Analysis for Dynamic of Analog Circuits by using HSPN

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Abstract: Since the ability to describe dynamic in high level, Petri Nets has already been used extensively in logic circuits and systems community for state analysis. As others formal methods, extending Petri Nets to analog circuits analysis requires continuous feature enhanced. In this paper, the object of study is dynamic of circuits with random nonlinear feature (such as random noise and random interface) by using a Hybrid Statistical Petri Net (HSPN). A formal definition of hybrid Petri Net is derived from hybrid automata to model the dynamic of nonlinear analog circuits. Furthermore, we improve the hybrid Petri Net to abstract the statistical feature by enhancing the ability of describe random continuous transition. And a formal definition of the novel Hybrid Statistical Petri Net is presented. Within this HSPN, the fundamental method of analyzing the dynamic of random analog circuits is studied. A sample circuit with random noise illustrates this technique.

Key–Words: Petri Nets, Random noise, Dynamic, Analog circuits, Hybrid Automata, Formal methods, Model

1 Introduction

Recently, As an alternative approach to verify circuits, formal method which bases on the finite state abstraction of analog circuits has been studied in several previous works [1] [2] [3] [4] [5]. They extend formal verification method to nonlinear analog circuits. In [1] [3], an analog system is approximated by a discrete system in which classical model-checking algorithms can be applied. [2] [4] present a potential extension of formal verification methodology in order to deal with time-domain properties of nonlinear circuits whose dynamic behavior is described by differential algebraic equations. An other significative paper [5] introduces the algorithm for approximating the reachable sets of analog system.

From the point of manufacturing technology, the fluctuation caused by random features in circuits becomes more and more significant. This stochastic dynamic has to be considered and evaluated in early level of design works. Since formal verification methodologies have been introduced into the field of analog nonlinear circuits, they can model this random fluctuation by enhancing in nonlinear statistical characters. However the statistical evaluation needs to utilize some statistical natured methods such as the Petri Net with statistical transition factors. In this paper, we propose and study a novel Hybrid statistical Petri Net(HSPN) in this research corner. HSPN is derived from hybrid automata, and extended to the continuous statistical transition. HSPN has the ability that analyze the hybrid dynamic system with nonlinear statistic.

The remainder of this paper is organized as follows. In section 2, we propose a brief introduction of hybrid automata, since this method is derived from hybrid automata. Then we propose and study the approach to model nonlinear analog circuits by using hybrid automata. And we propose the Hybrid Petri Net which is based on the hybrid automata. This Petri Net operates with a variable continuous transition velocity. In section 3, the Hybrid Petri Net is extended to stochastic analog transition of which velocity follows the given stochastic distribution. And the definition of HSPN is presented in this section. Since the nonlinear noise in oscillator circuits affects its phase and duty cycle in random, in the fourth section, the HSPN based method is explained by analyzing the statistical feature of relaxation oscillator's phase and duty cycle. Finally, we drawn a conclusion in section 5.

2 Hybrid Petri Net Model Derived from Hybrid Automata

2.1 Introduction to Hybrid Automata Based Model

2.1.1 Definition

A hybrid automata H is assigned as a 6-tuple $H = \langle Q, X, Init, f, Dom, Reset\rangle$ [6], in which

- Q is the finite set of discrete variable with values in $\boldsymbol{Q}$;
- X is the finite set of continuous variables with values in $\mathbf{X} = \mathbb{R}^2$;
- $Init \subseteq \mathbf{Q} \times \mathbf{X}$is the set of initial states;
- f: $\mathbf{Q} \times \mathbf{X} \to T\mathbf{X}$is the vector field;
- $Dom \subseteq \mathbf{Q} \times \mathbf{X}$ is the domain of H;
- Reset: $\mathbf{Q} \times \mathbf{X} \to P(\mathbf{Q} \times \mathbf{X})$ is the reset relation.

From the definition, hybrid automata is an efficient model to abstract the system with both continuous and discrete dynamics. Some analog circuits have proved to be a system with both dynamics. The method to model analog circuits by using hybrid automata is presented as follows.

