Cooperating Systems Based on Maximal Graphs in Semantic Schemas

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Abstract: In this paper we introduce the concept of cooperating system of semantic schemas. This structure is a tuple $(\{\mathcal{S}_i\}_{i=1}^n, E)$ $(n \geq 2)$ such that $\mathcal{S}_1,\ldots,\mathcal{S}_n$ are distinct regular semantic schemas and E is a distinguished component. The formal computation in $\mathcal{S}_i$ is a usual computation in a semantic schema and this kind of computation was defined in [3]. From the structural point of view E satisfies the rules of a semantic schema. An appropriate computation for E is defined and this computation describes the cooperation between $\mathcal{S}_1,\ldots,\mathcal{S}_n$. We exemplify these computations and several possible applications of this concept are discussed. Finally some open problems are shortly described.

Key–Words: Peano algebra, strict partial order, semantic schema, interpretation, derivation in a semantic schema, cooperating system

1 Introduction

The concept of semantic schema was introduced in ([3]) as a structure extending the concept of semantic network ([4]). A semantic schema is an abstract structure. In order to represent knowledge an appropriate interpretation is used. Various applications of these concepts were described: distributed representation in logic programming with constraints ([3]), knowledge management ([5]) and reasoning by analogy ([6]).

In this paper we define a partial order between the elements of the last component of a semantic schema, we consider the maximal elements with respect to this relation and based on these concepts we introduce the concept of **cooperating system**. Such a system contains several semantic schemas and one of them is a distinguished entity because this schema controls the cooperating between the other components of the system. Finally an application is presented and some open problems are relieved.

2 Semantic schemas

We consider a finite and nonempty set A_0 and we denote by θ an operator symbol of arity 2. We denote by $\overline{A}_0$ *the Peano θ-algebra generated by* A_0, therefore $\overline{A}_0 = \bigcup_{n\geq 0} A_n$ where A_n is defined recursively by $A_{k+1} = A_k \cup \{\, \theta(u,v) \mid u,v \in A_k\}$, $k \geq 0$ ([2], [9]). If we take

$$B_0 = A_0,\ B_{n+1} = A_{n+1} \setminus A_n \qquad (1)$$

then $\overline{A}_0 = \bigcup_{n\geq 0} B_n$ and $B_i \cap B_j = \emptyset$ for $i \neq j$. For $u \in \overline{A}_0$ we write $length(u) = n$ if $u \in B_n$.

In what follows we recall the main results concerning the concept of θ-semantic schema introduced in [3] and developed in [5], [6], [7] and [8]. We mention in this section only those results that are used in this paper.

A θ-**semantic schema** (shortly, θ-schema) is a system $\mathcal{S} = (X, A_0, A, R)$, where

• X is a finite non-empty set of symbols and its elements are named *object symbols*

• A_0 is a finite non-empty set of elements named *label symbols* and $A_0 \subseteq A \subseteq \overline{A}_0$, where $\overline{A}_0$ is the Peano θ-algebra generated by A_0

• $R \subseteq X \times A \times X$ is a non-empty set which fulfills the following conditions:

$$(x, \theta(u,v), y) \in R \Longrightarrow \exists z \in X : (x,u,z) \in R, (z,v,y) \in R \qquad (2)$$

$$\left.\begin{array}{l}\theta(u,v) \in A,\\ (x,u,z) \in R,\\ (z,v,y) \in R\end{array}\right\} \Rightarrow (x,\theta(u,v),y) \in R \qquad (3)$$

$$u \in A \Longleftrightarrow \exists (x,u,y) \in R \qquad (4)$$

We denote $R_0 = R \cap (X \times A_0 \times X)$.

Proposition 1 *If $\theta(u, v) \in A$ then $u \in A$ and $v \in A$.*

Proof. If $\theta(u, v) \in A$ then by (4) and (2) we deduce that there are $(x, u, y) \in R$ and $(y, v, z) \in R$. Using again (4) we obtain $u \in A$ and $v \in A$.

Let $\mathcal{S} = (X, A_0, A, R)$ be a θ-schema. If h is a symbol of arity 1 then we consider the set:

$$M = \Big\{h(x, a, y) \quad | \quad (x, a, y) \in R_0\Big\}$$

where we use the notation $h(x, a, y)$ instead of $h((x, a, y))$.

