Method of navigation of autonomous moving object

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Abstract: The paper describes the implementation of the Kalman filter for control of object moving in the plane. The object is described through system of non-linear equations. The system is put under a feedback linearization in the case that managing influences are constants during the period of discretization. As result is obtained an extended Kalman filter that increases the precision of the object managing, through the inaccuracies of its mathematical description and the disturbances in the observation.
Keywords: Kalman filter, object moving, non-linear systems, feedback linearization

1 Formulation of the problem

Kalman filter evaluates systems with casual disturbances, as in the entrance signal, so in the supervision, in the case that not all quantities in the position vector are available to be measured directly. It helps to be terminated the evaluation position problem in the linear dynamics, linear supervision and Gaussian distribution of the initial state.

It is known that the Kalman filter can be used for linear systems. The paper presents implementation of the Kalman filter which is used in the algorithm for control of moving object. The precise, efficient and fast control of the movement is impossible if its dynamics have not correct mathematical presentation. It is chosen three-tired architecture. Each coordinate that described the state of the object is presented with signal. The third derivative of this signal is white noise w(t). Its supervision is additionally skew from noise v(t). This architecture allows the method of feedback linearization to be applied in the synthesis of the control algorithm. The common diagram that ensures the control of the moving object is shown on fig.1.

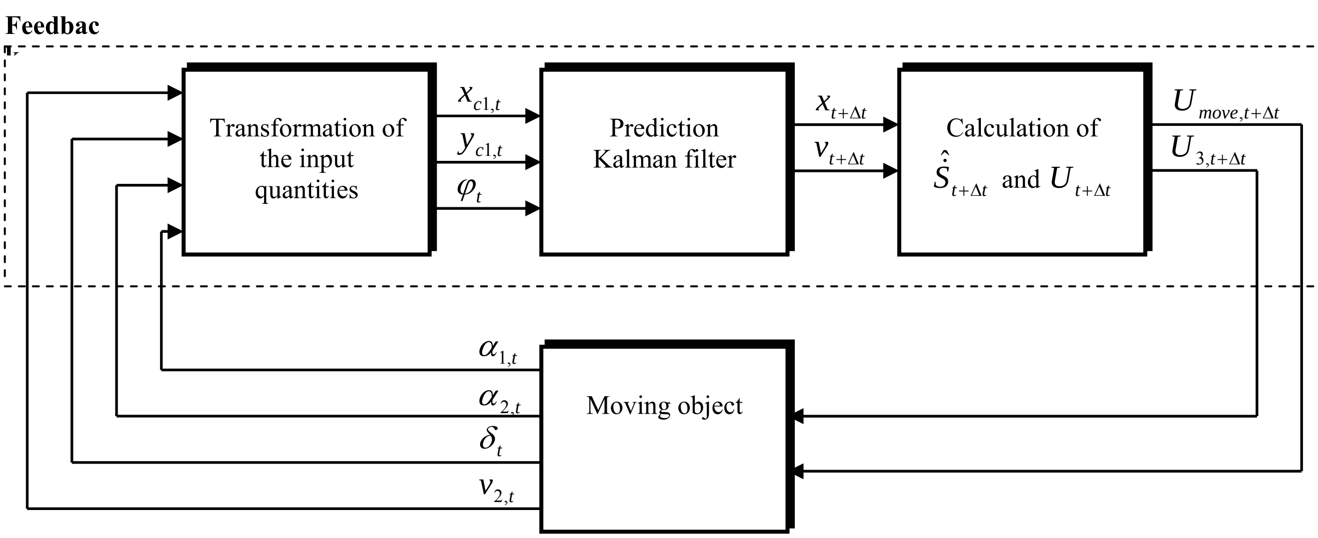

Fig. 1 Common diagram of the control of the moving object

Kalman filter is used after local feedback linearization in the case that input influences are constant during the discretization period Δt. The absence of influence between the different inputs of the system and equal number of the input and output quantities are an additional condition for the using of the filter [1, 2].

