Design of Robust PI Controllers for a Hydrogenerator Unit

A.K.BOGLOU[1] and K.G.ARVANITIS[2]
[1] School of Applied Technology
Technological Educational Institute of Kavala
Agios Loucas, 65404 Kavala
GREECE

[2] Department of Natural Resources Management and Agricultural Engineering, Laboratory Division of Agricultural Machinery and Automation
Agricultural University of Athens
Iera Odos 75, Botanikos 11855, Athens
GREECE

Abstract: - In this paper, the sixteen plants theory (SPT) is applied to a hydrogenerator system model, which incorporates parametric uncertainty, in order to design a robustly stabilizing PI controller, which enhances the stability characteristics of the plant over a variety of ope-rating points.

Key-Words: - Power systems, parametric uncertainty, robust stabilization, Kharitonov theorem.

1 Introduction

Simple linear models are often used in order to analyze and design power control systems. Such models are generally obtained through linearization and simplifica-tion of the highly nonlinear and complex models des-cribing the true behavior of the power system. Therefore, uncertainties naturally arise in the reduced models. In addition to the simplification aspects, model uncertainty may also arise from the behavior of the power plant itself which changes with time. Thus, the facing of model uncertainties is a common task in power control systems. The designer must ultimately insure stability and performance of the actual closed-loop system and the designed controller must be robust to the model uncertainty.

During the last decade several techniques have been developed to deal with model uncertainty and robust control [1]-[7]. One of the most powerful methods for robustness analysis and control is the so-called Sixteen Plant Theory (SPT) [6], [7], which is based on the well know Kharitonov's theorem [5].

In the present work, the aforementioned robust design procedure is used in order to design a specific first order compensator (in fact, a PI controller), which robustly stabilizes a hydrogenerator system, for the purpose of enhancing its dynamic stability characteristics over a wide range of operating conditions of the plant. The particular hydrogenerator studied here, is a 117 MVA hydrogenerator unit of the Greek Electric Utility Power System, which is installed in Sfikia, near Veria, Emathia, Greece and which supplies power through a step-up transformer and a transmission line to an infinite grid. The proposed robust control design relies on an uncertain linear transfer function model of the hydrogenerator, obtained by linearizing its nonlinear Park's equations [8], about several operating points. Simulation study clearly shows that the designed robust PI controller retains a satisfactory closed-loop response in cases of load distur-bances as well as set point changes.

2 Review of SPT theory

SPT deals with plant transfer functions with parametric uncertainty structure of the form

$$H(s,\mathbf{a},\mathbf{b}) = \left(\sum_{i=0}^{m} a_i s^i\right) / \left(s^n + \sum_{i=0}^{n-1} b_i s^i\right) \qquad (1)$$

$$\mathbf{a} \in \boldsymbol{A} = \left\{\mathbf{a} : a_i^- \le a_i \le a_i^+ \;,\; i = 0,\ldots,m\right\}$$

$$\mathbf{b} \in \boldsymbol{B} = \left\{\mathbf{b} : b_i^- \le b_i \le b_i^+ \;,\; i = 0,\ldots,n-1\right\}$$

where, the bounds a_i^-, a_i^+, b_i^-, b_i^+ are specified a priori. The uncertain plant (1) is controlled using a general first-order controller having the rational transfer function

$$C(s) = \frac{c_0 + c_1 s}{d_0 + d_1 s} \qquad (2)$$

As it has been shown in [6], the stability of the closed-loop system is guaranteed if and only if the controller (2) stabilizes the following *sixteen* Kharitonov plants, hence the naming *sixteen plant theory*

