# Optimal Control of a Three Phase Hydrogenerator using a Class of Sampled-Data Controllers

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*Abstract:* - An optimal control strategy based on Two-Point-Multirate Controllers (TPMRCs), is used to design a desirable excitation controller of a hydrogenerator system, in order to enhance its dynamic stability characteristics. In the TPMRCs based scheme, the control is constrained to a certain piecewise constant signal, while each of the controlled plant outputs is detected many times over a fundamental sampling period $T_0$. The proposed control strategy is readily applicable in cases where the state variables of the controlled plant are not available for feedback, since TPMRCs provide the ability to reconstruct exactly the action of static state feedback controllers from input-output data, without resorting to state estimators, and without introducing high order exogenous dynamics in the control loop. On the basis on this strategy, the original problem is reduced to an associate discrete-time LQ regulation problem for the performance index with cross product terms (LQRCPT), for which a fictitious static state feedback controller is needed to be computed. Simulation results for the actual 117 MVA hydrogenerator unit in Sfikia, Greece, show the effectiveness of the proposed method which has a quite satisfactory performance.

*Key-Words:* - Multirate controllers, optimal control, power systems, hydrogenerators.

## 1 Introduction

The dynamic stability enhancement of an open-loop power system model, linearized about its nominal operating point may be achieved by designing a suitable excitation controller and thus obtaining a closed-loop system with desired dynamic stability characteristics. The actual design of such controllers may be accomplished by using various modern control methods. Among them, LQ optimal control methods have received considerable attention in the past (see e.g. [1]-[8]). Most of these optimal control techniques, however, suffer from several serious disadvantages.

More precisely, they are applicable, under the condition that full state feedback of the plant under control is measurable and available for feedback, a fact that is rarely satisfied in practice. In the case where only output measurements are available, one can use, instead of the system states, their estimates, obtained from anyone of the classical state estimation methods. However, estimator based controllers, have their own disadvantages. First of all, the number of states of the estimator and of the system must be, in most cases, the same, and the estimator must run on-line. When the controlled system is of high order, this implies high computation in the controller. On the other hand, whereas continuous state feedback methods are able to meet the robustness objective, it has been shown that the introduction of state estimator negates this advantage. Even though some level of robustness may be recovered, robustness using and estimator based controllers is still an open research topic. Furthermore, most of the traditional LQ optimal regulation methods are directly applicable to the continuous-time system under control. However, digital control design through microprocessor is, today, the state of the art in control systems technology.

Thus, in order to incorporate the recent advantages of the digital computer technology in optimal techniques for power systems control, on has either to make the controller synthesis in continuous time and then to discretize the control law or to make the controller synthesis in discrete time after discretizing the continuous-time system under control. However, neither of these approaches take into account the intersample behavior of the continuous-time signals. Their use is, therefore, mainly restricted to cases when the sampling frequency is significantly higher than the design bandwidth.

From the previous analysis, it is clear that optimal control techniques, which do not need the state variables of the system under control or their estimates and which are directly applicable in digital environment, are of great importance in power systems control. In the present paper, a new feedback strategy of this kind is presented. We refer to this novel control strategy as a Two-Point-Multirate Controller (abbreviated here as TPMRC). TPMRC based control is a rather typical multirate control strategy where control (actuators) updates are performed at different rates than the output samples.

Multirate sampling schemes have long been the focus of interest by many control designers. There are several reasons to use such a sampling scheme in digital control systems. First of all, in complex, multivariable control systems, often it is unrealistic, or sometimes impossible, to sample all physical signals uniformly at one single rate. In such situations, one is forced to use multirate sampling. Furthermore, in general, one gets better performance if one can sample and hold faster. But faster A/D and D/A conversions mean higher cost in implementation. For signals with different bandwidths, better trade-offs between performance and implementation cost can be obtained using A/D and D/A converters at different rates.

On the other hand, multirate controllers are in general time-varying. Thus multirate control systems can achieve what singlerate cannot; e.g. gain improvement, simultaneous stabilization and decentralized control. Finally, multirate controllers are normally more complex than singlerate ones; but often they are finite-dimensional and periodic in a certain sense and hence can be implemented on microprocessors via difference equations with finitely many coefficients. Therefore, like singlerate controllers, multirate controllers do not violate the finite memory constraint in microprocessors. In particular, the control strategy presented here is essentially a combination of the control strategies reported in [9], [10]. The control is constrained to a certain piecewise constant signal, while the controlled plant output is detected many times over a fundamental sampling period. The proposed control strategy relies on solving the continuous LQ regulation problem.