We consider a symbol σ of arity 2 and let $\mathcal{H}$ be *the Peano σ-algebra generated by M*.

We denote by Z the alphabet including the symbol σ, the elements of X, the elements of A, the left and right parentheses, the symbol h and comma. We denote by Z^* the set of all words over Z. We define the following binary relation on Z^*:

Let be $w_1, w_2 \in Z^*$.

• If $a \in A_0$ and $(x, a, y) \in R$ then $w_1(x, a, y)w_2 \Rightarrow w_1h(x, a, y)w_2$

• Let be $(x, \theta(u, v), y) \in R$. If $(x, u, z) \in R$ and $(z, v, y) \in R$ then $w_1(x, \theta(u, v), y)w_2 \Rightarrow w_1\sigma((x, u, z), (z, v, y))w_2$

We denote by $\Rightarrow^*$ the reflexive and transitive closure of the relation $\Rightarrow$.

The **mapping generated** by $\mathcal{S}$ is the mapping $\mathcal{G}_\mathcal{S} : R \longrightarrow 2^\mathcal{H}$ defined as follows:

• $\mathcal{G}_\mathcal{S}(x, a, y) = \{h(x, a, y)\}$ for $a \in A_0$

• $\mathcal{G}_\mathcal{S}(x, \theta(u, v), y) = \{w \in \mathcal{H} \mid (x, \theta(u, v), y) \Rightarrow^* w\}$

We denote $\mathcal{F}(\mathcal{S}) = \bigcup_{(x,u,y)\in R} \mathcal{G}_\mathcal{S}(x, u, y)$.

An **interpretation** ([8]) of $\mathcal{S}$ is a system $\mathcal{I} = (Ob, ob, \{Alg_u\}_{u\in A})$, where

• Ob is a finite set of elements named the **objects** of $\mathcal{I}$

• $ob : X \rightarrow Ob$ is a bijective function

• $\{Alg_u\}_{u\in A}$ is a set of algorithms such that each algorithm has two input parameters and an output parameter.

Consider an interpretation $\mathcal{I} = (Ob, ob, \{Alg_u\}_{u\in A})$ of $\mathcal{S}$. The **output space** Y of $\mathcal{I}$ is the set $Y = \bigcup_{u\in A} Y_u$, where

$Y_a = \{Alg_a(ob(x), ob(y))|(x, a, y) \in R_0\}$

if $a \in A_0$ and otherwise

$Y_{\theta(u,v)} = \{Alg_{\theta(u,v)}(o_1, o_2)|o_1 \in Y_u, o_2 \in Y_v\}$

We define recursively the **valuation mapping**

$$Val_\mathcal{I} : \mathcal{F}(\mathcal{S}) \longrightarrow Y$$

as follows:

• $Val_\mathcal{I}(h(x, a, y)) = Alg_a(ob(x), ob(y))$

• $Val_\mathcal{I}(\sigma(\alpha, \beta)) = Alg_{\theta(u,v)}(Val_\mathcal{I}(\alpha), Val_\mathcal{I}(\beta))$ if $\sigma(\alpha, \beta)$ is derived from an element of the form $(x, \theta(u, v), y) \in R$ (in fact this element is uniquely determined, [7]).

3 Maximal graph of a semantic schema

A *labeled graph* is a tuple $G = (S, L_0, T_0, f_0)$, where

• S is a finite set, an element of S is a *node* of G;

• L_0 is a set of elements named *labels*;

• T_0 is a set of binary relations on S;

• $f_0 : L_0 \longrightarrow T_0$ is a surjective mapping.

Such a structure admits a graphical representation. Each element of S is represented by a rectangle specifying the corresponding node. We draw an arc from $n_1 \in S$ to $n_2 \in S$ and this arc is labeled by $e \in L_0$ if $(n_1, n_2) \in f_0(e)$. If we proceed in this manner for each element of $\bigcup_{e\in L_0} f_0(e)$ then we obtain a graphical representation of the whole structure.