$$H_{i,j}(s) = \frac{A_i(s)}{B_j(s)} \quad , \text{ i,j=1,2,3,4} \tag{3}$$

The eight Kharitonov polynomials associated with the uncertain plant are

$$A_1(s) = a_0^- + a_1^- s + a_2^+ s^2 + a_3^+ s^3 + a_4^- s^4 + a_5^- s^5 + a_6^+ s^6 + \cdots$$
$$A_2(s) = a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + a_4^+ s^4 + a_5^+ s^5 + a_6^- s^6 + \cdots$$
$$A_3(s) = a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + a_4^+ s^4 + a_5^- s^5 + a_6^- s^6 + \cdots$$
$$A_4(s) = a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + a_4^- s^4 + a_5^+ s^5 + a_6^+ s^6 + \cdots$$
$$B_1(s) = b_0^- + b_1^- s + b_2^+ s^2 + b_3^+ s^3 + b_4^- s^4 + b_5^- s^5 + b_6^+ s^6 + \cdots$$
$$B_2(s) = b_0^+ + b_1^+ s + b_2^- s^2 + b_3^- s^3 + b_4^+ s^4 + b_5^+ s^5 + b_6^- s^6 + \cdots$$
$$B_3(s) = b_0^+ + b_1^- s + b_2^- s^2 + b_3^+ s^3 + b_4^+ s^4 + b_5^- s^5 + b_6^- s^6 + \cdots$$
$$B_4(s) = b_0^- + b_1^+ s + b_2^+ s^2 + b_3^- s^3 + b_4^- s^4 + b_5^+ s^5 + b_6^+ s^6 + \cdots$$

The closed-loop characteristic polynomial associated with
the plant $H_{i,j}(s)$ is given by

$$p_{c,i,j}(s) = (c_0 + c_1 s)A_i(s) + (d_0 + d_1 s)B_j(s) \tag{4}$$

To determine if a controller with fixed structure and parameters robustly stabilizes a plant with known para-metric uncertainty, all sixteen characteristic polynomials of the form (4), corresponding to the sixteen Kharitonov plants (3) have to be set up and checked for stability (using, for example, the Routh test or the Bode criterion). If all sixteen plants are stable, then, we conclude that the controller robustly stabilizes the uncertain plant.

3 Hydrogenerator System Model

The hydrogenerator system studied, is an 117 MVA hydrogenerator unit of the Greek Electric Utility Power System, which is installed in Sfikia, Himathia, Greece and which supplies power through a step-up transformer and a transmission line to an infinite grid. A linear model of the hydrogenerator can be obtained by linearizing its nonlinear Park's equations [8], [9] about various opera-ting points. By mathematically eliminating the damper circuit currents i_D and i_Q and the field current i_f from the standard Park's equations one obtains, after some algebraic manipulations, the following modified practical form of these equations in state variable form

$$\frac{d\delta}{dt} = \omega - \omega_0 \tag{5a}$$

$$\begin{aligned}\frac{d\omega}{dt} = \frac{\omega_0}{2H}\Bigg\{ T_m &- \left[\frac{x_{ad}(x_{ad}-x_D)}{x_{ad}^2 - x_D x_f}\Psi_f + \frac{x_{ad}(x_{ad}-x_f)}{x_{ad}^2 - x_D x_f}\Psi_D\right] i_q + \frac{x_{aq}}{x_Q}\Psi_Q i_d \\ &+ \left(x_d - \frac{2x_{ad}^3 - x_{ad}^2 x_D - x_{ad}^2 x_f}{x_{ad}^2 - x_D x_f} - x_q - \frac{x_{aq}^2}{x_Q}\right) i_d i_q \Bigg\}\end{aligned} \tag{5b}$$

$$\frac{d\Psi_f}{dt} = \left(\frac{\omega_0 R_f x_D}{x_{ad}^2 - x_D x_f}\right)\Psi_f - \left(\frac{\omega_0 R_f x_{ad}}{x_{ad}^2 - x_D x_f}\right)\Psi_D + \frac{\omega_0 R_f}{x_{ad}} E_{fd} - \left[\frac{\omega_0 R_f x_{ad}(x_{ad} - x_D)}{x_{ad}^2 - x_D x_f}\right] i_d \tag{5c}$$

$$\frac{d\Psi_D}{dt} = \left(\frac{\omega_0 R_D x_{ad}}{x_f x_D - x_{ad}^2}\right)\Psi_f - \left(\frac{\omega_0 R_D x_f}{x_f x_D - x_{ad}^2}\right)\Psi_D + \left[\frac{\omega_0 R_D (x_{ad} - x_f)}{x_f x_D - x_{ad}^2}\right] i_d \tag{5d}$$

