Candidate Spectral Estimation for Cognitive Radio

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Abstract: Introducing a different view point of traditional filter bank approach for spectral estimation it is derived a spectral estimation method able to detect a given spectral shape forming part or contributing to a given data record. The method provides an estimated power level of the spectral shape contribution to the data record, and peaks close to the frequency where the spectral shape is located. Basically, the filter-bank, instead of being tuned to a single carrier, is tuned to the spectral shape. The spectral shape used to tune the filter-bank is refereed as the candidate spectrum. The major motivation for this procedure was the proper spectrum labelling of licensed users and interferers in cognitive radio applications.

Key-Words: Spectral estimation, cognitive radio, filter bank, spectrum labeling.

1 Introduction

Filter bank signal analysis has been the major tool for spectral estimation and the dominant technology for spectrum analyzers. In general, filter bank represents in the past the way to implement Fourier analyzers from acoustics to optics. The periodogram, or any DFT based procedure, move filter banks to domains of applications where A/D converters cannot afford, and being DFT-based technology dominant in those cases where the number of filters was high and complexity dictates the application. Nevertheless, most communications systems as well as spectrum analyzers remain in the filter-bank philosophy. For instrumentation issues the so-called poly-phase networks provide the manner to upgrade a Fourier analysis to a filter-bank and they are currently used for trans-multiplexers and communication systems matching strong mask constrains. Furthermore, it seems that filter-bank analysis, as an upgrade of OFDM, is well positioned to replace completely DFT technology in modern communications systems.

The upgrade to the DFT approach in spectral estimation (i.e. the Periodogram) was reported by Capon where, instead of designing a low pass prototype (data window) that was modulated to the steering frequency of the analyzer, propose to design a dedicated filter which depends on the steering frequency and the data record under analysis. Since this is just an upgrade of the Fourier analysis, the Capon's method can be easily extended to multidimensional and cross-spectral estimation yet preserving the superior performance with respect any version based on Fourier analysis. Filter bank analysis always estimate spectral density trough estimating power in a given frequency band, in consequence, constant bandwidth analysis scales the power level independently of the frequency; on the other hand, dedicated filter bank techniques may match the analysis bandwidth to better estimate spectral densities.

Motivated by the interest of, let us say, dedicated spectral estimation for cognitive radio applications, the so-called traditional spectral estimation represented by the dedicated filter bank approach of Capon is revisited in order to find fundamentals that allow to move the scanning by pure frequencies to scanning or tuning the filter to a specific spectral shape or correlation signature. This spectral shape is refereed herein as the candidate spectrum, which is assumed to be known in shape and bandwidth but remaining the power level and the frequency location unknown. In finding the autocorrelation signature or candidate in the autocorrelation matrix of data it is shown that the notion of geodesic distance among them becomes relevant, concluding that the detection and power level estimation depends directly on the geodesic distance of the candidate and the and data correlations matrix.

The motivation of this work was the interest that for cognitive radio air-interfaces has the ability to detect the presence or absence of a licensed user regardless other interferers, non-licensed users and noise. The reported method shows good performance for moderate data lengths and low filter order in SNR ranges above those demanded by the application.

Next section revisit the filter bank method for spectral estimation and explores the possibility of changing the traditional single frequency scanning by a shape scanning obtaining also power level and density in the data signal. The spectral shape and its correlation signatures are named as candidate spectrum. Section 3 is dedicated to connect frequency estimators and parametric methods for spectral estimation with the so-called candidate spectral estimation. Section 4 attempts to formalise, in terms of distance between correlation matrixes, the formulation of the candidate spectral estimation. Next, Section 5 describes briefly the cognitive radio motivation and reports the procedure: Finally, some simulation and performance evaluation is included on the simulations section.