In this paper we use the *union* of two labeled graphs. In order to define this operation we consider the labeled graphs $G_1 = (S, L_0, T_0, f_0)$ and $G_2 = (Q, M_0, K_0, g_0)$, where $T_0 \subseteq 2^{S\times S}$ and $K_0 \subseteq 2^{Q\times Q}$. The union of G_1 and G_2 is the labeled graph $G_1 \cup G_2 = (S \cup Q, L_0 \cup M_0, W_0, h_0)$, where

$$h_0(\alpha) = \begin{cases} f_0(\alpha) \text{ if } \alpha \in \mathrm{L}_0 \setminus \mathrm{M}_0 \\ g_0(\alpha) \text{ if } \alpha \in \mathrm{M}_0 \setminus \mathrm{L}_0 \\ f_0(\alpha) \cup g_0(\alpha) \text{ if } \alpha \in \mathrm{L}_0 \cap \mathrm{M}_0 \end{cases}$$

Obviously we have $W_0 = h_0(L_0 \cup M_0)$.

For a θ-semantic schema $\mathcal{S} = (X, A_0, A, R)$ we can build the labeled graph $G_\mathcal{S} = (X, A, T, f)$, named the **labeled graph associated** to $\mathcal{S}$, where

• $f(\alpha) = \{(x, y) \in X \times X \mid (x, \alpha, y) \in R\}$

• $T = \{f(\alpha) \mid \alpha \in A\}$

We introduce now a partial relation on the component R of $\mathcal{S}$.

Definition 2 *For two elements $(y_1, u_1, y_2) \in R$ and $(x_1, v_1, x_2) \in R$ we write $(y_1, u_1, y_2) \prec (x_1, v_1, x_2)$ if one of the following conditions is verified:*

• $v_1 = \theta(u_1, u_2)$, $y_1 = x_1$, $(y_2, u_2, x_2) \in R$

• $v_1 = \theta(u_2, u_1)$, $y_2 = x_2$, $(x_1, u_2, y_1) \in R$

The transitive closure of $\prec$ is denoted by $\prec^+$. This means that $\alpha \prec^+ \beta$ if there are $\alpha_1, \ldots, \alpha_n \in R$ such that $\alpha = \alpha_1$, $\alpha_n = \beta$ and $\alpha_i \prec \alpha_{i+1}$ for every $i \in \{1, \ldots, n - 1\}$.

Remark 3 *Suppose $\alpha = (y_1, u, y_2)$ and $\beta = (x_1, v, x_2)$. If $\alpha \prec \beta$ then $length(u) < length(v)$. Consequently, if $\alpha \prec^+ \beta$ then $length(u) < length(v)$.*

Proposition 4 *The relation $\prec^+$ is a strict partial order. In other words, for every $\alpha, \beta, \gamma \in R$ the following properties are satisfied:*

$\alpha \not\prec^+ \alpha$
$\alpha \prec^+ \beta \Rightarrow \beta \not\prec^+ \alpha$
$\alpha \prec^+ \beta, \beta \prec^+ \gamma \Rightarrow \alpha \prec^+ \gamma$

Proof. The first two conditions are verified by Remark 3. The last condition is verified by the transitivity of the relation $\prec^+$. □

Definition 5 *An element $\alpha \in R$ is a* **maximal element** *if $\alpha \not\prec^+ \beta$ for all $\beta \in R$. We denote by R^{max} the set of all maximal elements of R.*

Consider an arbitrary set $M \subseteq X_1 \times \ldots \times X_n$ and $i \in \{1, \ldots, n\}$. By $pr_i M$ we denote the following set:

$$\{y \in X_i \mid \exists(x_1, \ldots, x_{i-1}, y, x_{i+1}, \ldots, x_n) \in M\}$$

Definition 6 *If $\mathcal{S} = (X, A_0, A, R)$ is a θ-semantic schema then the labeled graph $G_{\mathcal{S}}^{max} = (Y, L, T, h)$ is the* **maximal graph** *associated to $\mathcal{S}$ if the following conditions are verified:*
- $Y = pr_1 R^{max} \cup pr_3 R^{max}$
- $L = pr_2 R^{max}$
- $h(\alpha) = \{(x, y) \mid (x, \alpha, y) \in R^{max}\}$ *for* $\alpha \in L$
- $T = \{h(\alpha) \mid \alpha \in L\}$

4 Cooperating systems based on maximal graphs

Based on several semantic schemas we define in this section a *cooperating system*, we discuss the intuitive aspects of this representation and we relieve several remarks concerning the components of this structure.

