

Linear Model Based Diagnostic Framework of Three Tank System

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Abstract:-In this paper, a linear model based FDI framework of nonlinear Three-tank system is developed. The nonlinear model [16] is analytically linearized using perturbation theory. Simulations are carried out to verify the linearization and effectiveness of the proposed framework, for fault detection, isolation and estimation of abrupt, incipient in the presence of model uncertainties as well as for simultaneous multiple faults. A comparison of results with existing nonlinear FDI schemes is presented, showing the effectiveness of proposed FDI framework for uncertain nonlinear systems.

Key words: Fault Detection and Isolation (FDI), Linearization, Linear Observers.

1 Introduction.

Modern control systems are becoming more and more complex, sophisticated with increasingly demanding performance goals. These systems must be highly reliable and secure. The complexity and sophistication of the new generation of aircrafts, automobiles, satellites, chemical plants and manufacturing lines, along with growing demands for higher performance, efficiency, reliability and safety, is being met by more automated control and monitoring systems. An effective way to assure their reliability and security is to swiftly detect and isolate their sensor and actuator failures, as well as failures in the systems components. The development of fault detection and diagnosis tools to help the correction of abnormal behavior during operating processes or off-line is a very active research area in automation and controls.

In this paper, a linear model based FDI framework of nonlinear benchmark, three-tank system is proposed. The nonlinear model is analytically linearized using perturbation theory. The novelty of proposed FDI framework is that it uses all the concepts of linear theory with nominal computational resources and achieves performance gains almost equivalent to nonlinear FDI techniques [16, 20, 21] which needs extensive computational resources. This framework uses linear model based approach to generate the residuals and subsequently detect, isolate and estimate faults. Different cases of abrupt, incipient and simultaneous multiple faults in the presence of model uncertainties are considered. The results obtained are compared with results of Li and Zhou [16]. Their technique is based on a modified robust fault diagnosis strategy from Polycarpou's online approximator [20,21]

using adaptive sliding mode observers with boundary layer control. These techniques need a lot more computational resources as compared with our scheme.

The three-tank system is a bench mark experimental facility developed for the research purposes for process and aerospace industry. This system has different variants in three tank configuration, as well as four tank systems is also quoted in the literature [15, 16, 17]. It is mainly used for the development and experimentation of complex linear as well nonlinear control and diagnostic algorithms. In this system actuator faults such as biases or loss of sensitivity of actuator and component faults such as leakage in tanks and clogs in pipes can be easily created [12]

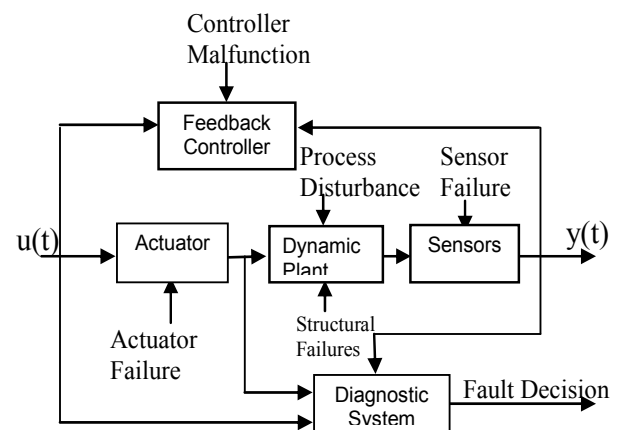


Fig. 1: General diagnostic framework

2 Model Based Diagnostic Framework

A typical fault diagnosis framework is shown in Fig. 1. In recent decades there have been significant research activities in the development of the new methodologies for on-line automated fault diagnosis and fault tolerant control. However, unlike the fault

detection problems, which have been investigated extensively in the literature, the fault isolation problem has received less attention, especially in the case of nonlinear uncertain systems. Indeed, the fault information generated in the fault diagnosis procedures can be very useful for fault tolerant control. Most of the issues related to the fault detection and fault tolerant control have been discussed in the review and survey papers by V. Venkatasubramanian et. al. [1~3], Rolf Iserman [4], J. J. Gertler [5], C. Angeli [6], R. Patton et al [7,8], and Y. Zhang & J. Jiang [9].