TPMRCs provide the ability of the exact reconstruction of action of the state feedback without resorting to the design of state estimators, and without introducing high order exogenous dynamics in the control loop. Based on this strategy, the original problem is reduced to an associated discrete-time LQ regulation problem for the performance index with cross product terms, for which a fictitious static state feedback controller is needed to be computed. Thus, the present technique essentially resort to the computation of simple gain controllers in a digital environment, rather than to the computation of state observers, as compared to known techniques. Finally, the designed TPMRCs based LQ optimal regulators can possess any prescribed degree of stability, since there is the possibility to choose the transition matrices of the controllers arbitrarily.

In this paper, the proposed optimal control strategy is used to design a desirable excitation controller of a hydrogenerator system, for the purpose of enhancing its dynamic stability characteristics. The particular hydrogenerator studied in the paper, is a 117 MVA hydrogenerator unit of the Greek Electric Utility Power System, which is installed in Sfikia, near Veria, Emathia, Greece and which supplies power through a step-up transformer and a transmission line to an infinite grid. The proposed optimal control design is based on linear state space models of the hydrogenerator, obtained by linearizing its nonlinear Park's equation [11], [12], about a particular operating point. Simulation results regarding the application of the proposed technique to the linearized stae-space model of the hydrogenerator unit clearly show the effectiveness of the method and a significant improvement of the dynamic performance of the system.

## 2 LQ Regulation using Two-Point Multirate Controllers

Consider the continuous-time, linear, time-invariant, multi-input, multi-output (MIMO) system described in state-space by the following equations

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \ , \ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \qquad (1)$$

where, $\mathbf{x}(t) \in \mathbf{R}^n$ is the state, $\mathbf{u}(t) \in \mathbf{R}^m$ is the input and $\mathbf{y}(t) \in \mathbf{R}^p$ is the output of the system and where all the matrices have real entries and

appropriate dimensions. It is further assumed, that system (1) is controllable (stabilizable) and observable (detectable).
The following definitions will be useful in the sequel.

**Definition 1.** Let $\mathbf{c}_i^T, i = 1,2,...,p$, be the ith row of the matrix **C**. For an observable matrix pair $(\mathbf{A},\mathbf{C})$, a collection of p integers $\{n_1, n_2, \cdots, n_p\}$, is called an *observability index vector* of the pair $(\mathbf{A},\mathbf{C})$, if the following relationships simultaneously hold

$$\sum_{i=1}^{p} n_i = n, \operatorname{rank}\left[\mathbf{c}_1 \;\cdots\; (\mathbf{A}^T)^{n_1-1}\mathbf{c}_1 \;\cdots\; \mathbf{c}_p \;\cdots\; (\mathbf{A}^T)^{n_p-1}\mathbf{c}_p\right] = n$$

**Definition 2.** The *generalized reachability Grammian of order N* on the interval $[0, T_0]$ is defined by

$$\mathbf{W}_N(T_0,0) = T_N^{-1}\sum_{\mu=0}^{N-1}\Delta_\mu\Delta_\mu^T \tag{2}$$

where

$$T_N = \frac{T_0}{N}, \quad \Delta_\mu \triangleq \hat{\mathbf{A}}_N^{N-\mu-1}\hat{\mathbf{B}}_{T_N} \tag{3a}$$

and where, in (3a),

$$\hat{\mathbf{A}}_N = \exp(AT_N), \quad \hat{\mathbf{B}}_{T_N} = \int_0^{T_N}\exp(\mathbf{A}\tau)\mathbf{B}d\tau \tag{3b}$$

Now, define $p_N = \operatorname{rank}\mathbf{W}_N(T_0,0)$.

Since $\mathbf{W}_N(T_0,0) \geq \mathbf{0}$, we can always find (perhaps not uniquely) an $n \times p_N$ full rank matrix $\mathbf{B}_N$ such that

$$\mathbf{W}_N(T_0,0) = \mathbf{B}_N\mathbf{B}_N^T \tag{4}$$

It is worth noticing at this point that matrix $\mathbf{B}_N$, can be computed as follows:

a) If $\mathbf{W}_N(T_0,0)$ is positive definite, then $\mathbf{B}_N$ can be obtained from the Clolesky factorization of $\mathbf{W}_N(T_0,0)$. That is, if $\hat{\mathbf{S}}$ is an upper triangular full rank matrix obtained from the Cholesky factorization of $\mathbf{W}_N(T_0,0)$ (i.e. $\hat{\mathbf{S}}$ is such that $\hat{\mathbf{S}}^T\hat{\mathbf{S}} = \mathbf{W}_N(T_0,0)$) and $\mathbf{U}_S$ is a unitary matrix (i.e. $\mathbf{U}_S^T\mathbf{U}_S = \mathbf{I}$), then