2 Revisiting the filter bank approach

The filter bank approach for spectral estimation is based on a dedicated filter design, which steered to a given frequency, aims to estimate the spectral density of the input signal at this frequency. The estimate is given by the output power of the filter divided by the bandwidth of the analysis filter. Independently of the frequency, including optical communications, spectrum analyzers use this principle currently and, being dependent on technology available, cost and complexity the filter bank could be data independent or data dependent. From the second class the most popular approach is due to Capon, which set a variational design of the analysis filter setting an objective that was to minimize the filter output power constrained to a cero dB. response at the steered frequency. The formulation of this design, also know as minimum variance, is shown in (1).

$$\underline{A}^H.\underline{\underline{R}}.\underline{A}\Big|_{MIN} \qquad \underline{A}^H.\underline{S}=1 \tag{1}$$

Where $\underline{A}$ contains the FIR Q filter coefficients, $\underline{\underline{R}}$ is the correlation matrix of the input signal, $\underline{S}=[1 \quad \exp(jw) \quad \ldots \quad \exp(j(Q-1).w]^T$ is the steering frequency vector and w $(2.p.f)$ is the frequency where the estimate of the spectral density of the input signal is going to be produced.

The output power is given by (2), the noise bandwidth of the analysis filter is given by (3), and the spectral estimate is (4).

$$P_x(w)=\frac{1}{\underline{S}^H.\underline{\underline{R}}.\underline{S}} \tag{2}$$

$$B_N(w).\left(\underline{A}^H.\underline{S}.\underline{S}^H.\underline{A}\right)=\underline{A}^H.\underline{A} \tag{3}$$

$$S_x(w)=\frac{P_x(w)}{B_N(w)}=\frac{\underline{S}^H.\underline{\underline{R}}^{-1}.\underline{S}}{\underline{S}^H.\underline{\underline{R}}^{-2}.\underline{S}} \tag{4}$$

And it is easy to prove (5) that evidences how resolution scales among normalized estimate, power estimate and periodogram estimate.

$$\frac{\underline{S}^H.\underline{\underline{R}}^{-1}.\underline{S}}{\underline{S}^H.\underline{\underline{R}}^{-2}.\underline{S}}\le\frac{Q}{\underline{S}^H.\underline{\underline{R}}^{-1}.\underline{S}}\le\frac{\underline{S}^H.\underline{\underline{R}}.\underline{S}}{Q} \tag{5}$$

The filter-bank framework can be used also for n-dimensional spectral estimation as well as for cross-spectral densities.

As concerns with the design equations, it is clear that the response of cero dB. At the steering frequency refers to the magnitude only and not necessarily over the filter phase at the steering frequency. When setting the magnitude constrain only (1) is reformulated as (6).

$$\underline{A}^H.\underline{\underline{R}}.\underline{A}\Big|_{MIN} \qquad \underline{A}^H.\left[\underline{S}.\underline{S}^H\right].\underline{A}=1 \tag{6}$$

The solution to this problem is formulated in (7), where l is the Lagrange multiplier. And the solution for the multiplier and the null-eigenvector is shown in (8).

$$\left(\underline{\underline{R}}-l.\underline{S}.\underline{S}^H\right).\underline{A}=\underline{0} \tag{7}$$

$$\underline{A}=\frac{\underline{\underline{R}}^{-1}.\underline{S}}{\left(\underline{S}^H.\underline{\underline{R}}^{-2}.\underline{S}\right)^{0.5}} \qquad l=\frac{1}{\underline{S}^H.\underline{\underline{R}}^{-1}.\underline{S}} \tag{8}$$

Regardless this solution may appears different, it produces exactly the same estimates than the traditional formulation. In addition it provides additional viewpoints on the filter-bank framework. First note that in (7) there is a spectral substraction from the correlation matrix of the original data and the correlation matrix of, let us say, the candidate spectrum. In other words, the formulation find out how much candidate spectrum is contained in the original data. The candidate spectrum is the one that corresponds to a single line at the steered frequency. Clearly the Lagrange multiplier is basically how much power seems to be contained in the original data from the candidate spectrum. Furthermore, the power can be reformulated as the maximum value of the candidate spectrum that can be subtracted from the original data correlation. The solution is that l is set in order that the minimum eigenvalue of the resulting matrix is cero. In summary, looking for a candidate spectrum, i.e. correlation matrix, the maximum eigenvalue of (9) provides the power estimate of the candidate spectrum in the data spectrum.