The fundamental problem fault detection and isolation lies in the generation of indication signals, usually called “residuals”, which point to the presence or absence of a fault. Most of the model based FDI techniques are based on the generation of residuals and their evaluation. The evaluation of residuals is based on many techniques ranging from observers, parity relations, directional residuals and stochastic methods such as sequential probability ratio tests, generalized likelihood ratio tests [4~9]. One of the challenging tasks of residual evaluation is the threshold analysis. Thresholding techniques are based on fixed as well as adaptive thresholding; their use in FDI is based on the availability of *a priori* knowledge of the process/ residuals.

A broader class of fault detection and isolation methods makes explicit use of mathematical model of the dynamic system, which is referred to as model based FDI. This approach is motivated by the fact that utilizing deeper knowledge of the system results in more reliable diagnostic decisions [11]. The model based FDI approach can be further segregated into two sections based on linear and nonlinear models being used for this purpose.

2.1 Fault Diagnosis of Linear System

Most model-based failure detection and isolation methods rely on linear dynamic models. In case of a nonlinear system, this implies a model linearization around an operating point. Although, dynamics of most of the systems are inherently nonlinear, these nonlinearities and other disturbance effects are generally smooth enough in the operating regions so that linear design techniques are applicable [11].

For modeling purposes, an open-loop system can be separated into three parts: actuators, system dynamics and sensors as illustrated in Fig. 1. In the fault free case, the system dynamics shown in Fig. 1 can be described by the state-space model as:

$$\dot{x}(t) = Ax(t) + Bu_R(t) \tag{1}$$

$$y_R(t) = Cx(t) + Du_R(t) \tag{2}$$

where $x(t)$ is the state vector and matrices A,B,C,D are matrices of proper dimensions, $y(t)$ the vector of measured output signals, $u_R(t)$ and $y_R(t)$ are signals corrupted by actuator and sensor faults. A linear observer is used to estimate the states of the system, thus generating residuals *signal* $r(t)$. These residuals signals are used for the detection, isolation and estimation of faults.

2.2 Fault Diagnosis in Nonlinear Systems

Although majority of model-based fault diagnosis approaches are based on linear system models. In real life most of the systems are nonlinear. The preferred way to address the nonlinearity is to deal with it directly and develop nonlinear fault detection and isolation techniques [14]. Consider a nonlinear dynamical system described by:

$$\begin{aligned} \dot{x}(t) &= g(x(t),u(t),f(x(t),u(t),t),\eta(x(t),u(t),t)) \\ y(t) &= h(x(t),u(t),f(x(t),u(t),t),\eta(x(t),u(t),t)) \end{aligned} \tag{3}$$

where $x(t)$ is the state vector, $y(t)$ is the output vector, $u(t)$ is the input vector, f and η represent the unknown fault and modeling uncertainty respectively. Both f and η are nonlinear functions of the system state $x(t)$, input vector $u(t)$ and time t .

2.3 Model based FDI

In model-based FDI, faults are detected by setting a threshold (fixed or variable), on a “residual” generated from the difference between real measurements and estimates of these measurements. The residual is a signal $r(t)$, that carries information on the time and location of the faults. It should be near zero in fault-free case and deviate from normal when a fault has occurred. The decision process evaluates the residuals and monitors if and where a fault has occurred. Let $J(r(t))$ and $T(t)$ denote the decision function and the threshold, a fault can be detected by the following test

$$\begin{aligned} J(r(t)) &\leq T(t) && \text{for } f(t) = 0 \\ J(r(t)) &> T(t) && \text{for } f(t) \neq 0 \end{aligned} \tag{4}$$

The isolation of the specific fault, say i^{th} out of q possible faults, requires

$$\begin{aligned} J(r_i) &\leq T_i && f_i(t) = 0 \\ J(r_i) &> T_i && \text{for } f_i(t) \neq 0; I=1,2,\dots, q \end{aligned} \tag{5}$$

where $J(r(t))$ is residual evaluation function. Similar residual enhancement techniques to the linear system case, i.e., structured residuals and directional residuals, can be used to facilitate fault isolation. As discussed before residual evaluation is to compare residual signal

$J(r(t))$ and the threshold $T(t)$, initially set in fault free case.