$$\frac{d\Psi_Q}{dt} = -\frac{\omega_0 R_Q}{x_Q}\Psi_Q - \frac{\omega_0 R_Q x_{aq}}{x_Q} i_q \tag{5e}$$

$$\begin{aligned}\frac{dE_{fd}}{dt} = \frac{K_E}{\tau_E} V_{ref} - \frac{1}{\tau_e} E_{fd} - \frac{K_e}{\tau_e v_t} \times \\ \Bigg\{ \left[-\frac{x_{aq}}{x_Q}\Psi_Q + \left(x_q - \frac{x_{aq}^2}{x_Q}\right) i_q - R_a i_d \right] v_d \\ + \Bigg[\frac{x_{ad}(x_{ad}-x_D)}{x_{ad}^2 - x_D x_f}\Psi_f + \frac{x_{ad}(x_{ad}-x_f)}{x_{ad}^2 - x_D x_f}\Psi_D \\ + \left(-x_d + \frac{2x_{ad}^3 - x_{ad}^2 x_D - x_{ad}^2 x_f}{x_{ad}^2 - x_D x_f}\right) i_d - R_a i_q \Bigg] v_q \Bigg\}\end{aligned} \tag{5f}$$

where, δ is the torque angle, ω and ω_0 are the ma-chine and synchronous speed, respectively, H is an inertia constant, T_m is the generator-shaft mechanical torque, x_{ad} and x_{aq} are the magnetizing reactances in d- and q-axis, x_D and x_Q are the damper circuit self-reactances in d- and q-axis, x_f is the field winding self-reactance, Ψ_f is the field flux linkage, Ψ_D and Ψ_Q are the damper circuit flux linkages in d- and q-axis, i_d and i_q are the stator currents in d- and q-axis circuits, x_d and x_q are the machine synchronous reactances in d- and q-axis, R_f is the field resistance, E_{fd} is the exciter output voltage, R_D and R_Q are the damper circuit resistances in d- and q-axis, τ_e is the exciter time constant, K_e is the exciter gain, v_t is the machine terminal voltage, R_a is the pha-se stator resistance, v_d and v_q are the stator voltages in d- and q-axis and V_{ref} is the voltage reference.

The principal data of the three phase hydrogenerator system under control is given in Table 1. Note that, in Table 1, all unspecified data is in p.u. on machine ratings, the time constants and the inertia constant of the generator and the prime-mover are in secs, while the synchronous speed is in rad/sec. Note also that the linkage reactances in d- and q-axis are given by x_{ld} =0.095 p.u. and x_{lq} =0.076 p.u.. The resistance and the reactance of the external system, consisting of the step-up transformer and the double-circuit transmission line are given by $R_e = 0.015$ p.u. and $X_e = 0.40$ p.u..

MVA = 117	$R_D = 0.014$
kV = 15.75	$R_Q = 0.008$
RPM = 125	$R_a = 0.002$
$x_d = 0.935$	$H = 3$
$x_q = 0.574$	$K_e = 50$
$x_{ad} = 0.827$	$\tau_e = 0.05$
$x_{aq} = 0.475$	$\omega_0 = 314.1593$
$x_f = 0.221$	$i_q = 0.6652$
$x_D = 0.992$	$i_d = 0.7467$
$x_Q = 0.551$	$v_q = 0.9242$
$R_f = 0.0006$	$v_d = 0.3819$

Table 1. Principal hydrogenerator system data.

	v_t p.u.	P_t p.u.	Q_t p.u.	δ_{nom} rad	ω_{nom} rad/sec
O.P. I	1.0	0.9	0.436	0.8024	100π
O.P. II	1.0	1.1	0.5	0.9604	100π
O.P. III	1.0	0.5	0.58	0.4592	100π
O.P. IV	1.0	0.4	-0.68	0.4914	100π