Definition 7 *A* **cooperating system of semantic schemas** *is a pair $(\{\mathcal{S}_i\}_{i=1}^n, E)$, where*
- *$\mathcal{S}_i = (X_i, A_{0i}, A_i, R_i)$ is a θ_i-semantic schema for $i \in \{1, \ldots, n\}$;*
- *$E = (X, L_0, L, R)$ is a θ-semantic schema such that*

$i)$ X and L_0 are the nodes and respectively the labels of the graph $\bigcup_{i=1}^n G_{\mathcal{S}_i}^{max}$

$ii)$ R satisfies the condition

$$(x, \theta(u, v), y) \in R, (x, u, z) \in R_i^{max}, (z, v, y) \in R_j^{max} \Rightarrow i \neq j \quad (5)$$

At this point we emphasize an aspect concerning the formal computations performed in a semantic schema. Let us denote by $\mathcal{S} = (X, A_0, A, R)$ an arbitrary θ-semantic schema and $R_0 = R \cap (X \times A_0 \times X)$. If $R_0 = R$ then $A = A_0$ and in this case no deduction is modeled by $\mathcal{S}$. Such a schema can be used only to store the facts of a knowledge piece and to retrieve this information. In view of this remark one might say that a semantic schema $\mathcal{S} = (X, A_0, A, R)$ satisfying the property $A = A_0$ (or equivalently, $R = R_0$) is a **trivial** semantic schema.

The concept introduced in Definition 7 can be analyzed from various points of view. As a particular case we can consider a cooperating system containing only trivial semantic schemas. Obviously such a system becomes a θ-semantic schema. In order to specify this case we consider the trivial schemas defined as follows:
- $\mathcal{S}_1 = (\{x, y, z_1\}, \{a, b\}, \{a, b\}, \{(x, a, y), (y, b, z_1)\})$
- $\mathcal{S}_2 = (\{x, y, z_2\}, \{a, b\}, \{a, b\}, \{(x, a, y), (y, b, z_2)\})$

Only two cooperating systems can be obtained by means of these schemas:

1. The trivial system given by $E = (\{x, y, z_1, z_2\}, \{a, b\}, \{a, b\}, R_0)$, where $R_0 = \{(x, a, y), (y, b, z_1), (y, b, z_2)\}$.

2. The non trivial cooperating system given by $E = (\{x, y, z_1, z_2\}, \{a, b\}, \{a, b, \theta(a, b)\}, R)$, where $R_0 = \{(x, a, y), (y, b, z_1), (y, b, z_2)\}$ and $R = R_0 \cup \{(x, \theta(a, b), z_1), (x, \theta(a, b), z_2)\}$. The structure E is obviously a θ-schema.

Remark 8 *If $\mathcal{S}_i = (X_i, A_{0i}, A_i, R_i)$ and $E = (X, L_0, L, R)$ then $X \subseteq \bigcup_{i=1}^n X_i$ and $L_0 \subseteq \bigcup_{i=1}^n A_i$. Really, if $G_{\mathcal{S}_i}^{max} = (Y_i, L_i, T_i, h_i)$ then by Definition 6 we have $Y_i = pr_1 R_i^{max} \cup pr_3 R_i^{max} \subseteq X_i$ and $L_i = pr_2 R_i^{max} \subseteq A_i$ for every $i \in \{1, \ldots, n\}$.*

Proposition 9 *If $\mathcal{C} = (\{\mathcal{S}_i\}_{i=1}^n, E)$ is a cooperating system then either $n \geq 2$ or $\mathcal{C}$ is a trivial schema.*

Proof. We can write $L = \bigcup_{k \geq 0}(L \cap B_k)$, where B_k is defined as in (1). If $n = 1$ then (5) can not be applied, therefore $L \cap B_1 = \emptyset$. Using Proposition 1 we can verify by induction on k that $L \cap B_k = \emptyset$. It follows that $L = L \cap B_0 = L_0$ and $\mathcal{C}$ is a trivial schema. □

In connection with Definition 7 we relieve the following aspects:

1. A cooperation system is based on several *distinct* semantic schemas because each schema $\mathcal{S}_i$ is built by means of a symbol θ_i and $\theta_i \neq \theta_j$ for $i \neq j$.