2.4 The Observer Design

The brief mathematical model of the classical (Luenberger type) observer along with residual generation used in this research note is described in this section. The purpose of the observer is estimate an output $\hat{x}(t)$ that asymptotically estimates the state $x(t)$ of the observed system [18]. The mathematical model of a linear system is given as under:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \\ \dot{\hat{x}}(t) &= F\hat{x}(t) + Hu(t) + Ly(t) \end{aligned} \tag{6}$$

The state estimation error is then,

$$\dot{e}(t) = x(t) - \hat{x}(t) \tag{7}$$

Using equations (1) and (3), we get can obtain the state estimation error dynamics of the form,

$$\dot{e}(t) = Fe(t) \tag{8}$$

This equation indicates that, if the eigenvalues of F are stable, then the state estimation error approaches to zero asymptotically and thus $\hat{x}(t)$ approaches to $x(t)$. The delectability of the system is ensured by the use of observable pair (A, C) for the system [12].

3 Process Description: Three Tank System

One of the popular experimental systems in controls community, the three-tank water process is the bench mark for the development and experimentation of complex linear as well nonlinear control and diagnostic algorithms. The three tank system model is depicted in Fig. 3 is written using the well known ‘‘mass balance’’ equations, as in [15] by:

$$\begin{aligned} S \frac{dL_1}{dt} &= q_1 - q_{13} - q_{1leak} \\ S \frac{dL_2}{dt} &= q_2 + q_{32} - q_{20} - q_{2leak} \\ S \frac{dL_3}{dt} &= q_{13} - q_{32} \end{aligned} \tag{9}$$

where q_{ij} represents the water flow rate from tank i to j . $i, j = 1, 2, 3$, which, according to Torricelli’s rule is given by:

$$q_{ij} = \mu_i S_p \text{sign}(L_i - L_j) \sqrt{2g|L_i - L_j|}$$

Notice that q_{20} represents the outflow rate with $q_{20} = \mu_2 S_p \sqrt{2gL_2}$ and q_{1leak}, q_{2leak} are leakages from tank-1

and tank-2 representing systems structural/component faults. The full system model is then obtained as follows:

$$\begin{aligned} \dot{x}_1 &= C_1 \text{sign}(x_1 - x_3) \sqrt{|x_1 - x_3|} + (u_1 + w_1) / S \\ \dot{x}_2 &= C_3 \text{sign}(x_3 - x_2) \sqrt{|x_3 - x_2|} - C_2 \text{sign}(x_2) \sqrt{|x_2|} + (u_2 + w_2) / S \\ \dot{x}_3 &= C_1 \text{sign}(x_1 - x_3) \sqrt{|x_1 - x_3|} - C_3 \text{sign}(x_3 - x_2) \sqrt{|x_3 - x_2|} \\ y_1 &= x_1, y_2 = x_2, y_3 = x_3 \end{aligned} \tag{10}$$

where $x_i(t)$ is the liquid level in tank i and $C_i = (1/S)\mu_i S_p \sqrt{2g}$. The two control signals are $u_1(t), u_2(t)$ respectively, the input flow $q_1(t)$ and $q_2(t)$. w_1 and w_2 are actuator faults which perturb the behavior of the system. These actuator faults must be detected and isolated.