$$\underline{\underline{R}}_{DATA}.\underline{e} = l.\underline{\underline{R}}_{CANDIDATE}.\underline{e} \tag{9}$$

Next section will use the same framework to encompass well-known spectral estimation procedures for line spectrum and parametric models.

3 MUSIC and Parametric Spectral Estimation.

Assuming that the problem is to find out if a given spectral candidate $S_c(w)$ (unity area) with autocorrelation matrix given by $\underline{\underline{R}}_c$ in data contaminated by other sources with different spectral shape, applying the framework of the previous section the solution will be to solve for the maximum eigenvalue of (10).

$$\underline{\underline{R}}.\underline{e} = l.\underline{\underline{R}}_c.\underline{e} \tag{10}$$

Once the maximum eigenvalue is found, it is the power estimate of the candidate spectrum. At the same time, denoting the eigenvector with the filter notation, i.e. changing $\underline{e}$ by $\underline{A}$, (10) can written as (11).

$$\left(\underline{\underline{R}} - l.\underline{\underline{R}}_c\right).\underline{A} = \underline{0} \tag{11}$$

Clearly, assuming exact spectral subtraction (11) implies that the eigenvector is orthogonal to the rest of components of the original data.

It is clear that when the candidate spectrum is coloured noise and the rest of the components are spectral lines, the inverse of the response of the filter peaks at the frequency location of the spectral lines. Using the multiplicity of the maximum eigenvector we arrive to Music.

To extent the same framework to parametric models, starting with the case of a pure AR process, being $\underline{A}$ the vector containing the denominator coefficients of the model, $\underline{\underline{R}}$ the autocorrelation matrix of the data and s^2 the power of the input white noise to the AR model, and b(0) the coefficient of the numerator, equation (12) holds.

$$\underline{A}^H.\underline{\underline{R}}.\underline{A} = s^2.b(0)*b^*(0) \tag{12}$$

At the same time,

$$\left[h(0) \quad 0 \quad \ldots \quad 0\right].\underline{A} = b(0) \tag{13}$$

Where h(.) is the impulse response of the model. In summary (12) changes to (14), where vector $\underline{1}$ is equal to $\left[1 \quad 0 \quad \ldots \quad 0\right]^H$.

$$\underline{A}^H.\underline{\underline{R}}.\underline{A} = s^2.\left|h(0)\right|^2.\underline{A}^H.\left(\underline{1}.\underline{1}^H\right).\underline{A} \tag{14}$$

Clearly, matrix $\left(\underline{1}.\underline{1}^H\right)$ plays the role of the candidate correlation matrix and the AR problem reduces to solve (15), and the solution is provided in (16) which, of course, coincides with the correct one.

$$\left(\underline{\underline{R}} - g.\underline{1}.\underline{1}^H\right).\underline{A} = \underline{0} \tag{15}$$

$$g = \frac{1}{\underline{1}^H.\underline{\underline{R}}^{-1}.\underline{1}} \qquad \underline{A} = \frac{\underline{\underline{R}}^{-1}.\underline{1}}{\left(\underline{1}^H.\underline{\underline{R}}^{-2}.\underline{1}\right)^{0.5}} \tag{16}$$

For ARMA models, just changing equation (13) by the relationship between the impulse response and the model coefficients we have:

$$\begin{bmatrix} h(0) & 0 & 0 & .. \\ h(1) & h(0) & 0 & .. \\ .. & .. & .. & .. \\ h(P) & h(P-1) & .. & .. \end{bmatrix}.\underline{A} = \underline{\underline{H}}.\underline{A} = \underline{B}$$

Where vector $\underline{B}$ contains the P+1 numerator coefficients.