2. By Remark 8 we observe that L is a subset of the Peano θ-algebra generated by a finite set that contains some elements taken from the Peano θ_i-algebras of the schemas $\mathcal{S}_1, \ldots, \mathcal{S}_n$.

Remark 10 *The condition (5) was introduced because a cooperating system $(\{\mathcal{S}_i\}_{i=1}^n, E)$ is not able to extend the deduction of some component $\mathcal{S}_i$. As a matter of fact the task of E is to model the collaboration of its components.*

5 Formal computations in a cooperating system

We consider a cooperating system $\mathcal{C} = (\{\mathcal{S}_i\}_{i=1}^n, E)$, where $E = (X, L_0, L, R)$. In order to describe the computation in $\mathcal{C}$ we consider the symbols $\sigma, \sigma_1, \ldots, \sigma_n$ of arity 2. Two kinds of computations can be described in $\mathcal{C}$:

• A regular formal computation for the θ_i-schema $\mathcal{S}_i$. This computation was described in Section 2 for the general case of a semantic schema, with the remark that for $\mathcal{S}_i$ the symbol σ_i instead of σ is used.

• A proper formal computation for the θ-schema E. The derivation in E is given in the next definition.

Definition 11 *Suppose* $(x, \theta(u, v), y) \in R$. *If* $(x, u, z) \in R$ *and* $(z, v, y) \in R$ *then*

$$w_1(x, \theta(u, v), y)w_2 \vdash w_1\sigma((x, u, z), (z, v, y))$$

for every words w_1, w_2. *We denote by* $\vdash^*$ *the reflexive and transitive closure of* $\vdash$. *We denote by* $\mathcal{H}_E$ *the Peano* σ*-algebra generated by* $R_0 = R \cap (X \times L_0 \times X)$. *We define*

$$\mathcal{F}(E) = \{w \in \mathcal{H}_E \mid \exists (x, u, y) \in R : (x, u, y) \vdash^* w\}$$

Remark 12 *Because* $\mathcal{H}_E$ *is generated by* R_0 *and* $\vdash^*$ *is a reflexive relation we have* $\mathcal{F}(E) \supseteq R_0$. *This inclusion is used further to define the valuation mapping of a cooperating system.*

In order to exemplify this computation and other concepts which follow in this section we consider the semantic schemas $\mathcal{S}_1$ and $\mathcal{S}_2$ represented respectively in Figure 1 and Figure 2. We remark that (x_2, b, x_3) is a maximal element both in $\mathcal{S}_1$ and $\mathcal{S}_2$. In other words we have $R_1^{max} \cap R_2^{max} \neq \emptyset$.

Remark 13 *The general case,* $R_i^{max} \cap R_j^{max} \neq \emptyset$ *for some* $i \neq j$*, implies some feature of the valuation mapping given in Definition 16.*

The graph $G_1^{max} \cup G_2^{max}$ is represented in Figure 3. From this figure we deduce that the following entities are used to specify E:

• $X = \{x_1, x_2, x_3, x_4, y_1\}$

• $L_0 = \{b, \theta_1(a, a), \theta_1(b, a), \theta_2(a, b), \theta_2(b, b), \theta_2(b, a), \theta_2(a, \theta_2(a, b))\}$

• $R_0 = \{(x_1, \theta_1(a, a), x_2), (x_1, \theta_2(a, b), x_2), (x_2, \theta_2(b, a), y_1), (x_2, b, x_3), (x_3, \theta_1(b, a), y_1), (x_3, \theta_2(b, b), x_4), (y_1, \theta_2(a, \theta_2(a, b)), x_4)\}$