Since the system is inherently unstable, a controller is required to regulate the flow rates and levels of the tanks, so that steady state condition can be achieved. A standard PI controller is used and is adapted from [15] with minor gain adjustments, since purpose of this work is not to design a controller rather use it as a component for establishing the FDI framework. The details are not can be found in [15]. This controller in [15] produces some unwanted (negative) levels of tanks, which are removed by adjusting the PI gains, such that no negative output levels are produced and system is stabilized in a reasonable time.

4 Analytic Linearization

Since the system (10) is nonlinear it is not possible to apply linear observer based techniques for fault diagnosis and isolation. The system is linearized by using small signal linearization or perturbation theory. The different types of linearization techniques, their merits and demerits/limitations along with linearization process are discussed in [19]. The small signal linearization is briefly discussed below:

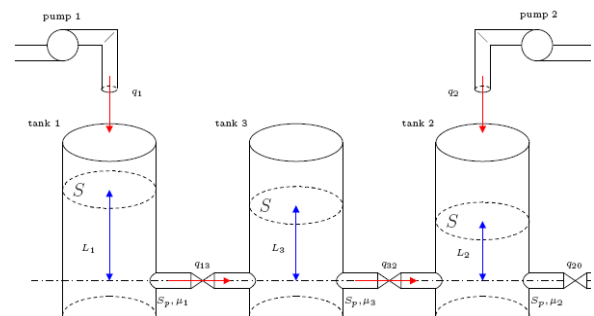


Fig. 3. The Three-Tank System [15, 16, 17]

In small signal linearization, equilibrium point of a fault-free nonlinear function is first identified [19], such that:

$$\dot{x} = f(x_0, u_0) = 0$$

Then the perturbed nonlinear function is expanded about equilibrium point i.e., $x = x_0 + \delta x$ and $u = u_0 + \delta u$, so that $\dot{x}_0 + \delta \dot{x} \cong \dot{x} = f(x_0, u_0) + g(\delta x, \delta u)$, where $g(\delta x, \delta u)$ is the linear equivalent of nonlinear function $f(x, u)$ at x_0 and u_0 . By simplifying the above equation we get:

$$\delta \dot{x} \cong g(\delta x, \delta u)$$

This is linear system gives the approximate dynamics of the nonlinear system about equilibrium point (x_0, u_0) .

The four operating regions corresponding to different states are $x_1 \geq x_3$ or $x_1 < x_3$ and $x_2 \geq x_3$ or $x_2 < x_3$. Let $S_{13} = \text{sign}(x_1 - x_3)$, let us introduce variable constants as: $S_{32} = \text{sign}(x_3 - x_2)$ and $S_{02} = \text{sign}(x_2)$. These constant variables represent the signs of the differences of states. These variable are not continuous functions, they act as switching function and switches their value between [-1,1] depending upon the sign of difference of two states. So there is no need for the linearization of these terms. Using these constants, the system represented by (10) can be written as:

$$\begin{aligned} \dot{x}_1 &= C_1 S_{13} \sqrt{|x_1 - x_3|} + (u_1 + w_1) / S \\ \dot{x}_2 &= C_3 S_{32} \sqrt{|x_3 - x_2|} - C_2 S_{02} \sqrt{|x_2|} + (u_2 + w_2) / S \\ \dot{x}_3 &= C_1 S_{13} \sqrt{|x_1 - x_3|} - C_3 S_{32} \sqrt{|x_3 - x_2|} \end{aligned} \quad (12)$$

The above mentioned linearization framework is applied to system represented by (16). Taking the \dot{x}_1 and expanding it using Taylor series expansion, neglecting the higher order terms, we get:

$$\begin{aligned} \dot{x}_1 &= -C_1 S_{13} \sqrt{x_1} \left(1 - \frac{x_3}{x_1}\right)^{1/2} + (u_1 + w_1) / S \\ &= -C_1 S_{13} \sqrt{x_1} + 1/2 C_1 S_{13} x_1^{-1/2} (x_3 - x_1) + (u_1 + w_1) / S \end{aligned} \quad (13)$$