2.1.2 Hybrid Automata Model of Analog Circuits

An analog system (including analog circuits) can be described as a DAE (differential algebraic equations) [2] [7].

$$f(\dot{x}(t), X(t), u(t)) = 0 \quad (1)$$

in which *x(t)* is the vector of system variables and *u(t)* denotes the vector of input variables.

We denote $\gamma(t, x_0, u(t))$ as the solution of (1) at *t*, and Φ as the space of whole solutions In a time slice $T = \{t|0 \leq t \leq t'\}$, the solution vector space can be denoted as (2).

$$\Phi(X_0, t) = \{\gamma(t, x_0, u(t))|t \in T and x_0 \in X_0 and u(t) \in U\} \quad (2)$$

where X_0 is the initial condition set of DAE and T is the set of time slice, and $\Phi(X_0, t) \in \Phi$.

If there are some hard nonlinear boundaries in the vector space of solutions, this continuous space can be divided into several continuous sub-spaces with the boundary. Each of the continuous sub-space can be represented by a discrete state $q \in Q$ in semantic (Q denotes the discrete variable in hybrid automata definition). The continuous dynamic, which continuous variables in space Φ change from one sub-space to another, can be represented by the discrete states transition, i.e. if we refer that discrete state q_1 transit to state q_2, it will means that the vector γ changes from sub-space q_1 to sub-space q_2. The discrete state such q represents a set of continuous variables X in semantic(X is defined in hybrid automata).

The transition of discrete states is a result that continuous variables change with time, e.g. two discrete states q_1 and q_2 map to the continuous variable set $\Phi_1(X_0, t_1)$ and $\Phi_2(X_1, t_2)$ separately. These two sets form a consequence $\Phi_1(X_0, t_1), \Phi_2(\phi_1(X_0, t_1), t_2)$, i.e. these two sets have the same convex polyhedron when q_1 transits to q_2.

The hybrid automata model provides a fundamental abstraction to analog circuits, and some important circuit property can be analysis.

2.2 Hybrid Petri Nets Derived from Hybrid Automata

In [8], a Timed Hybrid Petri Net(THPN) is introduced into circuits verification. However, the velocity of continuous transitions is constant, i.e. the dynamic response reacts in the fixed velocity. From the view of the general definition of hybrid automata [2], the velocity of continuous transition is a continuous function. And the fire rate of discrete transitions is restricted by this function. It is more significant for mixed-signal circuits. Besides that, in analog circuits, the flow of electric charge can be abstracted as the flow of continuing token in hybrid Petri net. Reviewing the hybrid automata defined in section 2.1, the hybrid Petri net can not model the hybrid dynamic system until it is expanded the fixed firing velocity of continuous transition to a variable.

According to the traditional hybrid Petri Net [9], we define a Hybrid Petri Net(HPN) with timed variable firing velocity as a 7-tuple $HPN =< P, T, F, B, M, \delta, \nu >$ where:

- ***P***: $P_D \cup P_C$, P_D is discrete place, and P_C is the continuous place;
- ***T***: $T_D \cup T_C$, T_D is discrete transition , and T_C is the continuous transition;
- $F \subseteq (P \times T) \cup (T \times P)$ is a relation specifying the arcs from place to transition and vice versa. $F \subseteq (P_X \times T_Y) \cup (T_X \times P_Y)$ for $X, Y \in (D, C)$;
- B: $(P_C \times T_D) \to (-\infty \cup \mathbb{Q}) \times (\mathbb{Q} \cup \infty)$ assigns a set of bound $[b_l(p, t), b_u(p, t)]$ for all continuous places with arcs to discrete transition where $b_l(p, t) \leq b_u(p, t)$ and $\mathbb{Q}$ is a rational. If the value of continuous place current marking exceed the bound (upper or lower), the discrete transition with a arc from the continuous place will enable;

- M is the set of marking where m_0 is assigned to be the initial marking of hybrid Petri net. The initial marking must be within the bound B;
- δ: time delay variable associating to each discrete transition;
- ν: is the firing velocity of continuous transition.