The solution is given by (17) which is just a generalization of the AR formula.

$$\left(\underline{\underline{R}} - g.\left(\underline{\underline{H}}.\underline{\underline{H}}^H\right)\right).\underline{A} = \underline{0} \tag{17}$$

This procedure to estimate ARMA spectrums is useful in those cases where the impulse response of the model is available for a few samples, close to the origin since the SNR use to be higher and very low for samples above P. At the same time, it uses the correlation values close to the origin with better results than estimating the denominator coefficients from extended Yules-Walker equations.

4 A distance criterion.

Concerning the problem of finding a candidate autocorrelation in the data autocorrelation it can be formulated in terms of distance between autocorrelation matrixes. Where (18) defines the line that connects the data autocorrelation with the candidate autocorrelation, the distance between these two matrixes is given by (19), sub index F indicates Frobenius norm.

$$\tilde{\underline{\underline{R}}} = \underline{\underline{R}} + t.\left(\underline{\underline{R}}_c - \underline{\underline{R}}\right) \qquad (18)$$

$$d^2\left(\underline{\underline{R}}, \underline{\underline{R}}_c\right) = \left(\underline{\underline{R}} - \underline{\underline{R}}_c\right)_F \qquad (19)$$

When the data matrix is equal to the candidate plus a single line with autocorrelation given by $g.\underline{S}.\underline{S}^H$, then is clear that maximizing g is equivalent to maximise the distance between the original correlation matrix and the resulting one from the subtraction of the line contribution. Nevertheless, it is not adequate to define the distance between matrixes from (18), since the line defined does not guarantee that any matrix in the line is positive definite.

Using geodesics instead of (18), the geodesic from the two matrixes is (20) and the geodesic distance is (21).

$$\tilde{\underline{\underline{R}}} = \underline{\underline{R}}^{1/2}.\exp\left[t.Ln\left(\underline{\underline{R}}^{-1/2}.\underline{\underline{R}}_c.\underline{\underline{R}}^{-1/2}\right)\right]\underline{\underline{R}}^{1/2} \qquad (20)$$

$$d^2_{geo}\left(\underline{\underline{R}}, \underline{\underline{R}}_c\right) = \sum \left|Ln\left(l(q)\right)\right|^2 \quad ; \quad \underline{\underline{R}}.\underline{e} = l.\underline{\underline{R}}_c.\underline{e} \qquad (21)$$

Again, when the original data contains the candidate plus a rank-one contribution, only one of the eigenvalues of the generalized problem is different from one, reducing the geodesic distance to the log square of the maximum eigenvalue.

In general, finding for candidate correlation $g.\underline{\underline{R}}_c$ in the original data, the resulting square of the geodesic distance between the two matrixes will be (22)

$$d^2_{geo}\left(\underline{\underline{R}}, g.\underline{\underline{R}}_c\right) = \sum \left|Ln\left(\frac{l(q)}{g}\right)\right|^2 \qquad (22)$$

This distance has a minimum when g is equal to the geometric mean of the eigenvalues and increases monotonically when the parameter is above the geometric mean. At the same time, g is bounded by the maximum eigenvalue. In consequence the maximum distance is obtained when g coincides with the maximum eigenvalue.

In summary, in order to perform spectral subtraction of the candidate contribution from the original data correlation the maximum eigenvalue of the generalized problem maximizes the geodesic distance between the original matrix and the contribution of the candidate matrix.