Now applying perturbation theory, we obtain,

$$\dot{x}_1 = -C_1 S_{13} (x_{10} + \delta x_1)^{1/2} + 1/2 C_1 S_{13} (x_{10} + \delta x_1)^{-1/2} (x_3 - x_{10} - \delta x_1) + (u_1 + w_1) / S$$

Again using Taylor series expansion,

$$\begin{aligned} \dot{x}_1 + \delta \dot{x}_1 &= -C_1 S_{13} \left[\sqrt{x_{10}} \left(1 + \frac{\delta x_1}{x_{10}}\right)^{1/2} + \frac{x_3}{2} \left(1 + \frac{\delta x_1}{x_{10}}\right)^{-1/2} \right] (x_{10})^{-1/2} \left(1 + \frac{\delta x_1}{x_{10}}\right)^{-1/2} \\ &\quad + (u_1 + w_1) / S \end{aligned} \quad (14)$$

after simplification we get

$$\begin{aligned} \dot{x}_1 + \delta \dot{x}_1 &= -S_{13} C_1 \left[\sqrt{x_{10}} + \frac{x_3}{\sqrt{x_{10}}} \right] \\ &\quad + S_{13} C_1 \left[\left\{ -\frac{1}{2\sqrt{x_{10}}} - \frac{x_3}{4(x_{10})^{3/2}} \right\} \delta x_1 + \frac{1}{2\sqrt{x_{10}}} \delta x_3 \right] + (u_1 + w_1) / S \end{aligned} \quad (15)$$

Elimination of the constant terms from both sides gives

$$\begin{aligned} \delta \dot{x}_1 &= S_{13} C_1 \left[\left\{ -\frac{1}{2\sqrt{x_{10}}} - \frac{x_3}{4(x_{10})^{3/2}} \right\} \delta x_1 + \frac{1}{2\sqrt{x_{10}}} \delta x_3 \right] \\ &\quad + (u_1 + w_1) / S \end{aligned} \quad (16)$$

Similarly the linearized expression for the \dot{x}_2 and \dot{x}_3 is obtained as follows:

$$\begin{aligned} \delta \dot{x}_2 &= C_3 S_{32} \left\{ \frac{1}{2\sqrt{x_3}} + \frac{x_2}{4(x_3)^{3/2}} \right\} \delta x_3 - \left(\frac{C_3 S_{32}}{2\sqrt{x_3}} + \frac{C_2 S_{02}}{2\sqrt{x_2}} \right) \delta x_2 \\ &\quad + (u_2 + w_2) / S \\ \delta \dot{x}_3 &= S_{13} C_1 \left[\left\{ \frac{1}{2\sqrt{x_{10}}} + \frac{x_3}{4(x_{10})^{3/2}} \right\} \delta x_1 - \frac{1}{2\sqrt{x_{10}}} \delta x_3 \right] \\ &\quad + S_{32} C_3 \left[\left\{ \frac{1}{2\sqrt{x_3}} + \frac{x_2}{4(x_3)^{3/2}} \right\} \delta x_3 - \frac{1}{2\sqrt{x_3}} \delta x_2 \right] \end{aligned} \quad (17)$$

This is a linearized version of nonlinear system about an operating point. The state space representation of the linearized system (16~18) can be written as:

$$\begin{aligned} \delta \dot{x}(t) &= A \delta x(t) + B u(t) \\ \delta y(t) &= C \delta x(t), \end{aligned} \quad (19)$$

