Fuzzy Membership, Partial Aggregation and Reinforcement in Multi-Sensor Data Fusion

S. Banerjee

West Bengal University of Technology
Kolkata 700064, INDIA

Abstract

Information fusion plays an important role in decision support support systems as this procedure can provide a wealth of information by integrating data obtained from multiple sources. However, this data fusion is a challenging problem owing to the uncertainty and reliability of the sources involved, as well as the incompleteness of the data obtained. In this paper we propose a framework to aggregate information from different sensors imaging the same view of the same object. After classification is performed to give hard membership values to well defined pixels, a fuzzy membership is assigned to 'mixed' pixels using a reliability criteria of the sensor to a particular object class. Once this classification process is complete, a fusion procedure is outlined utilizing concepts of compatibility, partial aggregation and reinforcement. Thus, the fused data sets will contain contributions from individual sensors based on the reliability of the individual sensors and the compatibility of the sensors compared to the most reliable sensor for the particular object class, using an appropriate distance measure. When the reliability of the sensors fall within acceptable distance measures, Ordered Weighting Average (OWA) operators and fuzzy measures are used to decided weightage of the data from different sensors to be fused. When the sensors are equally reliable, a reinforcement procedure is adopted, and finally, a partial aggregation is performed.

Keywords: multisensor data fusion, fuzzy membership, OWA operator, partial aggregation, compatibility, reinforcement

1 Introduction

Fusion of information from several independent sources owes its importance due to the fact that a wealth of information is obtained by exploring the salient features of these different sources, thus overcoming the limitations of the individual sources. However, fusion of information obtained from these sources poses a challenging problem due the amount of uncertainty and reliability of the sources as well as the incompleteness of the data involved.

In particular, image data fusion is very useful in displaying composite images of objects imaged by different sensors, thus rendering the salient features of each sensor in the composite image. Applications have been manifold, with one of the important areas being in medical diagnosis and therapy planning, where even surgical procedures can be aided. The need for data fusion arises from the fact that the sensors which image a particular object rely on a particular physical property of the object to obtain the image. This imposes several restrictions on image quality rendered by a single sensor as the constituents of a given object can vary greatly in physical properties (e.g. soft and hard tissues in a brain scan). Thus the limitations obtained in the image obtained from one sensor can be obtained by fusing it with the image obtained from another sensor which is strong in imaging that particular constituent, thereby increasing the global information content of the composite image. Many image properties such as brightness[1], boundaries, regions, etc., are context dependent (CD) and so a data fusion operator would have to use a CD operation representing the degree of reliability of the individual sources of information to be combined as well as the global knowledge or measure on these sources. This measure can be a conflict between the sources, or a reliability between them or a mutual compromise. Thus, two sources of fuzziness[2] are introduced: fuzziness due to classification by a given sensor and fuzziness due to the reliability of the sensor. Consequently, region classification and subsequent fusion from the different modalities is a fuzzy event.

In this paper we propose a fusion scheme that incorporates concepts of compatibility, Ordered Weighting Average (OWA) operators, fuzzy measures and reinforcement. A fuzzy c-means method is used to obtain hard memberships for well defined pixels belonging to a particular object (e.g. tissue in a medical image) class. 'Mixed' pixels, i.e. pixels that lie in a region of overlap exist for objects that have very close grey level values, and such pixels are assigned fuzzy membership values. The present approach consists of introducing triangular fuzzy membership functions for each of the classes of data. By taking a triangular shaped membership function which includes the overlap between neighbouring regions we are considering a fuzzy membership process[3]. This fuzzy membership process is used here as a surrogate for a hard membership process, which, within suitable limits in our domain D approximates a hard membership process. Hence, in regions where pixels lie within one triangle, the membership can be considered to be hard, whereas in regions of overlap, a fuzzy membership value is assigned, based on the reliability of the sensor to the overlapping object regions. Following classification, a fusion process is performed. For each object class, a compatibility function[4] is considered which allows for the inclusion of consideration of whether the classified region is reasonable to fuse or too conflicting to combine, using a distance measure based on the reliability of the sources. For sources which fall within the acceptable distance limit, OWA operators[5] derived from entropy maximization [6], are used as fusion functions to ascertain the weightage of the data to be fused from the classified images of the different object regions. For sensors which are equally reliable for a particular object class, a reinforcement procedure is adopted in evaluating the weightage. Finally, a partial aggregation of classified data from the different sensors is performed, utilizing fuzzy measures wherever appropriate. This is an improvement over an earlier work using a fuzzy c-means (FCM) tissue classification of transverse CT and MR images into five (Air, CSF and ventricles, brain matter, fatty tissues and skull) and four (all the same tissues excluding skull) tissue classes, respectively and a subsequent fusion of these two sets of images [7]. The improvements are threefold. In the earlier work, membership functions were considered as simple probabilities (hard). Secondly, this work treats the global reliability of the sensors more objectively, by attributing numerical measures of belief. Thirdly, the treatment presented here is based on more rigorous mathematical premises. The underlying theory, an example and discussions and future work are included in the following sections.

