposed scheme to the point that the complexity per one iteration is independent of the number of nodes in the network. Through a numerical example we show the advantages of the proposed scheme.

The rest of the paper is organized as follows. The system model and proposed scheme are described in Section 2. Section 3 overviews the related work. Next, in Section 4 we propose and analyze the suboptimal detection rule and propose the evaluation scheme of the detector performance parameters. The results of simulations are provided in Section 5. Finally, Section 6 contains the conclusions of this work.

2 Proposed Scheme Description

In this paper we consider a sensor network with a simple structure where all K sensors are directly connected to the fusion center. The observations of each sensor at the t^{th} time stage are given as the n-length sequence $\bar{x}_k(t) = \{x_{k,i}(t)\}_{i=1}^{n}$, $k = 1 \dots K$. The data are quantized to $\chi + 1$ levels, and $x_{k,i}(t)$ obtains values from the discrete alphabet $\widetilde{A} = \{a_0, a_1, \dots, a_\chi\}$. We assume that all the observations are identically and independently distributed according to a probability distribution Q, which is either Q_1 with probabilities $p_{Q_1}(a_m) = p_{1,m}$, or Q_0 with probabilities $p_{Q_0}(a_m) = p_{0,m}$, $m = 0 \dots \chi$. After n observations are completed, the k-th node produces the type-information of the observations, $\bar{T}_k(t) = \{T_{k,m}(t)\}_{m=1}^{\chi}$, $k = 1 \dots K$. The value of $T_{k,m}(t)$ can be represented as the relative frequency of $a_m \in \widetilde{A}$ in the sequence $\bar{x}_k(t)$, such that

$$T_{k,m}(t) = \frac{N_{\bar{x}_k(t)}(a_m)}{n}, \tag{1}$$

where $N_{\bar{x}_k(t)}(a_m)$ is the number of occurrences a_m in $\bar{x}_k(t)$. The type-information is then sent to the fusion center. As a channel, we consider an additive white Gaussian noise channel model with multiple access (MAC), where all the nodes share the same channel. The influence of such a channel on detection and estimation accuracy has been studied in [5]-[6], where it was shown that MAC yields an significantly better detection performance, as compared to traditional access methods. The χ-length type information arrays are transmitted through MAC with χ number of channel uses, and an individual node power P_{ind} assigned for each channel use. As a result, the received signal $\bar{r}(t) = \{r_m(t)\}_{m=1}^{\chi}$ takes the form [5]

$$\bar{r}(t) = \frac{1}{K}\left[\sum_{k=1}^{K} \bar{T}_k(t) + \bar{\omega}(t)\right], \tag{2}$$

where $\bar{\omega}(t)$ is channel Gaussian noise with a zero mean, and a covariance matrix $1/P_{ind}\mathbf{I}$ i.e. the noise samples are independent of each other. Note, that the size of the used alphabet is $\chi + 1$, but the size of $\bar{r}(t)$ and $\bar{T}_k(t)$ is χ.

Based on the received signals, the fusion center processes the hypothesis testing $H_1 : Q = Q_1$ versus $H_0 : Q = Q_0$ using a sequential decision rule, which consists of a stopping rule and a final decision rule. During the test, if the number of observations is not sufficient for making a final decision, the sensors send one more series of observations, made in the same manner as the previous one. Finally, when the conditions of the stopping rule are satisfied, the observations are stopped and the hypothesis testing is held by the final decision rule. Wald [9] has proved that such a test terminates with probability of 1.

The desired accuracy of detection is given by the probabilities of misdetection and false alarm, P_m and P_{fa}, respectively. The performance is evaluated in terms of average number of observations and average energy expense, which in general depends on the desired accuracy. First, the average number of observations under each hypothesis, $E(L_\theta)$, can be counted as

$$E(L_\theta) = nKE(l_\theta), \tag{3}$$

where l_θ is a random variable for counting the number of data transmissions, and $E(\cdot)$ represents the mean of a random variable.

In this work, we have assumed that the sensors spend energy in two main areas: to acquire the data, and to transmit information from the nodes to the fusion center. However, the amount of energy consumed for data acquisition is an application-dependent quantity that varies for different problems. Additionally, this quantity is generally proportional to the average number of observations. Therefore, in this study we take into account only the energy spent for data transmission, P_θ, which is dependent on the number of sensors K in the network, the size of the used alphabet $\chi+1$, and the number of data transmissions l_θ; but not dependent on the observation sequence size n. Consequently, we can define the average energy expense $E(P_\theta)$ as

$$E(P_\theta) = KP_{ind}E(l_\theta)\sum_{m=1}^{\chi} E\left[T_{k,m}^2(t)|H_\theta\right], \quad (4)$$

where $\theta = 0$ and $\theta = 1$ are the indexes of the hypotheses H_0 and H_1, respectively. Note, that with knowledge of the correlation between the energy expense for data acquisition and the number of observations, we can consider the performance in terms of total energy consumption; however, this is out of the scope of this work.