3 Statistical Factor Set in Hybrid Petri Net

The HPN defined in section 2.2 abstracts the both continuous and discrete dynamics of analog circuits. Because the noise affects the dynamic in random, the HPN needs to be extend to statistic field.

It is assumed that all the nonlinear random noise can be isolated from the both continuous and discrete transition. We add the statistical continuous transition to HPN defined in section 2.2. And this transition following some given probability density function is abstracted as the statistical effect of noise.

The Hybrid Petri Net is extended to the Hybrid Statistical Petri Net(HSPN) as a 8-tuple $HSPN =< P, T, F, B, M, \delta, \nu_C, \nu_S >$ in which:

- P is the same with the one defined in section 2.2;
- T: $T_D \cup T_C \cup T_S$, T_D is discrete transition , T_C is the continuous transition, and T_S is the statistical transition;
- $F \subseteq (P_X \times T_Y) \cup (T_X \times P_Y) \cup (P_C \times T_S) \cup (T_S \times P_C)$ for $X, Y \in (D, C)$, statistical transition is added to the arc relation set.
- B, M and δ are the same with the definition in section 2.2;
- ν_C denotes the absolute velocity function without statistical effect;
- ν_S denotes the statistical velocity function (nonlinear function) which follows a given stochastic distribution (such as Gauss Distribution).

Since variables disturbed by random noise are always continuous, and the statistical distribution is continuous, the statistical transition is defined to be continuous. And it is the statistical continuous transition that causes the fluctuation of discrete states. It is illustrated in Fig. 1, the HSPN of the relaxation oscillator which we will explain in section four.

$P_{\phi 1}$ and $P_{\phi 2}$ are denote the discrete places. e.g. the charging state and discharging state. Due to the control of the fire condition by continuous place P_{C1}

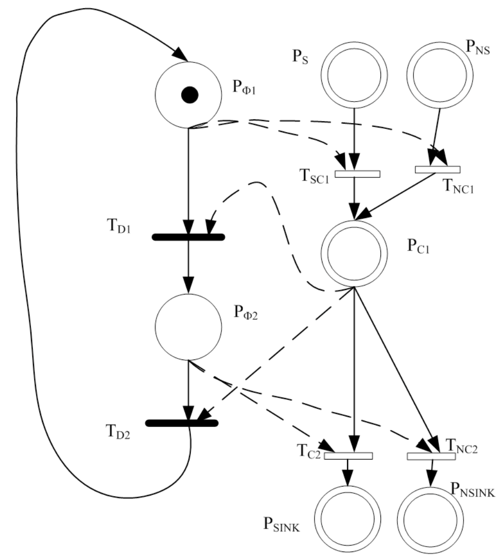

Figure 1: Hybrid Petri Net Model of Relaxation Oscillator

(its boundary conditions controls the discrete transition), the change of discrete states is restricted by continuous state. Transitions T_{NC1} and T_{NC2} are two statistical transition. They cause the random fluctuation of variable in place P_{C1}. Therefor a model of state transition with random fluctuation is abstracted by HSPN.

4 Transient Noise Analysis of Relaxation Oscillator

The output voltage of relaxation oscillator changes periodically with the charging and discharging of capacitor. Such as Fig. 2, where V_1 and V_2 denote two reference voltages of Schmitt Comparator. If the voltage of capacitor (V_C) is lower than reference voltage V_1, Schmitt circuit output a high level voltage V_{dd}. On the other hand, if V_C is higher than reference voltage V_2, Schmitt circuit output a low GND.