In order to finish the definition of E we take

$$R \setminus R_0 = \{(x_1, \theta(\theta_1(a, a), b), x_3), (x_1, \theta(\theta(\theta_1(a, a), b), \theta_1(b, a)), y_1), (x_1, \theta(\theta_2(a, b), b), x_3), (x_1, \theta(\theta(\theta_2(a, b), b), \theta_1(b, a)), y_1), (x_1, \theta(\theta(\theta_2(a, b), b), \theta_2(b, b)), x_4)\}$$

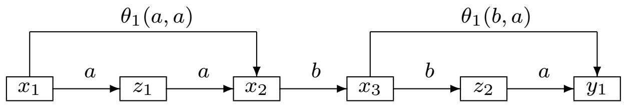

Figure 1: Schema $\mathcal{S}_1$

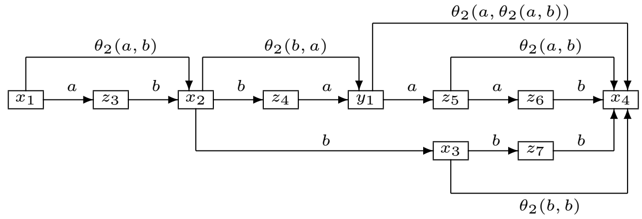

Figure 2: Schema $\mathcal{S}_2$

and therefore

$$L \backslash L_0 = \{\theta(\theta_1(a, a), b), \theta(\theta(\theta_1(a, a), b), \theta_1(b, a)), \theta(\theta_2(a, b), b), \theta(\theta(\theta_2(a, b), b), \theta_1(b, a)), \theta(\theta(\theta_2(a, b), b), \theta_2(b, b))\}$$

We observe the condition (5) is satisfied by R. As an example of derivation in $\mathcal{C}$ we have the following sequence:

$$(x_1, \theta(\theta(\theta_1(a, a), b), \theta_1(b, a)), y_1) \vdash$$

$$\sigma((x_1, \theta(\theta_1(a, a), b), x_3), (x_3, \theta_1(b, a), y_1)) \vdash$$

$$\sigma(\sigma((x_1, \theta_1(a, a), x_2), (x_2, b, x_3)), (x_3, \theta_1(b, a), y_1))$$

By a similar computation we obtain also

$$(x_1, \theta(\theta(\theta_2(a, b), b), \theta_1(b, a)), y_1) \vdash$$

$$\sigma((x_1, \theta(\theta_2(a, b), b), x_3), (x_3, \theta_1(b, a), y_1)) \vdash$$

$$\sigma(\sigma((x_1, \theta_2(a, b), x_2), (x_2, b, x_3)), (x_3, \theta_1(b, a), y_1))$$

In order to define the valuation mapping of a cooperating system $\mathcal{C} = (\{\mathcal{S}_i\}_{i=1}^n, E)$ we denote $\mathcal{S}_i = (X_i, A_{0i}, A_i, R_i)$ and consider an interpretation $\mathcal{J}_i = (Ob_i, ob_i, \{Alg_u^i\}_{u \in A_i})$ of $\mathcal{S}_i$, $i \in \{1, \ldots, n\}$. We suppose that for $x, y \in X_i \cap X_j$ we have $x = y$ if and only if $ob_i(x) = ob_j(y)$.

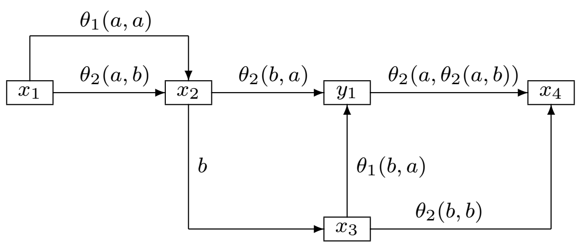

Figure 3: $G_1^{max} \cup G_2^{max}$

Definition 14 *An* **interpretation** *of the cooperating system $\mathcal{C}$ is a system $\mathcal{I} = (Ob, ob, \{Alg_u\}_{u\in A})$ such that $Ob = \bigcup_{i=1}^{n} ob_i(X \cap X_i)$, $ob(x) = ob_i(x)$ if $x \in X \cap X_i$, $ob : X \longrightarrow Ob$ and Alg_u is an algorithm accepting two input arguments and one output argument.*