	$\Psi_{f,nom}$ p.u.	$\Psi_{D,nom}$ p.u.	$\Psi_{Q,nom}$ p.u.	$E_{fd,nom}$ p.u.
O.P. I	1.44005	1.0062	-0.3160	1.6123
O.P. II	1.4737	1.0001	-0.3645	1.7720
O.P. III	1.4802	1.0508	-0.1740	1.6069
O.P. IV	1.0107	0.8842	-0.2911	0.4734

Table 2. Some operating points of the hydrogenerator system

Defining the following vectors

$$\mathbf{x} = \begin{bmatrix}\Delta\delta & \Delta\omega & \Delta\Psi_f & \Delta\Psi_D & \Delta\Psi_Q & \Delta E_{fd}\end{bmatrix}^T, \ \mathbf{u} = \Delta V_{ref}$$

and after linearization of the nonlinear equations (5a)-(5f), with respect to a nominal operating point of the system, we obtain a linear state space model for the hy-drogenerator. Note that, Δ defines incremental changes of the variables, involved in the description, around the particular operating point chosen for the linearization procedure. Some of the operating points considered in this study are given in Table 2, wherein P_t and Q_t de-note the active and the reactive generator power.

By considering the torque angle δ as the output of the system, a linear transfer function model of the hydrogene-rator is given by

$$H(s) = \frac{a_0 + a_1 s + a_2 s^2}{b_0 + b_1 s + b_2 s^2 + b_3 s^3 + b_4 s^4 + b_5 s^5 + s^6}$$

where

$$a_0 \in \begin{bmatrix}a_0^- & a_0^+\end{bmatrix} = \begin{bmatrix}-1640301.5287 & -808056.6756\end{bmatrix}$$
$$a_1 \in \begin{bmatrix}a_1^- & a_1^+\end{bmatrix} = \begin{bmatrix}-245697.5166 & -122573.4966\end{bmatrix}$$
$$a_2 \in \begin{bmatrix}a_2^- & a_2^+\end{bmatrix} = \begin{bmatrix}-4541.6972 & -2270.5455\end{bmatrix}$$
$$b_0 \in \begin{bmatrix}b_0^- & b_0^+\end{bmatrix} = \begin{bmatrix}764282.2624 & 1039049.8310\end{bmatrix}$$
$$b_1 \in \begin{bmatrix}b_1^- & b_1^+\end{bmatrix} = \begin{bmatrix}361280.3908 & 391101.3798\end{bmatrix}$$
$$b_2 \in \begin{bmatrix}b_2^- & b_2^+\end{bmatrix} = \begin{bmatrix}63842.2612 & 77335.8423\end{bmatrix}$$
$$b_3 \in \begin{bmatrix}b_3^- & b_3^+\end{bmatrix} = \begin{bmatrix}8787.1623 & 10123.0723\end{bmatrix}$$
$$b_4 \in \begin{bmatrix}b_4^- & b_4^+\end{bmatrix} = \begin{bmatrix}839.9738 & 872.7566\end{bmatrix}, \ b_5 = 48.0680$$

As it can be easily checked, linearization about opera-tional point II, leads to an unstable linear open-loop model of the hydrogenerator. Thus robust stabilization is a primary objective in controlling such a power system.

4 Simulation Study

In this section, the SPT theory is applied to the hydro-generator system presented in the previous section, in order to design a robustly stabilizing PI controller of the form