Where system matrix A, B and C are linear matrices given as under:

$$A = \begin{bmatrix} -S_{13} C_1 \left\{ \frac{1}{2\sqrt{x_{10}}} + \frac{x_3}{4(x_{10})^{3/2}} \right\} & 0 & \frac{S_{13} C_1}{2\sqrt{x_{10}}} \\ 0 & -\left(\frac{C_3 S_{32}}{2\sqrt{x_3}} + \frac{C_2 S_{02}}{2\sqrt{x_2}} \right) & C_3 S_{32} \left\{ \frac{1}{2\sqrt{x_3}} + \frac{x_2}{4(x_3)^{3/2}} \right\} \\ S_{13} C_1 \left\{ \frac{1}{2\sqrt{x_{10}}} + \frac{x_3}{4(x_{10})^{3/2}} \right\} & -\frac{S_{32} C_3}{2\sqrt{x_3}} & S_{32} C_3 \left\{ \frac{1}{2\sqrt{x_3}} + \frac{x_2}{4(x_3)^{3/2}} \right\} - \frac{S_{13} C_1}{2\sqrt{x_{10}}} \end{bmatrix}$$

$$B = \begin{bmatrix} 1/S & 0 \\ 0 & 1/S \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\delta x = [\delta x_1 \quad \delta x_2 \quad \delta x_3]^T \quad \text{and} \quad u = [u_1 \quad u_2]^T$$

5 Simulation Results

Simulations have been carried out for the linearized observer, along with nonlinear controller and plant. The operating point selected for the simulations is given in Table 1. The system matrices A, B, and C corresponding

to this operating point are given in appendix. A classical Luenberger type observer (8) is designed with stable eigenvalues. The linearized system model is observable at selected operating point. The simulation parameters are used as in [16], which is being considered as reference for comparison of the simulation results. In [16] authors have developed a fast and robust fault diagnosis strategy for a class of nonlinear systems based on Polycarpou's [20,21] online approximator using adaptive sliding mode observers with boundary layer control. They have proved that, fault detection time of their proposed strategy is much shorter than the Polycarpou's scheme. The singularity issue in the state trajectories is handled by introducing a small constant ϵ (10^{-10}), so that the system does not become unstable. Also the initial condition conditions of observer are set to '0', and that of plant are close to operating point. The parameters used in simulation studies are taken from [16]:

Table 1
Parameters used for Simulation

Parameter	Value
S , Area of the Tanks	0.0154 m^2
S_n , Area of pipes, $n=1,2,3$	$5 \times 10^{-5} \text{ m}^2$
q_{1max} , q_{2max} (input flow rates)	100 ml/s
L_{i-max} , x_i , Level in Tanks, $i=1,2,3$	0.62 m
C_1 , C_3 and C_2	$0.0072, 0.0097$
The Operating point x_{10} , x_{30} and x_{30}	$0.60, 0.40, 0.25$

The fault size is estimated using simple relationship as: $f_i(t) = L(i,i)r_i(t)$, $L(i,i)$ is the i^{th} diagonal element of the observer gain matrix, L .

All the fault cases discussed in [16] are simulated in this research. The results for different cases of fault are given in figures below. It may be noted that the simulation time selected for simulations can be different from time used in [16]. The reason is obvious, because of the observer dynamics and gains used for the nonlinear PI controller and slow dynamics of Three Tank System. The main focus of this research is the fault detection and isolation and fault detection time, which is compared with fault detection times for different faults given in [16]. Fig. 4 shows the states $x(t)$ of the nonlinear plant, estimated states $\hat{x}(t)$ and error. It is quite evident that the error reduces to zero exponentially in 0.4 sec, showing the perfect working of the linearized observer.

In the 1st three cases, leakage in tank 1 is taken as simulated fault, the radius of hole in bottom of tank 1 is taken as r , the leakage rate is $q_{1_leak} = \mu\pi r^2 \sqrt{2gh_1}$. For the case 1 and case 2 there is an abrupt leakage tank 1 with

$r = 4.5 \cdot 10^{-2} \text{ m}$, in case 2 there is a set point change from 0.60 to 0.62 at 595 sec. The simulation results are shown in Fig. 5. The residual is shown in Fig. 5(b) indicating the location of fault in tank 1 and corresponding estimated and actual fault in Fig. 5(c), which clearly indicates that our proposed scheme has fairly estimated the actual fault magnitude, and the estimation errors of tank 2 and tank 3 remains within threshold. For the case 2, there is no significant change in fault estimation and fault detection time, except there is a large residual peak of about 0.5 sec width at 595 sec. The fault detection time is given in Table 2.