In the next sections, based on the scheme described above, we investigate the performance of the suboptimal detection rule for type-based sequential detection.

3 Related Work

In this section we briefly overview the related work. At first, we recall the theory of sequential detection, and then the optimal detection rule for sequential type-based detection is presented.

Let $\{\bar{r}(t); t = 1, 2, \dots\}$ is a sequence of independent and identically distributed random variables, distributed according to two possible hypotheses

$$\begin{array}{ll} H_1: \ \bar{r}(t) & \sim f_{1,\bar{r}(t)}(x), \quad t = 1, 2, \dots \\ \text{versus} & \\ H_0: \ \bar{r}(t) & \sim f_{0,\bar{r}(t)}(x), \quad t = 1, 2, \dots \end{array} \quad (5)$$

Now let us define the sequential log-likelihood ratio

$$\Lambda(t) = \Lambda(t-1) + \log\frac{f_{1,\bar{r}(t)}(\bar{r}(t))}{f_{0,\bar{r}(t)}(\bar{r}(t))}, \quad (6)$$

with $\Lambda(0) = 0$ and $t = 1, 2, \dots$ Then the sequential log-likelihood test is given

$$\left\{\begin{array}{lllll} & & \Lambda(t) & \leqslant & A, \quad \text{Accept } H_0, \\ A & < & \Lambda(t) & \leqslant & B, \quad \text{Take more observations,} \\ B & < & \Lambda(t) & , & \quad \text{Accept } H_1; \end{array}\right. \quad (7)$$

The optimality of the detector presented in (7) in terms of size is justified by the Wald-Wolfowitz theorem [9]. Next, in order to maintain the required accuracy, regardless of the probability distributions Q_1 and Q_0, the thresholds should be set [9], [10] as

$$A = \log P_m, \quad (8)$$

and

$$B = -\log P_{fa}. \quad (9)$$

Now the optimal detection rule for sequential type-based detection is provided. When the all information at the t^{th} time stage is received by the fusion center, the value of the received signal for a symbol a_m can be represented according to (1) and (2) as

$$r_m(t) = \frac{N_{X(t)}(a_m)}{Kn} + \frac{\omega_m(t)}{K}, \quad (10)$$

where $N_{X(t)}(a_m)$ is the number of occurrences, when the symbol a_m has appeared in the array of all observations $X(t)$ with a length of Kn. That is, $N_{X(t)}(a_m)$ is the number of a_m occurrences in Kn Bernoulli trials, where for each trial the probability of a_m is $p_{\theta,m}$ under the hypothesis H_θ. This implies that $N_{X(t)}(a_m)$ is a random variable with the binomial distribution and parameters Kn and $p_{\theta,m}$, denoted by $B(Kn, p_{\theta,m})$. In addition, the random variable $\omega_m(t)$ has Gaussian distribution with a zero mean and variance $1/P_{ind}$, denoted by $N(0, 1/P_{ind})$. Since the random variables $N_{X(t)}(a_m)$ and $\omega_m(t)$ are mutually independent, the probability density functions (pdfs) of $r_m(t)$ under the hypothesis H_θ have the form [11]

$$f_{\theta,r_m(t)}(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\sum_{i=0}^{Kn}\binom{Kn}{i}p_{\theta,m}^i(1-p_{\theta,m})^{Kn-i} \exp\left(-\frac{\left(x-\dfrac{i}{Kn}\right)^2}{2\sigma^2}\right), \quad (11)$$

where

$$\sigma^2 = \frac{1}{K^2P_{ind}}. \quad (12)$$

4 Suboptimal Decision Rule for Sequential Type-based Detection

In the previous section the optimal sequential log-likelihood ratio test for type-based detection is provided. However, practical implementation of this detection rule is quite complex, due to the required processing time and the amount of computations in the fusion center. Furthermore, the numerical integration methods to calculate the average number of data transmissions and corresponding performance are required. Taking these factors into account, we are motivated to propose a suboptimal detector with a similar performance, but one which is less computationally complicated than the optimal detector.

One of the best ways to simplify the optimal detection rule is to apply the Gaussian approximation. The first reason for this choice is that the accuracy of a sequential detector depends not on the probability distributions of the observations, but on specified thresholds. The second is that the normal distribution well approximates the binomial distribution with a large number of trials.