5 Cognitive Radio application

The advance of radio technology put in evidence that regulation rules become obsolete in some respects. Regulators did not envisage today's technology or, at least, not to cope with what can be done with advanced and intelligent radio interfaces. One the protection that regulation provides to a licensed user is the spectral mask. The design of the mask, most of the cases, represents a great wall to preclude any non-licensed user to access to the spectrum. Ultrawideband communications are able to provide reliable communications, mainly in the short range without violating the regulation mask. Nevertheless, the major drawback of current regulation is that the radio spectrum is over-licensed but not over-used. Licensed worldwide a given frequency band may be free of use depending on the location since within this location there is not interest for a communications operator. This is the case in low populated areas or hazardous scenarios among others. From the regulation point of view seems to be clear that the spectrum have to be licensed taking into account the location. It is non-sense to force to pay for spectrum in locations where is not going to be used by the bidder. Radio spectrum licensed in frequency and square meters (geographically) will allow frequency reuse in non-licensed sites where other communication demands exists.

Nevertheless, the current system for licensing radio spectrum implies that many time-frequency slots are free even in highly populated areas. This fact represents an opportunity for modern technology, which may use these free time-frequency slots, being able to shutdown the communication link when a licensed user enters to the pretended slot. A radio interface with such capabilities requires fast and reliable methods of spectrum labelling within the frequency band where it aims to transmit. Encompassed within the so-called cognitive radio, spectrum labelling is one of the major subsystems composing a cognitive radio transceiver.

The objective of spectrum labelling is to detect any spectral occupancy and, more important than this, to label spectral occupancies depending if they belong to a licensed user or to an opportunistic one. In this section, the concepts revisited previously will be used to propose a solution to this problem.

Let us assume that the licensed user when access to the radio spectrum is sensed without knowing the power level neither the carrier frequency. The first assumption is general since this level depends on the location of the sensing equipment. The second corresponds to the case where the cognitive radio search among different licensed bands and it is agile to move among them. The licensed user is assumed to use a known power spectral density, which mostly depends on the baud rate and the symbol shape. This candidate spectrum implies a spectral occupancy defined by F(w) for a carrier frequency cero.

$$F(w) = \begin{cases} k(w) & (-2pB) \leq w \leq (2pB) \\ 0 & elsewhere \end{cases} \qquad (23)$$

$$\frac{1}{2p}.\int F(w).dw = 1$$

From this candidate spectrum, the corresponding autocorrelation function is obtained by inverse DFT, and the Toeplitz matrix $\underline{\underline{R}}_c$ is derived in accordance with the order of the spectral estimation procedure. The greater the order is the best will be the performance of the procedure and, at the same time, it will increase the number of samples required to detect the presence or absence of the licensed user.

In order to explore the carrier frequency and the power level, the candidate correlation is scaled by a factor g and modulated using the component-product (the product of the two matrixes is the direct product of each corresponding entries) of the cero frequency correlation candidate with a rank-one matrix formed with the steering vector at the sensed carrier frequency. The candidate correlation is shown in (24), where $\otimes$ denotes the component-by-component product.

$$g.\left[\left(\underline{S}.\underline{S}^H\right)\otimes \underline{\underline{R}}_c\right] \qquad (24)$$

Note that the candidate spectrum reduces to traditional spectral estimation when only the cero frequency on (23) has a value different from cero (equal to one due to the power normalization.

The power level estimate is found as the maximum l of the following problem:

$$\left(\underline{\underline{R}} - l.\left[\left(\underline{S}.\underline{S}^H\right)\otimes \underline{\underline{R}}_c\right]\right)\underline{A} = \underline{0} \qquad (25)$$

As it is expected this power level estimate shows the same reliability and low resolution that it experiences for detecting a single line spectrum.