$$\mathbf{B}_N = \hat{\mathbf{S}}^T\mathbf{U}_S^T \tag{5a}$$

b). If $\mathbf{W}_N(T_0,0)$ is positive semidefinite, then we can proceed as follows: Let a Singular Value Decomposition of $\mathbf{W}_N(T_0,0)$ be defined as

$$\mathbf{W}_N(T_0,0) = \mathbf{U}\begin{bmatrix}\Sigma^* & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\mathbf{V}^T$$

where $\mathbf{U} \in \mathbf{R}^{n\times n}$, $\mathbf{V} \in \mathbf{R}^{n\times n}$ are unitary matrices and $\Sigma^* \in \mathbf{R}^{p_N \times p_N}$ is defined by $\Sigma^* = \operatorname{diag}\{\sigma_1, \sigma_2, ..., \sigma_{p_N}\}$, where $\sigma_j$, $j=1,2,\ldots,p_N$ are the nonzero singular values of $\mathbf{W}_N(T_0,0)$. Since, by definition, $\mathbf{W}_N(T_0,0)$ is a symmetric matrix, we have **U**=**V.** Let $\mathbf{M}_{\Sigma^*}$ be an upper triangular full rank matrix obtained from the Cholesky factorization of $\Sigma^*$ (i.e. $\mathbf{M}_{\Sigma^*}^T\mathbf{M}_{\Sigma^*} = \Sigma^*$).

Now, let $\mathbf{M}_W \in \mathbf{R}^{p_N\times n}$ be constructed as

$$\mathbf{M}_W = \left[\mathbf{M}_{\Sigma^*} \quad \mathbf{0}_{p_N\times(n-p_N)}\right]$$

Then,

$$\mathbf{M}_W^T\mathbf{M}_W = \begin{bmatrix}\Sigma^* & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}$$

and

$$\mathbf{W}_N(T_0,0) = \mathbf{U}\mathbf{M}_W^T\mathbf{M}_W\mathbf{U}^T$$

Consequently,

$$\mathbf{B}_N = \mathbf{U}\mathbf{M}_W^T\mathbf{V}^T \tag{5b}$$

In the sequel, let matrix $\mathbf{\Phi}$, be defined as

$$\mathbf{\Phi} = \exp(\mathbf{A}T_0)$$

Let, also, $\mathbf{B}_N^r$ be the $n \times p_N$ full rank matrix defined as the right pseudoiverse of $\mathbf{B}_N^T$, i.e.

$$\mathbf{B}_N^r = \mathbf{B}_N\left(\mathbf{B}_N^T\mathbf{B}_N\right)^{-1} \tag{6}$$

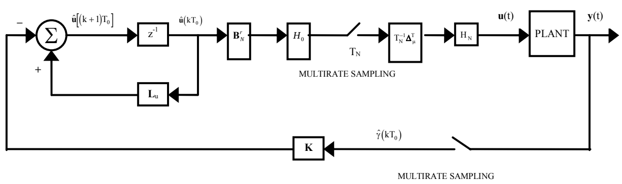


**Figure 1.** Control of linear systems using Two –Point Multi Rate Controllers.

Consider now applying to system (1) the multirate control strategy depicted in Figure 1. In particular, we assume that all samplers start simultaneously at t=0. The hold circuits $H_0$ and $H_N$ are zero order holds with holding times $T_0$ and $T_N$, respectively. The inputs of the plant are constrained to the following piecewise constant controls

$$\mathbf{u}(kT_0 + \mu T_N + \zeta) = T_N^{-1}\Delta_\mu^T\mathbf{B}_N^r\tilde{\mathbf{u}}(kT_0), \quad \tilde{\mathbf{u}}(kT_0) \in \mathbf{R}^{p_N} \tag{7}$$

for

$t = kT_0 + \mu T_N, \mu = 0,1,...,N-1, k \geq 0$ and for $\zeta \in [0, T_N)$.

The ith plant output $y_i(t)$, is detected at every $T_i = T_0 / M_i$, such that

$$y_i(kT_0+\rho T_i)=\mathbf{c}_i^T\mathbf{x}(kT_0+\rho T_i)\ ,\ \rho=0,1,...,M_i-1 \tag{8}$$

where, $M_i \in \mathbf{Z}^+, i=1,2,...,p$, are the so-called output multiplicities of the sampling.

It is worth noticing that, in general, $M_i \neq N$. That is, multirate sampling of the plant inputs and outputs may be performed at a different rate.