According to chosen three-tired architecture the system of equations that describes moving object is as follows:

$$\dot{x} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix} x + \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} v$$

$$z^i = \begin{vmatrix} 1 & 0 & 0 \end{vmatrix} x + w$$

Here v and w are white noises (indeterminateness) in the object and in the supervision whit different spectral densities q and r, z^i is a quantity that present supervision and x is a state vector:

$$x = \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} position \\ velocity \\ acceleration \end{vmatrix}$$

The matrix description of the system is:

$$\begin{vmatrix} \dot{x} = Ax + Bv \\ z^i = Cx + w \end{vmatrix} \qquad (1.1)$$

2 Mathematical substantiation of the noise influence adaptation method during a movement

The system (1.1) is linear and it affects all raised noise influences. According to it for every inner quantity is constructed a Kalman filter. It is supposed that managing of the movable object is happening in real time, so a static Kalman filter is used. It ensures that recalculation of the co-variation noise matrix is eluded.

It is important to notice that the traditional Kalman filter proceeds to established work conditions after certain time and it becomes insusceptible to outer changes. So, it is developed a version of Kalman filter with limited memory, which helps control of the object to be globalized. According to some results [2] co-variation matrix for the filter is:

$$P = r\begin{vmatrix} 2\mu & 2\mu^2 & \mu^3 \\ 2\mu^2 & 3\mu^3 & 2\mu^4 \\ \mu^3 & 2\mu^4 & 2\mu^5 \end{vmatrix}$$

where $\mu = \sqrt[6]{\frac{q}{r}}$

It could be written for the equation of the static filter:

$$\hat{\dot{x}} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix} \hat{x} + \begin{vmatrix} 2\mu \\ 2\mu^2 \\ \mu^3 \end{vmatrix} \left(z^i - \hat{x}_1 \right)$$

(1.2)

We replace y=x and the equation (1.2) is as follows:

$$\hat{\dot{y}} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix} \hat{y} + \begin{vmatrix} 2\mu \\ 2\mu^2 \\ \mu^3 \end{vmatrix} \left(z^i - y_1 \right)$$

Until this moment the system is examined as continuous one. Through the discrete characteristic of the managing and because of the connections between continuous and discrete processes with outer disturbances [3], it could be written:

$$\hat{y}^i_{(t+\Delta t)} = A\hat{y}^i_t + \Delta t A m^i \left(z^i_t - \hat{y}_{1,t} \right)$$

where

$$A = \begin{vmatrix} 1 & \Delta t & \frac{\Delta t^2}{2} \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{vmatrix}$$

$$m^i = \begin{vmatrix} 2\mu_i & 2\mu_i^2 & \mu_i^3 \end{vmatrix}^T$$

or

$$\hat{y}_{(t+\Delta t)} = \left(\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} + \Delta t \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix} + \frac{\Delta t^2}{2} \begin{vmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \right) \left(\hat{y}_t + \Delta t \begin{vmatrix} 2\mu \\ 2\mu^2 \\ \mu^3 \end{vmatrix} \left(z_t - y_{1,t} \right) \right) \qquad (1.3)$$

The evaluations for every inner signal and it's derivatives for $\Delta t + 1$ moment, is received from equation (1.3). These evaluations allow computation of the quantities that are necessary for control synthesis. The spectral densities of the noise signal r and q are unknown quantities. They are connected with the co-variation matrixes of the relevant signals, and dispersions D_1^i and D_2^i of the two signals that are received from z_t^i, after their division from digital filter from the type [2]:

$$F_i(p) = \frac{p^3}{(1 + T_i p)^4} \qquad i = 1, 2 \text{ и } p = \frac{d}{dt}$$

Time-constants are selected according to the spectral density of the noise. They have recommendable values [2].