In (25) the filter $\underline{A}$ proves that we can measure at its output a power level equal to $\underline{A}^H.\underline{\underline{R}}.\underline{A}$ which, thanks to the frequency response of the filter is proportional to the output power when the candidate spectrum is the only contribution to the input spectrum.

$$\underline{A}^H.\underline{\underline{R}}.\underline{A} = l.\underline{A}^H\left[\left(\underline{S}.\underline{S}^H\right)\otimes \underline{\underline{R}}_c\right]\underline{A} = l.\underline{A}^H.\underline{\underline{y}}\left(\underline{S}\right).\underline{A} \qquad (26)$$

Thus, the power level estimate can be viewed as the power output of the filter, normalized by the response to the candidate spectrum.

$$\hat{g} = \frac{\underline{A}^H.\underline{\underline{R}}.\underline{A}}{\underline{A}^H.\underline{\underline{y}}\left(\underline{S}\right).\underline{A}} = l \qquad (27)$$

In order to compute the noise bandwidth of the filter, we need to assume that the background noise of the input spectrum is white with power density No. In this case the power level estimate is formed by the contribution of the candidate plus the white noise density multiplied by the equivalent white noise bandwidth.

$$\hat{g} = \frac{\underline{A}^H.\underline{\underline{R}}.\underline{A}}{\underline{A}^H.\underline{\underline{y}}\left(\underline{S}\right).\underline{A}} = g + N_o.\frac{\underline{A}^H.\underline{A}}{\underline{A}^H.\underline{\underline{y}}\left(\underline{S}\right).\underline{A}} = g + N_o.B_N\left(\underline{S}\right) \qquad (28)$$

Since the filter is an eigenvector which norm is one. The noise bandwidth is $\frac{1}{\underline{A}^H.\underline{\underline{y}}\left(\underline{S}\right).\underline{A}}$.

In summary, the spectral estimation procedure is summarized below:

Definecandidateautocorrelationmatrix
atunitpowerleveland basebandfrequency $\underline{\underline{R}}_c$
Findthemaximumeigenvalueandtheeigenvector
asociatedof $l.\underline{\underline{R}}.\underline{e} = \left[\underline{S}.\underline{S}^H \otimes \underline{\underline{R}}_c\right].\underline{e}$
Power level estimate $\hat{g}\left(\underline{S}\right) = l^{-1}$
Densityestimate $\hat{s}_c\left(\underline{S}\right) = l^{-1}.\left(\underline{e}^H\left[\underline{S}.\underline{S}^H \otimes \underline{\underline{R}}_c\right].\underline{e}\right)$ (29)

Needless to say that the response, mainly of the density estimate, depends on how close in geodesic distance terms is the candidate spectrum with the actual component within the data matrix. At the same time, those components, which are far away from the candidate, will not be shown by the density estimates. Also, it is worthwhile to remark that the procedure reduces to classical spectral estimation when the candidate spectrum is a single line (i.e. correlation matrix of the candidate is all ones).

6 Simulations

In order to illustrate the performance of the procedure, a data record of 800 samples corresponding to the case of two unmodulated carriers in white noise is analyzed. The SNR of the carriers are 2 and 10 dB respectively, the frequencies of the carriers are 0.2667 and 0.4, and the filter is 8. The candidate spectrum is $g.ones(8,8)$. Note that this case corresponds with classic spectral estimation. The periodogram is also included in the plot for reference. Figure 1 contains the periodogram, power level and power density obtained with the procedure. As mentioned previously the periodogram is above the power level and the power level is above the density estimate.

Next, the unmodulated carrier at 0.2667 is replaced by a wideband spectrum uniformly occupying the frequency range of 0.1142 up to 0.4192 yet preserving the same signal to noise ratio of 2dB. Figure 2 illustrates the performance of traditional periodogram, power level and density estimates. As it can be viewed only the unmodulated carrier is detected with an increased floor on its left corresponding to the wideband spectrum.