To derive the suboptimal detection rule, we consider an approximated distribution of the received signal $r_m(t)$. In (10), $N_{X(t)}(a_m) \sim B(Kn, p_{\theta,m})$ and it can be approximately represented as $N(Knp_{\theta,m}, Knp_{\theta,m}(1-p_{\theta,m}))$ for $Kn \gg 1$. As a result, the received signal is approximately governed by the normal distribution $N(p_{\theta,m}, \sigma^2_{\theta,m})$, with the variance

$$\sigma^2_{\theta,m} = \frac{p_{\theta,m}(1-p_{\theta,m})}{nK} + \frac{1}{K^2 P_{ind}}. \quad (13)$$

The suboptimal detector is represented by the sequential log-likelihood ratio test (7). The approximated likelihood ratio, $\Lambda^a(t)$, is derived by substituting the pdfs of $N(p_{\theta,m}, \sigma^2_{\theta,m})$ into (7). With independence we have

$$\begin{aligned} \Lambda(t) &= \Lambda(t-1) + \sum_{m=1}^{\chi} \log \frac{f^a_{1,r_m(t)}(r_m(t))}{f^a_{0,r_m(t)}(r_m(t))} \\ &= \Lambda^a(t-1) + \sum_{m=1}^{\chi} \left(\frac{1}{2} \log \left(\frac{\sigma^2_{0,m}}{\sigma^2_{1,m}} \right) + \right. \\ &+ \left. \frac{(r_m(t)-p_{0,m})^2}{2\sigma^2_{0,m}} - \frac{(r_m(t)-p_{1,m})^2}{2\sigma^2_{1,m}} \right). \end{aligned} \quad (14)$$

In this test, we set the same thresholds as in the optimal detector. These are given by (8) and (9).

Finally, the suboptimal detection rule makes a decision based on the log-likelihood ratio, given by (14); the sequential test, given by (7); and the thresholds, given by (8) and (9). To perform the test with the suboptimal detection rule we only have to calculate square functions rather than exponential functions, and as such the proposed suboptimal detection rule markedly reduces the computational complexity of the fusion center.

In general, Wald's approximation can be employed to calculate the expected size of a sequential detector. However, for the considered scheme with a large number of sensor nodes and multiple access, Wald's approximation has insufficient accuracy because in this case the excess over one of the thresholds is not negligible (refer to [9] for details). Another method was suggested in [12]; that is, if the test that is performed with (7), (14), (8) and (9) starts at the point $\Lambda^a(0) = w$, we have

$$E(l^a_\theta | \Lambda^a(0) = w) =$$

$$1 + \int_A^B E(l^a_\theta | \Lambda^a(0) = v) f_{\theta,\Lambda^a(1)}(v-w) dv, \quad (15)$$

where $f_{\theta,\Lambda^a(1)}(x)$ is a pdf of likelihood ratio $\Lambda^a(1)$, under hypothesis θ. Eqn. (15) is a Fredholm integral equation of the second kind, and can be solved numerically. For simplicity, we approximate pdf $f_{\theta,\Lambda^a(1)}(x)$ by the pdf of the normal distribution $N(E(\Lambda^a(1)|H_\theta), \sigma(\Lambda^a(1)|H_1))$, where $E(\Lambda^a(1)|H_\theta)$ and $\sigma^2(\Lambda^a(1)|H_1)$ can be obtained without usage of numerical integration, as follows

$$E(\Lambda^a(1)|H_\theta) = \frac{1}{2} \sum_{m=1}^{\chi} \left(\log \left(\frac{\sigma^2_{0,m}}{\sigma^2_{1,m}} \right) \right.$$

$$\left. + (-1)^{1-\theta} \left(\frac{(p_{1,m} - p_{0,m})^2 + \sigma^2_{\theta,m}}{\sigma^2_{1-\theta,m}} - 1 \right) \right), \quad (16)$$

and

$$\sigma(\Lambda^a(1)|H_\theta) = \left(\sum_{m=1}^{\chi} \left(\frac{1}{2} \left(\frac{\sigma^2_{1,m} - \sigma^2_{0,m}}{\sigma^2_{1-\theta,m}} \right)^2 \right. \right.$$

$$\left. \left. + \frac{\sigma^2_{\theta,m}}{\sigma^4_{1-\theta,m}} (p_{1,m} - p_{0,m})^2 \right) \right)^{\frac{1}{2}}. \quad (17)$$

After numerically solving Eqn. (15), $E(l^a_\theta)$ is obtained as $E(l^a_\theta) = E(l^a_\theta | \Lambda^a(0) = 0)$. Then, the performance of the suboptimal detector can be calculated with (3) and (4).