Proposition 15 *The mapping $ob : X \longrightarrow Ob$ is well defined and is bijective.*

Proof. If $x \in X \cap X_i \cap X_j$ for $i \neq j$ then $ob(x) = ob_i(x)$ and $ob(x) = ob_j(x)$ by the definition of ob. But $ob_i(x) = ob_j(x)$, therefore ob is well defined. If $y \in Ob$ then by Definition 14 there is i such that $y \in ob_i(X \cap X_i)$. Thus there is $x \in X \cap X_i$ such that $y = ob_i(x)$. But $ob(x) = ob_i(x)$, therefore $y = ob(x)$.

In what follows we consider the following decomposition of R: $R = D_0 \cup D_1 \cup D_2$, where $D_0 = R_0$, $D_1 = \{(x, \theta(u, v), y) \in R \mid u, v \in D_0\}$ and $D_1 = R \setminus (D_0 \cup D_1)$. We obtain a corresponding decomposition for $\mathcal{F}(E)$: $\mathcal{F}(E) = R_0 \cup \mathcal{F}_1(E) \cup \mathcal{F}_2(E)$, where $\mathcal{F}_1(E) = \{w \in \mathcal{F}(E) \mid \exists(x, u, y) \in D_1 : (x, u, y) \vdash^* w\}$ and $\mathcal{F}_2(E) = \{w \in \mathcal{F}(E) \mid \exists(x, u, y) \in D_2 : (x, u, y) \vdash^* w\}$.

Definition 16 *The* **valuation mapping** *of the cooperating system $\mathcal{C}$ is the function $Val_{\mathcal{I}} : \mathcal{F}(E) \longrightarrow 2^Y$, where Y is the output space of the semantic schema E, defined as follows:*

- *If $(x, a, y) \in D_0 \cap \left(\bigcup_{j=1}^{n} R_{0j}\right)$ then $Val_{\mathcal{I}}(x, a, y) = \bigcup_{i=1}^{n} \{Alg_a^i(ob_i(x), ob_i(y))\}$*
- *$Val_{\mathcal{I}}(x, \theta_i(u, v), y) = \{Val_{\mathcal{I}_i}(\sigma_i(w_1, w_2)) \mid \sigma_i(w_1, w_2) \in \mathcal{F}(\mathcal{S}_i), (x, \theta_i(u, v), y) \Rightarrow_i^* \sigma_i(w_1, w_2)\}$*
- *Let be $\sigma(\alpha, \beta) \in \mathcal{F}_1(E)$. There is $(x, \theta(u, v), y) \in D_1$ such that $(x, \theta(u, v), y) \vdash^* \sigma(\alpha, \beta)$. We take*

$$Val_{\mathcal{I}}(\sigma(\alpha, \beta)) = \bigcup_{\substack{o_1 \in Val_{\mathcal{I}_i}(\alpha), \\ o_2 \in Val_{\mathcal{I}_j}(\beta), \\ i \neq j}} \{Alg_{\theta(u,v)}(o_1, o_2)\} \quad (6)$$

- *Let be $\sigma(\alpha, \beta) \in \mathcal{F}_2(E)$. There is $(x, \theta(u, v), y) \in D_2$ such that $(x, \theta(u, v), y) \vdash^* \sigma(\alpha, \beta)$. We take*

$$Val_{\mathcal{I}}(\sigma(\alpha, \beta)) = \bigcup_{\substack{o_1 \in Val_{\mathcal{I}}(\alpha), \\ o_2 \in Val_{\mathcal{I}}(\beta),}} \{Alg_{\theta(u,v)}(o_1, o_2)\}$$

Remark 17 *The condition $i \neq j$ in (6) is connected by Remark 13.*

6 An application

It is well known the interest of the great companies to design and implement proper *contact centers*. Among the tasks of this entity we find the applications that include the workforce management, quality monitoring and various applications allowing connectivity and collaboration with voice communications to provide a much richer customer experience ([1]). Various chapters of artificial intelligence can be implied to accomplish these tasks (natural language processing, voice recognition, speech technology, knowledge representation). According to [10] a company can use *customer service representatives* or equivalently *center agents*. They are people that respond to calls, chats or emails from customers and can be replaced by *virtual agents* whose tasks can be modeled by semantic schemas. Obviously in this case the speech technology can be used and even *interfaces by voice* can be successfully applied. The designers of contact centers can use the cooperating systems based on semantic schemas as a method of knowledge representation. In the remainder of this section we give a short description of this application. We treat the manner in which a customer service representative can be modeled as a cooperating system.