In case 3, there is an incipient fault in tank 1 from 630 sec, the radius of the leakage is time variant according to the following law:

$$r = \begin{cases} 0 & t < 630 \text{ sec} \\ 0.005(t - 630) & 630 \leq t < 700 \\ 0.0497 & t \geq 700 \end{cases} \text{ (units, m)} \quad (20)$$

The estimation results are shown in Fig. 6. Fig. 6(c) shows that the fault estimator has gracefully estimated the fault, but due the very small size of the actual fault, which grows very slowly with time fault detection time is large. This is due to the fact that the fixed thresholds set for fault detection is large as compared to size of incipient faults over short period of its inception. This fault can be detected at very early stage, if multiple

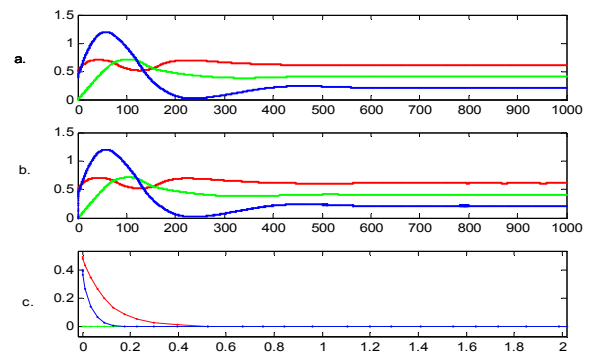


Fig. 4. Original $x(t)$, estimated $\hat{x}(t)$ states and residual, a.) Plant states, b.) Observer states, c.) estimation error.

thresholds are set for such faults and monitoring the fault after fault estimate has crossed the lower thresholds and fault decision is generated at higher threshold level. The other residuals remain within thresholds as in previous cases.

In case 4 and 5, there is an actuator 1 (pump 1) fault, the effectiveness of actuator is reduced by 90%, i.e., $q_1 = 90\% q_1^0$. In case 5, an uncertainty given below is introduced:

$$q_1 = (1 + 0.05 \sin(\omega t + \varphi))q_1^0 \quad (21)$$

where q_1^0 , is the nominal output of nonlinear PI controller. The simulation results are illustrated in Fig. 6, from which it is noted that the fault detection, isolation and estimation works perfectly, residuals for the other two tanks are within specified threshold. For the case 5, an uncertainty represented by (21) is introduced with $\omega_1 = 3, \varphi_1 = \pi/2$. Fig. 7(c) indicates a sinusoidal variation along with controller effects, still fault detection and estimation is satisfactory. The residual e_1 and e_2 vary due to introduced model uncertainties and the estimation errors of all states also remain in the thresholds.

To show the capability of our scheme to detect multiple faults simultaneously, case 6 includes the simultaneous multiple faults in the presence of uncertainty given by (21). There is an abrupt leakage in tank 1 from 566 sec to 622 sec, and simultaneously an abrupt leakage in tank 2 from 614 sec to 642 sec in the presence of model uncertainty. Simulation results shown in Fig. 8, the fault estimates of both tanks fluctuate like sine wave, but faults in tank 1 & tank 2 have been successfully detected, isolated and estimated simultaneously.

The fault detection times of our scheme are compared with [16] which compared the actual detection times, their bound estimates by Polycarpou [20, 21]. The comparison results are given in Table 2.

6 Conclusion

The results of proposed diagnostic framework are compared in table 2, with [16] and [20,21]. From table 2, it is clear that our fault detection times are better than Polycarpou's scheme, and slower than Li and Zhou's scheme.