$$G_c(s) = K_P + K_I / s$$

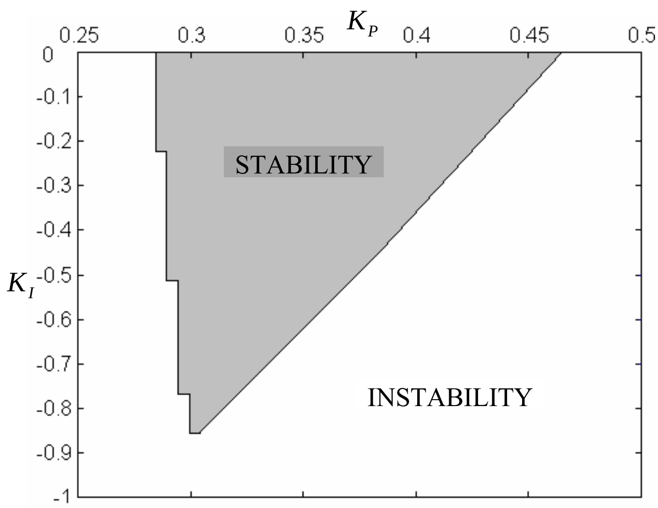

Figure 1. Set of robust PI stabilizers

In this case, the closed-loop characteristic polyno-mials corresponding to a Kharitonov plant $G_{ij}(s)$ is

$$p_{c,i,j}(s) = (K_I + K_P s)A_i(s) + sB_j(s)$$

where, in our case, the eight Kharitonov plants are given in the Appendix. Forming the Routh tables for the six-teen Kharitonov plants, we enforce positivity for each of the first columns. This leads to inequalities involving K_P and K_I. After certain manipulations on these Routh inequalities, we obtain their satisfaction set in the range

$$0.28 < K_P < 0.46 \ , \ -0.88 < K_I < 0$$

which is depicted in Figure 1. For implementation purpo-ses, one can use any (K_P, K_I) combination in this set, to generate a stabilizing controller. For example, a specific robust PI stabilizing controller is

$$C(s) = 0.35 - \frac{0.2}{s}$$

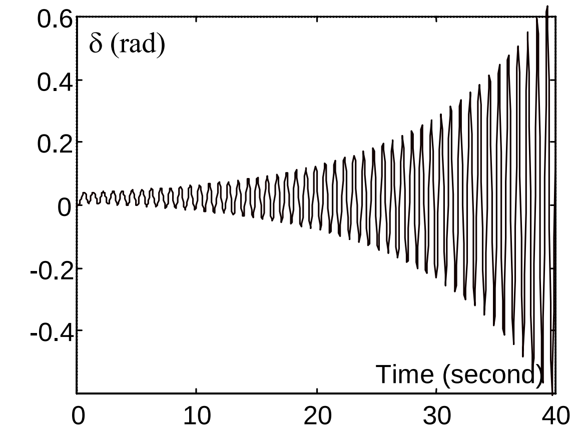

Figure 2. Open-Loop system response for O.P. II, to a set point step change

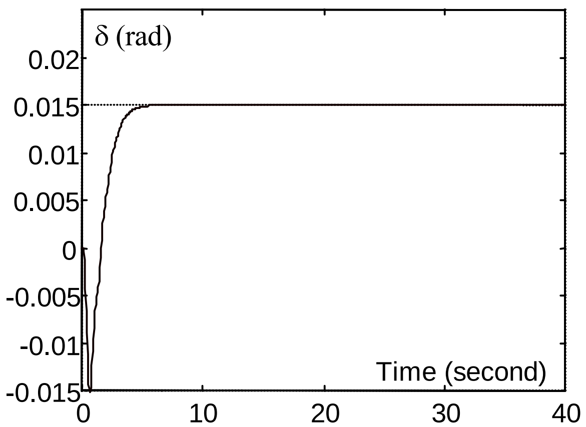

Figure 3. Closed-Loop system response for O.P. II, to a set point step change

Simulation results regarding the application of the above specific controller to the hydrogenerator system are given in Figures 2-4. In particular, in Figures 2 and 3, the open-loop and the closed-loop system response for operational point II, to a set point step change of 0.015 p.u., are depicted. In Figure 4, both the open-loop and the closed-loop system response for operational point III, to the same set point change and to a step load disturbance of 0.001 p.u., are given. From these Figures, it can be easily recognized that the designed PI controller is very effective in facing large parametric uncertainty, set point changes and load disturbances.