5 Simulation Results

In this section, we investigate the performance of the sequential type-based detection scheme with the suboptimal detection rule, and also compare the performance of the optimal and the suboptimal detection rules. Here we study a case of a binary alphabet $\widetilde{A} = \{0, 1\}$, so $\chi = 1$. The observations are distributed according to the Bernoulli distribution with $p_{Q_1}(1) = 0.6$ and $p_{Q_0}(1) = 0.4$. The system parameters are set as follows: the length of observation

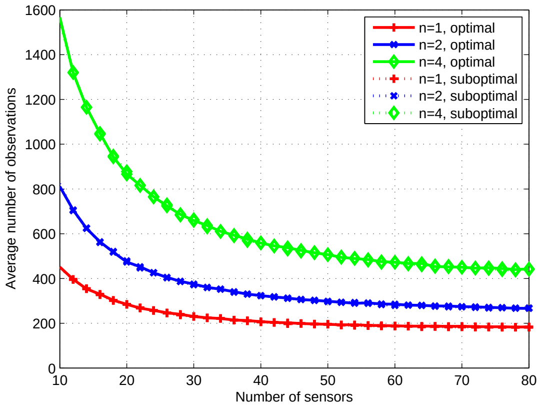

Figure 1: Average number of observations vs. Number of sensors.

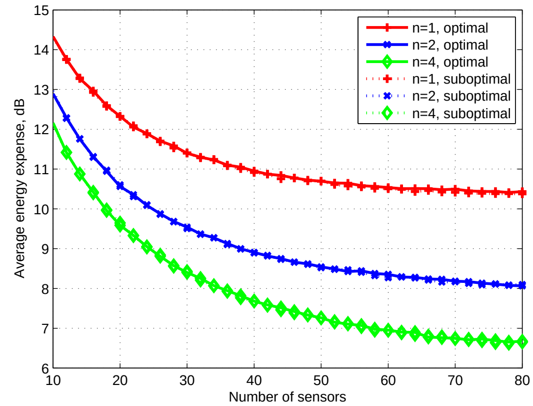

Figure 2: Average energy expense vs. Number of sensors.

sequence n is 1, 2, or 4; the desired probabilities of false alarm P_{fa} and misdedetection P_m are 0.001 and 0.0001 respectively.

Fig. 1 shows the average number of observations according to the number of sensor nodes in the network under the hypothesis H_1, where the solid lines and the dotted lines represent the simulation results for the optimal detector and the suboptimal detector respectively. From the figure, it can be seen that the average number of observations is a decreasing function of the number of sensors. The reason is as follows: if the number of sensors increases, the variance of channel noise decreases and the fusion center receives more precise type (histogram) information such that a lesser number of observations is required. Also, with a large number of sensors, it can be observed that an additional increase of number of sensors does not provide a significant gain in the number of observations. This is because with the large number of sensors the performance of the distributed detection scheme becomes similar to the performance of centralized sequential detection. Another observation is that a detector with longer sequences such as $n = 4$ does not have superiority over a detector with shorter sequences such as $n = 1$ in terms of the average number of observations. Intuitively, this can be explained by the fact that in the last received group of observations, more data can be regarded as being redundant after the log-likelihood ratio has exceeded one of the thresholds.

Fig. 2 displays the average energy expense according to the number of sensors under the hypothesis H_1, where the solid lines and the dotted lines represent the simulation results for the optimal detector and for the suboptimal detector, respectively. It is observed that more effective energy consumption can be achieved by using longer observation sequences (larger n). This justifies a benefit of the type-based approach for data transmission, which spends a similar level of power for each channel use, regardless of n. Also it is seen, that a network with a large number of sensors becomes more efficient in terms of energy consumption. This is also due to the advantages of TBMA.

From both Figs. 1 and 2, we finally observe that the difference between the performances of the optimal and the suboptimal detector is negligible. In the numerical example, the actual difference between the performances did not exceed 2 $\%$ in all cases. Furthermore, the figures illustrate that the simulation and the numerical results for the suboptimal detector almost coincide, which is evidence that the proposed approximation of the optimal detection rule, referred to as the suboptimal rule, and the method of calculation of performances are reasonable.

6 Conclusion

In this paper, we studied a type-based sequential detection in wireless sensor networks. Motivated by the computational and analytical complexity of the optimal detection rule and in order to reduce the detection delay, we proposed a new detection rule for sequential type-based detection with Gaussian approximation, referred to as the suboptimal detection rule. A performance comparison between the two detection rules showed that the optimal rule can be approximated by the suboptimal rule without essential losses in performance. At the same time, the suboptimal rule significantly reduces the complexity of the computations as well as the processing time to the point that processing time per one iteration does not depend on the number of sensor nodes in the network. Additionally, we showed that in the case of sequential type-based detection large-scale networks allows to save both the average number of observations and total energy consumption. These facts make the proposed suboptimal detector useful for real-time applications in networks performing both sequential data acquisition and detection.

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