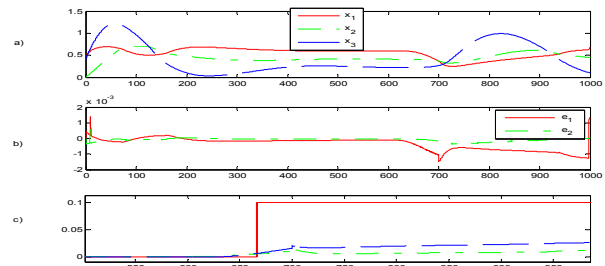


Fig. 6. Simulation for case 3 (incipient leakage in tank 1): Fault detection, isolation, and estimation. a.) Plant states, b.) Residuals, c.) Fault estimate

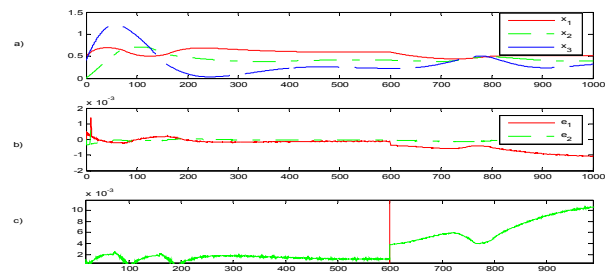


Fig. 7. Simulation for case 4 and 5 (actuator fault with model uncertainty): Fault detection, isolation, and estimation. a.) Plant states, b.) Residuals, c.) Fault estimate tank 1.

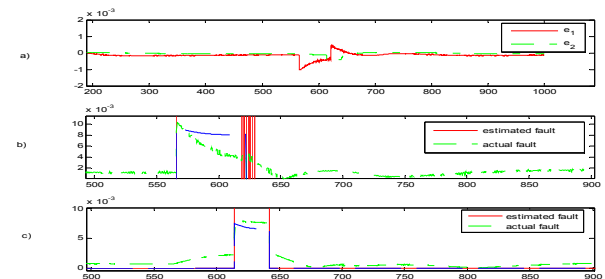


Fig. 8. Simulation for case 6: Simultaneous multiple fault detection, isolation, and estimation. a.) Residuals, b.) Fault estimate in tank 1, c.) Fault estimate in tank 2.

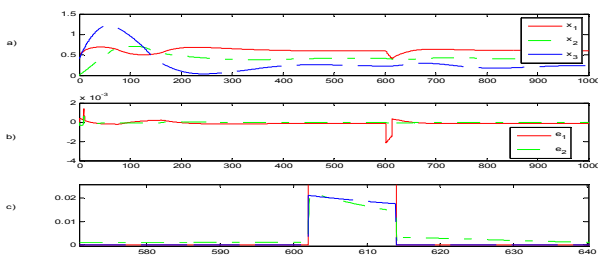


Fig. 5. Simulation results for case 1 (Leakage in Tank 1): Fault detection, isolation, and estimation. a.) Plant states. b.) Residuals. c.) Estimated fault

Table 2

Actual detection times, their upper bounds reproduced from Li, Zhou [16]

Cases	Upper bound by [20,21]	Actual time in [20,21]	Upper bound by Li, Zhou	Actual time by Li, Zhou	Actual time our's
Case 1	0.811	0.359	0.0016	0.0006	0.015
Case 2	5.66	3.57	3.740	2.750	15.08
Case 3	2.26	0.626	0.0059	0.0014	0.123
Case 4	0.245	0.129	0.00049	0.00024	0.148

For the case 2, i.e., incipient faults, our technique gives worst case times, but this can be improved by selection of better fixed thresholding scheme. However, the advantage of our proposed framework is that it uses simple linearized model, taking the advantage of linear theory leading to much less computational resources. Future work includes robust adaptive thresholding techniques to improve the response times especially for the case of incipient faults.

APPENDIX

The system matrices of linearized state space model of the system about operating point in Table 1 are given as under:

$$A = \begin{bmatrix} -0.000135739 & 0 & 0.000071573 \\ 0 & -0.000228975 & 0.001530144 \\ 0.0001357393 & -0.000110880 & -0.001601716 \end{bmatrix}$$

$$B = \begin{bmatrix} 64.935064935 & 0 \\ 0 & 64.935064935 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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