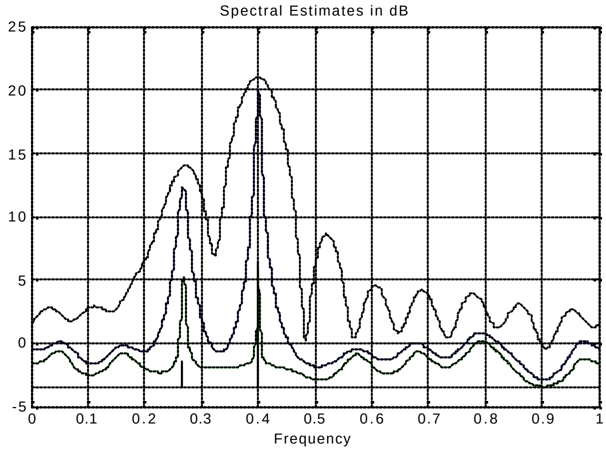

Fig. 1. Top-Down, Periodogram, Power level estimate, Spectral density estimate for a candidate spectrum equal to an unmodulated carrier.

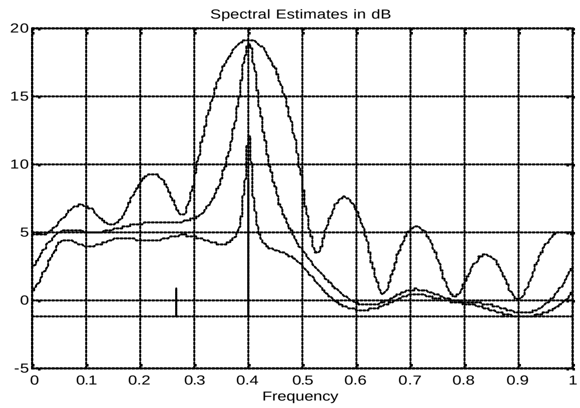

Fig. 2. Top-Down, Periodogram, power and density estimates (traditional spectral estimates) and with a wideband (Bandwidth 0.25) centred at 0.2667.

Changing the candidate spectrum to the wideband spectrum with the same bandwidth the situation changes and as can be viewed in Figure (3) the unmodulated carrier disappears meanwhile a clear peak appears in the central frequency of the candidate spectrum. Note that the power level indicates 1.9 dB. (Actual 2dB.) and the density peaks close to the actual location. In any case, the performance is remarkable.

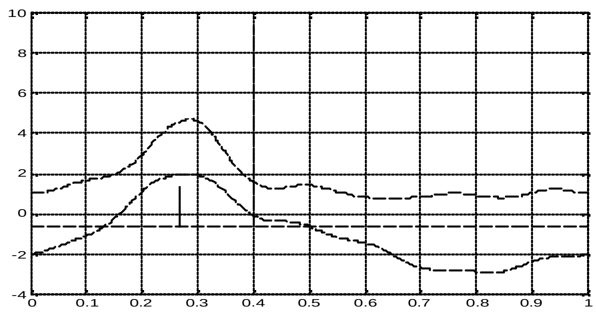

Fig. 3. Density estimate (solid) and power level estimate (dashed) when using a wideband candidate of the same bandwidth.

Figure 4 shows the performance of the procedure, compared with traditional estimates, when the power of the wideband signal increases up to 12 dB.