2 Theory

As is evident from the remarks made in the introduction, intelligent data fusion involves several considerations which are defined in the following subsections.

2.1 Compatibility

Following Yager[8], a relationship R: $X^2 \rightarrow$ I = [0,1] is called a compatibility relation if

(i) ∀ x ∈ X; R(x,x) = 1: a number fully compatible with itself,
(ii) ∀ (x,y) ∈ X^2; R(x,y) = R(y,x) : commutativity,
(iii) if d is a distance on X; if d(x,z) ≥ d(x,y) then R(x,z) ≤ R(x,y)
The negation of the compatibility, (1-R(x,y)) measures the degree of conflict of the data being fused.

The compatibility relationship is extremely problem dependent and a mechanism for including meta-knowledge is incorporated in evaluating the distance measure.

2.2 OWA operators in formulating fusion functions

Ordered Weighting Average (OWA) operators [5] provide a useful class of fusion functions for representing preferences in the fusion process.
Definition An aggregation operator F: R $^n \rightarrow R$ is called an OWA operator of dimension n if it has associated with it a weighting vector $W = [w_1, w_2, \ldots, w_n]^T$ such that
(i) $w_i \in [0, 1]$
(ii) $\sum_1^n w_i = 1$ and
(iii) $F(a_1, a_2, \ldots, a_n) = \sum_1^n b_j w_j$, where b_j is the largest element of the set $A = (a_{1,2}, \ldots, a_n)$ to be fused and $B = (b_1, b_2, \ldots, b_n)$ is the set after fusion.
Here, we choose the weights using a coefficient of optimism α. This coefficient of optimism has a high value if a sensor assigned a higher index is more reliable and vice versa. The weights of the OWA operator is obtained as the solution of the mathematical programming problem:
Maximize the entropy $-\sum_1^n w_j ln(w_j$ for each object class,
subject to
$\sum w_j \times r_(j) = \alpha$
$\sum w_j = 1$ and
$w_j \geq 0$ for all j, where
$r_(j) = \frac{actualreliability}{max.reliability}$
(e.g. in the example described below r_j correponds ro r(c): CT and r(m): MRI.) A similar method is used for selecting weights for constructing a fuzzy neuron has been suggested by Hagan[6].

2.2.1 Entropy Maximization

We consider region classification as a fuzzy event. Since we have used fuzzy c-means for classification, the membership of a pixel in a given class is considered as the maximum membership value for which the entropy is maximum, as discussed below. For a probability space (R^n, F, P) in which F is the σ-field of Borel sets in R^n and P is the probability measure over R^n , a fuzzy event [8] in R^n is a fuzzy set A in R^n whose membership function $\mu_A(\mu_A : R^n -> [0, 1])$ is Borel measurable. According to the maximum Entropy Principle, a fuzzy event contains maximum information when its associated entropy is maximum.

Let us consider an image that has been segmented into n regions ranging from a_0 to a_{n-1} . We can represent the image by a triplet (Ω ,F,P) with Ω = (a_0, a_1, ..., a_{n-1}), P being the probability measure indicating the fraction of pixels belonging to a certain region and F is a σ field of fuzzy subsets of Ω. Region segmentation of a given image can be represented of a given image can be regarded as a fuzzy event in Ω as it can vary according to the modality used depending on its physical limitations.

A membership function is used to denote the degree of an element in the sample space belonging to a fuzzy set.

In the majority of data fusion systems, information is either of of numerical or symbolic nature obtained from images or sensors can be represented as measures of belief in an event. These degrees of belief generally have values in a real closed interval live ([0,1],[1,-1], ...) and can be modeled in different mathematical models. Thus for instance, they can represent probabilities in data fusion methods based on probabilities and Bayesian theory, membership degrees to a fuzzy set in fuzzy set theory or even mass, belief or plausibility functions in Dempster-Shafer's evidence theory. A classification of data fusion operators into three classes have been made [10] based on their dependence on the nature of information and behavior, of which, the third class, namely context dependent (CD) operators is relevant for the purposes of the present discussion. This class is composed of operators which depend on the representation of information obtained from different sensors as well as global knowledge or measure on the sources to be fused like conflict between the sources or reliability of sources. Since the adaptive features of these operators enable them to combine information related to one class in one way and information related to another class in another way, they are particularly useful for classification.