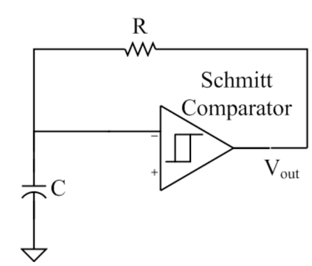

Figure 2: Schematic of Relaxation Oscillator

4.1 Analysis of Random Phase Noise

The time constant of RC based oscillator is denoted as $\tau = RC$ (the total resistance and total capacitance on V_{out} node). If the circuit keeps steady, the period of relaxation oscillator will be very accurate and stable in theory. Unfortunately, in non-ideal CMOS integrated circuits, the stochastic property of oscillator always exists. As for frequency, the noise source connected to node V_{dd} disturbs its phase and duty cycle. It makes the phase and duty cycle act in nonlinear feature with statistic. If it is assumed that the noise follows Gaussian distribution, the output of relaxation oscillator can be approximated to a stochastic square wave signal with mutually independent, Gaussian-distributed period jitter [10]. The period T_0 of relaxation oscillator is found to be (5).

$$T_1 = RC \times \ln(\frac{V_{dd} - V_1}{V_{dd} - V_2}) \tag{3}$$

$$T_2 = RC \times \ln(\frac{V_2}{V_1}) \tag{4}$$

$$T_0 = T_1 + T_2 = RC \times \ln(\frac{V_{dd} - V_1}{V_{dd} - V_2} \times \frac{V_2}{V_1}) \tag{5}$$

where T_1 is the first half period of oscillator, and T_2 is the second one. The jitter at each of period can be found in (5)

$$\overline{\Delta V^2{}_C(t)} = \frac{kT}{C}(1 - e^{-2t/RC}) \tag{6}$$

If we denote V_C as the voltage crossing capacitor and ΔV_C as the fluctuation of V_C. The variance of V_C fluctuation is given by (6).

in which T is circuit temperature and if we assume a duty cycle of 50%, $2t$ will be replaced by T_0. The detail about phase noise of relaxation oscillator can be found in [10].

We substitute ΔV_C as a fluctuation of the circuit in Fig. 2. Then the dynamic, noise involved, can be analyzed.

4.2 Hybrid Petri Net Model of Relaxation Oscillator

The circuit equation set is showed in (7).

$$V_C(t) = \begin{cases} V_{dd} + e^{-t/RC}(-V_{dd} + V_1) & if\, 0 \le t \le T_1 \\ V_2 \cdot e^{t/RC} & if\, T_1 \le t \le T_2 \end{cases} \tag{7}$$

The capacitor storing energy in the relaxation oscillator is the key in modeling.

In order to make discretion simple and clear, and focus on the nonlinear dynamic caused by statistic, we linearize (7) by standard PWL (Piecewise Linear) method [11]. As for preciseness, some more accurate algorithm can be used in this step.

According to (7), the relaxation oscillator circuit can be abstracted to two discrete states. One is capacitor charging state and another one is discharging state. The system operates as a hybrid automata with two discrete states.

In Fig. 1, this hybrid Petri net consists of two parts. The left part represents discrete transition. Discrete place $P_{\Phi 1}$ denotes the capacitor charging state and $P_{\Phi 2}$ is the discharging state. The transitions of discrete states are contributed by continuous states.

In the first half period of oscillator, we denote P_S as a power source, a continuous place which provides infinite continuous token, as an ideal power source provides infinite current (electric charge source). Continuous place P_{NS} denotes a ideal noise current source, providing infinite continuous token too. The last continuous place in the first half period is P_{C1} as the state of voltage accumulating in capacitor, of which bound is $[V_1, V_2]$. The upper bound is V_2, and its initial value is V_1. T_{C1} is continuous transition. T_{NC1} is the statistical transition. The firing velocity of T_{C1} follows (6), $dV_C(t)/dt$ can be view as the electric charge flow rate from the source P_S (i.e. the charge current $i_C(t) = (V_{dd} - V_C(t))/R$). And T_{NC1} follows the Gaussian distribution with zero mean and a variance of $\overline{\Delta V_C^2}$.

If the value of P_{C1} becomes higher than its upper bound, the oscillator will move into the second half period, discharging period. P_{SINK} is viewed as current sink to discharge the capacitor, and P_{NSINK} is another noise source. The firing velocity of transition T_{C2} also fellows (7), $dV_C(t)/dt$ in discharge period. Since the noise source in the second half period is independent, the firing velocity of T_{NC2} follows independent Gaussian distribution with the same parameters.