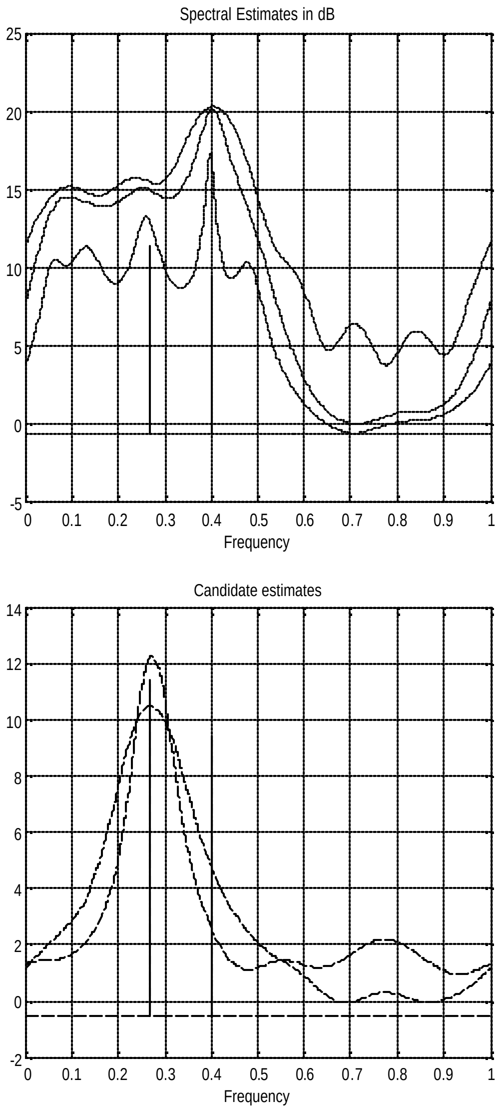

Fig. 4. Estimates corresponding with figures 2 (upper) and 3 (lower) when the power of the wideband signal is 12 dB.

Finally, for a candidate spectrum equal to a BPSK candidate of five samples per symbol using rectangular NRZ signalling, the candidate correlation is just a triangle of four samples per side and maximum one at the cero lag. Note that this spectral candidate is valid also for QPSK, MSK and QAM whenever the baud rate and the pulse shaping are the same. Figure 5 depicts the results for the case that the BPSK signal has a power of 12 dB. above white noise power level. The rest of the data for these graphics are the same as before.

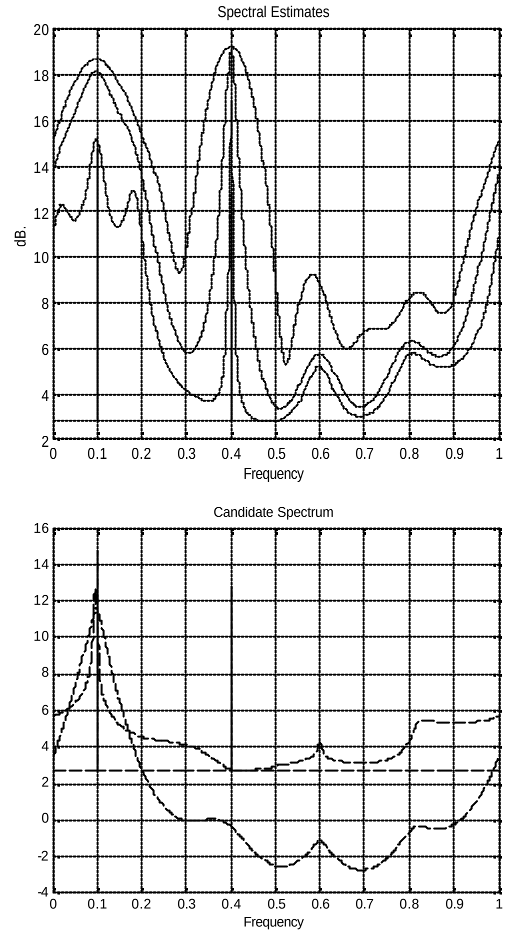

Fig. 5. Traditional (upper) Candidate (lower) for a BPSK signal centred at frequency 0.1 together with an unmodulated carrier at 0.4. Power levels above noise power of 12 and 10 dB. respectively.

7 Conclusions

A generalization of the filter bank analysis for spectral estimation has been generalized, providing power level and density estimates of spectral components. The shape and bandwidth of the spectral component, referred as the candidate spectrum, is introduced in the filter bank framework just in the same manner that single spectral lines are used to scan the input data in order to estimate power and density. The quality of the estimates reported depends directly on the geodesic distance of the correlation matrix of the spectral component present in data with the corresponding matrix of the candidate.

The procedure was motivated to solve the problem faced in cognitive radio systems where different spectral labelling has to be provided to licensed users neglecting, at the first stage other non-licensed users or interferers.

Finally, the framework reported provides a different view of major spectral estimation methods including frequency detectors and parametric spectral estimation procedures.

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