The sampled values of the plant outputs obtained over $[kT_0,(k+1)T_0)$, are stored in the $M^*$-dimensional column vector $\hat{\gamma}(kT_0)$ of the form

$$\hat{\gamma}(kT_0)=[y_1(kT_0)\ \cdots\ y_1(kT_0+(M_1-1)T_1)\ \cdots\ y_p(kT_0)\ \cdots\ y_p(kT_0+(M_p-1)T_p)]^T$$

where

$$M^*=\sum_{i=1}^{p}M_i\ .$$

The vector $\hat{\gamma}(kT_0)$ is used in the discrete dynamic control law of the form

$$\hat{\mathbf{u}}[(k+1)T_0]=\mathbf{L}_u\hat{\mathbf{u}}(kT_0)-\mathbf{K}\hat{\gamma}(kT_0) \tag{9}$$

where $\mathbf{L}_u \in \mathbf{R}^{p_N\times p_N}, \mathbf{K}\in\mathbf{R}^{p_N\times M^*}$.

The multirate control strategy described above will be mentioned in the sequel as a Two-Point-Multirate Controller (abbreviated here as TPMRC).

The multirate optimal scheme suggested in this paper is based on solving the continuous-time LQ regulation problem with the control strategy of Figure 1. More precisely, the control objective is to find an optimal $\mathbf{u}(t)$ constrained by (7) and (9), which, given the initial value $\mathbf{y}(0)$, minimizes the cost function

$$J=\frac{1}{2}\int_0^\infty[\mathbf{y}^T(t)\mathbf{Q}\mathbf{y}(t)+\mathbf{u}^T(t)\mathbf{\Gamma}\mathbf{u}(t)]dt \tag{10}$$

Note that in (10), $\mathbf{Q}\in\mathbf{R}^{p\times p}$ and $\mathbf{\Gamma}\in\mathbf{R}^{m\times m}$ are symmetric matrices with $\mathbf{Q}\geq\mathbf{0},\mathbf{\Gamma}>\mathbf{0}$, while $(\mathbf{A},\mathbf{C}^T\mathbf{QC})$ is an observable (detectable) pair.

To find a solution to the aforementioned LQ optimal regulation problem via TPMRCs, observe first that since $\mathbf{u}(t)$ is a function of $\hat{\mathbf{u}}(kT_0)$, the problem considered in this Section, is essentially the problem of finding an optimal $\hat{\mathbf{u}}_{opt}(kT_0)\in\mathbf{R}^{p_N}$, $k\geq 0$, which minimizes (10). Moreover, since $\hat{\mathbf{u}}(kT_0)$ obeys (9), the LQ optimal regulation problem considered here, can be reduced to the determination of the optimal gains $\mathbf{L}_u$ and $\mathbf{K}$ which minimize (10). These optimal gains can be determined using the following procedure:

Observe first that the following relationship hold (see [9] for its derivation)

$$\mathbf{x}(kT_0+\mu T_N+\zeta)=\exp\{\mathbf{A}(\mu T_N+\zeta)\}\mathbf{x}(kT_0)+\mathbf{B}_N^*(\mu,\zeta)\hat{\mathbf{u}}(kT_0) \tag{11}$$

where

$$\mathbf{B}_N^*(\mu,\zeta)=T_N^{-1}\{\exp(\mathbf{A}\zeta)\mathbf{V}_\mu^T+\hat{\mathbf{B}}_\zeta\Delta_\mu^T\}\mathbf{B}_N^r \tag{12}$$

In (12), the matrices $\mathbf{V}_\mu$ and $\hat{\mathbf{B}}_\zeta$ are defined as follows

$$\mathbf{V}_\mu^T=\hat{\mathbf{A}}_N^{N-\mu}\mathbf{\Theta}_\mu(T_N)\mathbf{\Theta}_\mu^T(T_N) \tag{13}$$

$$\hat{\mathbf{B}}_\zeta=\int_0^\zeta\exp(\mathbf{A}\lambda)\mathbf{B}d\lambda$$

where, in (13), matrix $\mathbf{\Theta}_\mu(T_N)$ is defined as follows

$$\mathbf{\Theta}_\mu(T_N)=[\hat{\mathbf{B}}_{T_N}\quad \hat{\mathbf{A}}_N\hat{\mathbf{B}}_{T_N}\quad\cdots\quad\hat{\mathbf{A}}_N^{\mu-1}\hat{\mathbf{B}}_{T_N}]$$