Once the circuit powers on, the Petri net can start from the initial mark m_0. The discrete token is stored in place $P_{\Phi 1}$, and the current marking of the continuous place P_{C1}(i.e. the initial value of place P_{C1}) satisfy bound condition, therefore the transition T_{C1} and T_{NC1} fires with the variable velocity $dV_C(t)/dt$ and ν_S until place P_{C1} reaches its upper bound V_2. The value of P_{C1} increases with T_{C1} and T_{NC1}. If the current marking of P_{C1} reaches upper bound V_2, the discrete transition T_{D1} will be fireable, and T_{D1} fires. The discrete token flows to next discrete place $P_{\Phi 2}$. And the transitions T_{C1} and T_{NC1} stop firing because lost the discrete token. The circuit step into the second half period.

If the token in place $P_{\Phi 2}$ and place P_{C1} satisfy its bound condition, T_{C2} and T_{NC2} will fire. The contin-

Table 1: Condition of Simulation

V_{dd}	T	R	C	Frequency in Theory
$1.8V$	$300K$	500Ω	$10pF$	$200MHz$

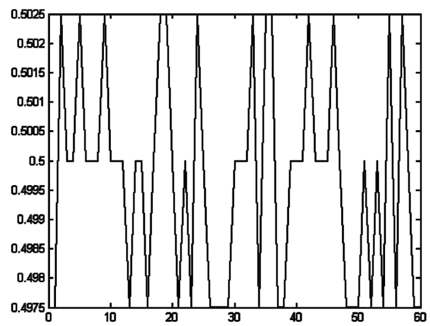

Figure 3: The Fluctuation of Duty Cycle

uous place P_{C1} decreases in value with T_{C2} and T_{NC2} till the marking of P_{C1} reaches its lower bound. If the lower bound of P_{C1} is reached, T_{D2} will be fireable and fire. Then the discrete token returns to $P_{\Phi 1}$. A period of relaxation oscillator is completed.

In order to obtain the timed information, we compute the time delay variable for the Petri net. Firstly, when P_{C1} increases in value, the time delay is calculated with $(m(P_{C1})-V_1)/\nu_{sum1}$ where $m(P_{C1})$ is the marking of P_{C1} and ν_{sum1} denotes the sum of the velocity of T_{C1} and T_{NC1}. Then in second half period, it is $(V_2 - m(P_{C1}))/\nu_{sum2}$ in which ν_{sum2} denotes sum of T_{C2} and T_{NC2}.

4.3 Simulation

Since this TSPN model abstracts the dynamic property of relaxation oscillator circuit with random noise, the stability of duty cycle is analyzed during the TSPN simulation. The parameters of relaxation oscillator we used are in Table 1 where the absolute temperature is approximated to 273 degrees. The theoretic frequency of this oscillator is 200MHz.

We use Matlab as the platform of simulation. In order to obtain the non-ideal duty cycle, we set $V_1 = 0.6V$ and $V_2 = 1.2V$. Fig. 3 illustrates the fluctuation of duty cycle caused by the random noise. The vertical axis represents the value of duty cycle, and the horizontal axis represents the time. If we change V_1 to $0.42V(V_1 = 0.24V_{dd})$, V_2 to $1.38V$. All the duty cycle approximates to 50%. This result corresponds with [10].

5 Conclusion

In this paper, we have highlighted hybrid automata as a formal method to model the hybrid nonlinear circuits. Both discrete and continuous dynamics of the circuit can be analyzed through hybrid automata. Deriving from it, we propose a novel hybrid Petri net, HSPN, to analyze the circuits with nonlinear random noise. HSPN utilizes the hybrid automata based theory and extension of continuous statistical transition to analyze the hybrid statistical dynamic system.

HSPN proposes a quickly approach to analyze the dynamic of circuits with random features before the generation of netlist files. It is important to mention that HSPN is not only limited in analog phase noise verification. The all statistical fluctuation of circuits, which exists in nonlinear dynamic and follows some given statistical distribution independently, can be analyzed by HSPN. In addition, we plan to find more fluctuation model to optimize the algorithm of HSPN simulation, and establish an automatic platform to recognize the circuits in hardware description language in the future.

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