The components of a cooperating system performing the tasks of a customer service representative can be thought as follows:

1. The component E receives a phrase in a natural language from the customer. This can be a sentence given by voice and in this case the speech recognition methods are used by E to obtain the associated text T. The phrase can be taken also from an email sent by a customer and in this case the component E disposes directly of the corresponding text T.

2. The text T is parsed by E to extract the semantics. A set of specific entities T_1, …, T_k are obtained. Each T_i requests a partial answer.

3. For each $i \in \{1, \ldots, k\}$ the component E selects some schema $\mathcal{S}_{j_i}$ to prepare an answer Ans_i corresponding to T_i. The entity Ans_i is sent to E by the component $\mathcal{S}_{j_i}$.

4. By an appropriate combination of the entities Ans_1, …, Ans_k an answer Ans is prepared by E and this answer is sent to customer. If the customer used the voice to send the message then the text-to-speech technology is used by E to send its answer back to customer. Otherwise the entity Ans is sent by e-mail.

In order to exemplify the computation we consider the case when E is represented in Figure 3, therefore E can use $\mathcal{S}_1$ and $\mathcal{S}_2$ from Figure 1 and Figure 2 respectively. Suppose that the interpretation $\mathcal{I}$ of E and the interpretation $\mathcal{I}_1$ of $\mathcal{S}_1$ specify that $ob(x_3) = ob_1(x_3) = Peter$, $ob(y_1) = ob_1(y_1) = Helen$ and $ob_1(z_2) = Mary$.
We denote by p(x,y)="x is the son of y", q(x,y)="x is the sister of y" and r(x,y)="x is the nephew of y" are sentential forms. This means that if x and y are substituted by proper names then these entities become sentences in a natural language. Suppose that the interpretation $\mathcal{I}_1$ for $\mathcal{S}_1$ includes the following algorithms:

Algorithm $Alg_b^1(o_1, o_2)$ {return $p(o_1, o_2)$};
Algorithm $Alg_a^1(o_1, o_2)$ {return $q(o_1, o_2)$};
Algorithm $Alg_{\theta_1(b,a)}^1(o_1, o_2)$ {if $o_1 = p(t_1, t_2)$, $o_2 = q(t_2, t_3)$ then return $r(t_1, t_3)$};

Suppose E receives the message *"I want to know if Peter is the nephew of Helen"*. Parsing this sentence the component E obtains the entity $(Peter, Helen)$. From its schema and using the interpretation $\mathcal{I}$ the component E discovers that $\mathcal{S}_1$ is able to find an answer corresponding to the entity $(Peter, Helen)$. We emphasize the fact that E can identify a connection between $Peter$ and $Helen$ but it does not know the information attached to this connection. Using its interpretation, the schema $\mathcal{S}_1$ finds the conclusion $r(Peter, Helen)$ and this sentence is sent to E. Thus E responds by the message "Yes, Peter is the nephew of Helen". Finally we remark that E can respond by a negative sentence without any consultation of the components $\mathcal{S}_i$. For example, if E receives the sentence "I want to know if Peter is the nephew of John" then the response of E is "No" because there is no path in the corresponding schema from x_3 to some node interpreted as $John$.

7 Conclusions and future work

In this paper we introduced a kind of cooperation between semantic schemas. I introduced the concept of *cooperating system*. This is an aggregation of several semantic schemas. The cooperation is guided by some of them and we defined the specific mechanism performing this task. A brief description of a possible application is given in Section 6. Another application is connected by the multi-agent systems. Such systems can be modeled by a cooperation system if some conditions are satisfied. Among these conditions we enumerate the following: the actions of each agent are represented by means of a semantic schema; each agent accomplishes several tasks such that each of them can be described by an entity of its maximal graph. This is a task of a future work.

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