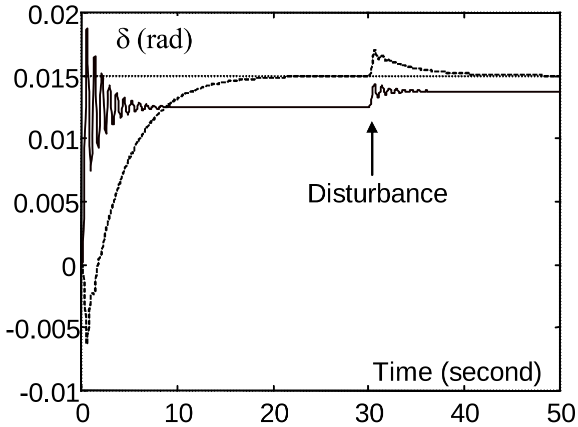

Figure 4. Open-Loop and Closed-Loop system response for O.P. II, to a set point step change and to a step load disturbance

5 Conclusion

The SPT has been applied, in this paper, to an uncer-tain hydrogenerator system, in order to design a robustly stabilizing PI controller. Simulation results concerning the closed-loop system response as well as the robust stability region of the control loop have been presented. As it has been shown by simulations, the designed robust PI controller retains a satisfactory closed-loop response in cases of load disturbances and set point changes.

Acknowledgment:
The work described in this paper has been funded by the Greek Ministry of National Education (25%) as well as the European Union (75%) in the frame-work of the project "ARCHEMIDES-Funding of re-search groups in Greek Technology Education Institutes", under the grant 04-3-0013.

References:

[1]. J.C.Doyle, "Analysis of feedback systems with structured ucertainty", *Proc. IEE-pt. D,* Vol. 129, 1982, pp. 242-250.

[2]. B.Francis, *A course on* H^∞ *control theory,* Springer-Verlag, New York, 1987.

[3]. J.C.Doyle, B.A.Francis and A.R.Tannenbaum, *Feedback Control Theory* McMillan Publishing

Company, New York, NY, U.S.A, 1992.
[4]. S.Skogestad and I.Postlethwaite, *Multivariable Feedback Control,* John Wiley and Sons, New York, NY, U.S.A, 1996.
[5]. V.L.Kharitonov, "Asymptotic stability of an equilibrium position of a family of systems of linear differential equations", *Differentsial'nye Uravneniya,* Vol. 14, 1978, pp. 2086-2088.
[6]. B.R.Barmish, C.V.Hollot, F.J.Kraus and R. Tempo, "Extreme point results for robust stabilization of interval plants with first order compensators", *IEEE Trans. Autom. Control,* Vol. AC-37, 1992, pp. 707-714.
[7]. B.R.Barmish, *New Tools for robustness of linear systems,* McMillan, New York, 1994.
[8]. R.H.Park, "Two reaction theory of synchronous machines: General method of analysis", Part I, *Trans. AIEE,* Vol. 48,1929, p.p. 716.
[9]. G.Shackshaft,"A general purpose turbo-alternator model", *Proc. IEE,* Vol. 110, 1963, pp. 703-713.

APPENDIX

$A_1(s) = -1640301.5287 - 245697.5166\,s - 2270.5455s^2$

$A_2(s) = -808056.6756 - 122573.4966\,s - 4541.6972s^2$

$A_3(s) = -808056.6756 - 245697.5166\,s - 4541.6972s^2$

$A_4(s) = -1640301.5287 - 122573.4966\,s - 2270.5455s^2$

$B_1(s) = 764282.2624 + 361280.3908\,s + 77335.8423s^2 + 10123.0723s^3 + 839.9738s^4 + 48.0680s^5 + s^6$

$B_2(s) = 1039049.8310 + 391101.3798\,s + 63842.2612s^2 + 8787.1623s^3 + 872.7566s^4 + 48.0680s^5 + s^6$

$B_3(s) = 1039049.8310 + 361280.3908\,s + 63842.2612s^2 + 10123.0723s^3 + 872.7566s^4 + 48.0680s^5 + s^6$

$B_4(s) = 764282.2624 + 391101.3798\,s + 77335.8423s^2 + 8787.1623s^3 + 839.9738s^4 + 48.0680s^5 + s^6$