Thus information fusion utilizing fuzzy set theory can be performed in two essential steps. In the first step we identify a membership function in a certain region of the image using the procedure outlined in [7] for both the sensors. These regions, which constitute a fuzzy set, are characterized by certain gray level values. The membership function μ_{region} (a_k) indicates the degree of belonging to a certain region. Thus, for an image classified into certain regions, the region segmented image can be written using fuzzy set notation as [11]

$$\text{region} = \mu(a_0)/a_0 + \mu(a_1)/a_1 + ... + \mu(a_{n-1})/a_{n-1}$$

here the subscript region for μ is omitted and +

means union. Thus, for the classified image memberships are given by

$\sum_{a_k} \mu(a_k) a_k$

(The probability for this fuzzy event A can be calculated using Lebesgue-Stieltjes integral

$P(A) = \int_{R^n} \mu_A(x) dP$

with a summation replacing integration for the discrete case.)

The entropy for the occurence of a fuzzy event A can be defined as [12]

H(A)= -P(A)log(P(A)) - (1-P(A))log(1-P(A))

2.3 Reinforcement and MICA operators

If all sensors 'agree' on a particular object class and are equally reliable then the concept of reinforcement becomes useful and the output membership degrgree can be reinforced. Monotonic Identity Commutative Aggregators (MICA) operators offer a means of reinforcement.

Definition: A bag A of a set X is a collection of elements in which ordering doesn't matter and duplication is allowed.

If two bags A and B have the same cardinality and if the elements of A and of B can be ordered in such a way that $\forall i, a_i \geq b_i$, then $A \geq B$.

$A \oplus B$ is a bag consisting of all elements in A and B.

In the following B^X indicates the set of all bags associated with the set X and I indicates the unit interval.

Definition: A bag mapping M: $B^I \rightarrow I$ is called a MICA operator if it has the following properties: (i) Monoticity: If $A \geq B$ then $M(A) \geq M(B)$ (ii)Identity Element: For every $A \in B^I$ there exists an element g, called the identity of A under M, such that $M(A) = M(A \oplus < g >)$.

(iii) Commutativity: M(A) is independent of the indexing of the elements in A.

Although the general properties of MICA operators have been listed above, our premises for reinforcement is based on the fact that reinforcement is applied only when the sensors are equally reliable, so that R(x,y) = 0 if $x \neq y$,and so the effect of MICA operator is simple aggregation, which means that pointwise aggregation is made and fusion can be anything. So for simplicity we can use the maximum value. subsectionPartial Aggregation Using considerations detailed above, a partial aggregation[13] of data is made using comaptibility criteria and including weighted data sets from different sensors with the help of OWA operators based on reliability of the sensors and reinforcement when the sensors are equally reliable.

Table 1: Reliabilities of sensors

Class	CT	MRI
Air	3	4
BM	2	5
V-CSF	2	5
Fat	2	4
Skull	5	2

Table 2: Probabilities and Memberships for CT

region	air	BM	VCSF	Fat	Skull
P (r)	.13	44	.03	.03	.38
$\mu(r)$	.06	.88	.03	.04	.06

3 Example: Fusion of data obtained from two sensors

We illustrate the above remarks by using an idealized example based on real data taken from a transverse supraorbital region of the human brain imaged by two sensors CT and MRI. The original images have been classified into a number of tissue classes using a fuzzy c-means algorithm that has been described in detail elsewhere [7]. Figure 1 represents the classified, transverse CT and MR images of the same supraorbital region of the human brain [7] and Figure 2 represents their corresponding histograms with the fuzzy membership classes depicted as dashed triangles. It is found that the CT images have five major classes corresponding to air, Cerebro Spinal Fluid (CSF) and Ventricles (V or V-CSF), Brain Matter (BM), Fatty tissue (Fat) and skull, whereas the skull class is omitted for MRI, thereby the four other classes. The reliabilities on a scale of 1-5 are given below, with 1 being poor and 5 being excellent. These reliability measures are based on statistics of usage of the particular modality for the given tissue class.

The distance limit between any two sources is taken to be 3 on a scale of 1-5, that is: Compatibility measure is

R(x,y) = 0 if $\|x - y\| \rangle 3$

R(x,y) = 1- $\frac{1}{3}\|x - y\|$if $\|x - y\| \leq 3$

The ratio of the areas of the non-overlapping regions of the triangles to the total area occupied by the image of the object is taken to be the membership value in the given class while the probabilities are estimated from the frequencies in the histogram. With these considerations, the quantities used for the calculation of the marginal entropies are listed in Table I and II.