The filters F_i have continuous characteristic. Their transmission function corresponds with the next presenting in the space of statements:

$$\dot{x}_1 = -\frac{1}{T_i} x_1 - \frac{1}{T_i} U$$

$$\dot{x}_2 = -\frac{1}{T_i}x_2 - \frac{1}{T_i}x - \frac{1}{T_i}U$$

$$\dot{x}_3 = -\frac{1}{T_i}x_3 - \frac{1}{T_i}x_2 - \frac{1}{T_i}x - \frac{1}{T_i}U$$

$$\dot{x}_4 = -\frac{1}{T_i}x_4 + x_3$$

$$y = -\frac{1}{T_i^4}x_4$$

where U is input influences.

In case of $a = \frac{-1}{T_i}$ it could be written in matrix mode:

$$\dot{x}(t) = Ax(t) + BU(t)$$

$$y(t) = \begin{vmatrix} 0 & 0 & 0 & \frac{1}{T_i^4} \end{vmatrix} x(t)$$

where $A = \begin{vmatrix} a & 0 & 0 & 0 \\ a & a & 0 & 0 \\ a & a & a & 0 \\ 0 & 0 & 1 & a \end{vmatrix}$ и $B = \begin{vmatrix} a \\ a \\ a \\ 0 \end{vmatrix}$

After discretization [3], the discrete model of the filter F_i is:

$$x_{t+1} = \left(E + \Delta t A + \frac{\Delta t^2}{2}A^2\right)\left(x_t + \Delta t \begin{vmatrix} a \\ a \\ a \\ 0 \end{vmatrix} U_t\right)$$

in which E is the single matrix. The dispersion (D_i) of the calculated quantity y, multiplied by a defined coefficient is equal to the necessary densities r and q in the Kalman filter.

The quantity μ defines the co-variation matrix of the noise in the Kalman filter and it depends on dispersions D_i of the signals from the filters F_i. The extension of the sensitivity of the Kalman filter could be achieved through continuous recalculation (through recurrent dependences) or renovation of D_i for each one of the object movement sections. The filter ensures adaptation to the object movement in unspecified curves, straight section or series of curves and straight sections. It is possible because of the periodical recalculation of the dispersions D_1^i and D_2^i.

3 Conclusion

The Kalman filter could be successfully applied to the Gaussian linear systems. In that case it is executed over a system, containing a non-linear movable object. The result of the paper is an extended Kalman filter that increases the precision of the object managing, through the inaccuracies of its mathematical description and the disturbances in the observation.

References:

[1] Levine, L. Un aperqu elementaire de la theorie moderne des systemes non lineares, R.A.I.R.O. Automatique/Systems Analisi and Control, 1993, №17.

[2] Piasco, J.M., Filtrage auto-adaptif de trajectories de mobiles manoeuvrants, Control and Computers, 2002, №1, 10-14.

[3] Diduk, G.A., Machine methods for researching of automatic systems, Emergoatomizdat, 1983.

[4] Steven M. LaValle, Planning Algorithms, Cambridge University Press, 2005.

[5] E. J. Rossetter J. Ryu and J. C. Gerdes. Vehicle sideslip and roll parameter estimation using gps. In AVEC 2002 6th International Symposium of Advanced Vehicle Control, 2002.

[6] C. Chen J. K. Hendrick R. Rajamani, A. S. Howell and M. Tomizuka. A complete fault diagnostic system for automated vehicles operating in a platoon. IEEE Transactions on Control Systems Technology,9(4):553–564, July 2001.

[7] Arvin R. Savkoor and C.T. Chou. Application of aerodynamic actuators to improve vehicle handling. Vehicle System Dynamics,32:345–374, 1999.

[8] Adam T. Simmons. Control allocation techniques using existing and novel quadratic programming algorithms. Master's thesis, Auburn University, August 2003.

[9] Thor I. Fossen Tor A. Johansen and Svein P. Berge. Constrained nonlinear control allocation with singularity avoidance using sequential quadratic programming. IEEE Trans. Control Systems Technology, 12, 2004.

[10] -Anton T. van Zanten. Evolution of electronic control systems for improving the vehicle dynamic behavior. In Proceedings of the 6th International Symposium on Advanced Vehicle Control, 2002.

[11] Anthony B. Will and Stanislaw H. Zak. Modelling and control of an automated vehicle. Vehicle System Dynamics, 27:131–155, 1997.