The fuzzy membership classes assigned to the mixed pixel data are given below for the two sensors, calculated from the ratio of overlapping area to total area using Figure 2 and weighted with reliabil-

Table 3: Probabilities and Memberships for MRI

region	air	BM	V-CSF	Fat
P (r)	.09	.80	.11	.03
$\mu(r)$	.02	.80	.11	.03

Table 4: Class(es) assigned for mixed pixels in CT

Classes	Fuzzy Membership
Air-BM	.002(BM)+.003(Air)
BM-VCSF	.005(BM)+.005(V')
VCSF-Fat	.002(BM)+.002(Fat)
Skull	no overlap

ity (The fractions indicate corrections to probability values in Tables 2 and 3).

where V' denotes V-CSF. The marginal entropies for the whole images are H(CT)=0.4199*ln(0.4199) and H(MR)=0.6342*ln(0.6342).

When both sensors are comparable for a particular tissue class, a pointwise aggregation is performed and the net values are reinforced to obtain a maximum membership value of unity. A composite image is depicted in Figure 3.

The composite image is obtained after registration using a simple pixel correlation scheme and fusion using partial aggregation involving compatibility between the two sensors for each tissue class as well as OWA operator weighting

4 Discussions and future work

In the idealized example considered, the fused image will predominantly display the MR image as the soft tissues are imaged prominently by MR. The skull from the CT sensor is also shown. According to our weight functions, MRI is more sensitive to soft tissues like brain matter, ventricles and CSF and fatty tissue while CT is the preferred modality for hard tissues like skull. This work presents an attempt to fuse classified data sets by emphasizing the salient features of each imaging sensor.

Future work will therefore focus on using different types of images and fine tuning of sensors that have comparable capability in imaging similar regions of objects so that more accurate estimates of OWA operators can be made. Also, experiments can be extended to situations were information from three or more sensors need to be fused. More rigorous quantificaion of measures of belief of different sensors are also the need of the hour.

Table 5: Class(es) assigned for mixed pixels in MRI

Classes	Fuzzy Membership
Air-BM	.004(BM)+.002(Air)
BM-VCSF	.005(BM)+.005(V')
VCSF-Fat	.003(V') +.001(Fat)

Table 6: Weighting and Optimism Coefficients

Class	r(c)	r(m)	α
Air	.04	.03	.25
BM	.16	.85	.25
V'	.15	.86	.25
Fat	.12	.89	.25
Skull	.88	.11	.75

5 References

1. H.D. Cheng and J.R. Chen, Automatically Determine the Membership Function Based on the Maximum Entropy Principle, Information Science 96, 163-182 (1997)
2. G. Beliakov, Fuzzy sets and membership functions based on probabilities, Information Sciences,91, 95-111 (1997).
3. J.T. Kent and K.V. Mardia, Spatial Classification using fuzzy membership, IEEE Trans. Patt. Anal. Mach. Intell. PAMI-10 659-671 (1988).
4. R. R. Yager, General Approach to the Fusion of Imprecise Information , Technical Report, no. MII 1512, Machine Intelligence Institute, Iona College, New Rochelle, New York 1995.
5. R. R. Yager, On the ordered Weighted Averaging Aggregation Operators in multicriteria decision making, IEEE Trans. Systems, Man and Cybernetics, 18, 183-190, 1988.
6. M. O'Hagan, Using Maximum-entropy ordered weighting averaging to construct a fuzzy neuron', Proceedings 24th Annual IEEE Asilomar Conf. on Signals, Systems and Computers, Pacific Grove, Ca, 618-623, 1990.
7. S. Banerjee, D.P. Mukherjee and D. Dutta Majumdar, Fuzzy c-means approach to tissue classification in multimodal medical imaging, Information Sciences, 115, 261-279 (1999).
8. R. R. Yager and A. Kelman, Fusion of Fuzzy Information with Considerations for Compatibility, Partial Aggregation and Reinforcement, Technical Report no. MII-1514, Machine Intelligence Institute, Iona College, New Rochelle, New York, 1995.
9. L.A. Zadeh, Probability measures odf fuzzy events, J. Math. Ana. and Appl. 23:421-427 (1968).
10.I. Bloch Information combination operators for data fusion, A Review with classification, IEEE Syst. Man Cybernet. 26, 52-67 (1996).
11. L.A. Zadeh, Outline of a new approach to the analysis of complex systems and decision processes, IEEE Trans. Syst. Man Cybern. SMC-3(1):28-44

(1973).
12. T. Terano, H. Tanaka and K. Asai, A formulation of fuzzy decision problems with fuzzy information using probability measures of fuzzy events, Inf. Contr. 38:135-147 (1978).
13. R.R. Yager, Aggregation operators and fuzzy systems modelling, Fuzzy Sets and Systems, 67, 129-146 (1994).

6 Figure Captions

Fig. 1 (a) and (b): CT and MR classified images of brain.
Fig. 2 (a) and (b): CT and MR histograms with membership functions indicated.
Fig. 3: Fused image .