It is pointed out that

$$\mathbf{B}_N^*(N-1,T_N)=\mathbf{B}_N$$

Therefore, at the sampling instants $t=kT_0$, we can easily obtain

$$\mathbf{x}[(k+1)T_0]=\mathbf{\Phi}\mathbf{x}(kT_0)+\mathbf{B}_N\hat{\mathbf{u}}(kT_0) \tag{14}$$

Note also that, at every $t=kT_0+\rho T_i, \rho=0,1,...,M_i-1$, we have

$$\mathbf{x}(kT_0+\rho T_i)=\hat{\mathbf{A}}_i^\rho\mathbf{x}(kT_0)+\mathbf{B}_{M_i}^*(\rho)\hat{\mathbf{u}}(kT_0) \tag{15}$$

where

$$\hat{\mathbf{A}}_i=\exp(\mathbf{A}T_i)$$

$$\mathbf{B}_{M_i}^*(\rho)=T_N^{-1}E_{M_i}(\rho)\mathbf{B}_N^r \tag{16}$$

In (16), matrix $E_{M_i}(\rho)$ is defined as follows

$$E_{M_i}(\rho)=\sum_{j=0}^{a(i,\rho)-1}\int_{jT_N}^{(j+1)T_N}\exp\left\{\mathbf{A}\left(\rho\frac{N}{M_i}T_N-\xi\right)\right\}\mathbf{B}d\xi\Delta_j^T+\int_{a(i,\rho)T_N}^{\rho\frac{N}{M_i}T_N}\exp\left\{\mathbf{A}\left(\rho\frac{N}{M_i}T_N-\xi\right)\right\}\mathbf{B}d\xi\Delta_{a(i,\rho)}^T \tag{17}$$

where

$$a(i,\rho)=\mathrm{INT}_S\left(\rho\frac{N}{M_i}\right)$$

and where $\mathrm{INT}_S(\nu)$ is the greatest integer that is less than or equal to $\nu\in\mathbf{R}^+$.

Now, define the following matrices

$$\widetilde{\mathbf{Q}}_N = \int_0^{T_0} \exp\left(\mathbf{A}^T\xi\right)\mathbf{C}^T\mathbf{Q}\mathbf{C}\exp\left(\mathbf{A}\xi\right)d\xi \equiv \sum_{\mu=0}^{N-1}\left(\hat{\mathbf{A}}_N^{\mu}\right)^T \Xi\left(T_N\right)\hat{\mathbf{A}}_N^{\mu} \tag{18}$$

$$\widetilde{\mathbf{G}}_N = T_N^{-1}\left\{\sum_{\mu=0}^{N-1}\left(\hat{\mathbf{A}}_N^{\mu}\right)^T\left[\Xi\left(T_N\right)\mathbf{V}_\mu^T + \Lambda\left(T_N\right)\Delta_\mu^T\right]\right\}\mathbf{B}_N^r \tag{19}$$

$$\widetilde{\Gamma}_N = T_N^{-2}\left(\mathbf{B}_N^r\right)^T\left\{\sum_{\mu=0}^{N-1}\left[\mathbf{V}_\mu \quad \Delta_\mu\right]\begin{bmatrix}\Xi(T_N) & \Lambda(T_N)\\ \Lambda^T(T_N) & \mathbf{N}(T_N)+T_N\Gamma\end{bmatrix}\begin{bmatrix}\mathbf{V}_\mu^T\\ \Delta_\mu^T\end{bmatrix}\right\}\mathbf{B}_N^r \tag{20}$$

where

$$\Xi\left(T_N\right) = \int_0^{T_N}\exp\left(\mathbf{A}^T\xi\right)\mathbf{C}^T\mathbf{Q}\mathbf{C}\exp\left(\mathbf{A}\xi\right)d\xi \tag{21}$$

$$\Lambda\left(T_N\right) = \int_0^{T_N}\exp\left(\mathbf{A}^T\xi\right)\mathbf{C}^T\mathbf{Q}\mathbf{C}\hat{\mathbf{B}}_\xi d\xi \tag{22}$$

$$\mathbf{N}\left(T_N\right) = \int_0^{T_N}\hat{\mathbf{B}}_\xi^T\mathbf{C}^T\mathbf{Q}\mathbf{C}\hat{\mathbf{B}}_\xi d\xi \tag{23}$$

It is pointed out that matrices $\Xi\left(T_N\right)$, $\Lambda\left(T_N\right)$ and $\mathbf{N}\left(T_N\right)$ (and consequently matrices $\widetilde{\mathbf{Q}}_N$, $\widetilde{\mathbf{G}}_N$ and $\widetilde{\Gamma}_N$), can be easily computed on the basis of the algorithm reported in [13].
Now, substituting (1), (7) and (11) in (10) and taking into account (18)-(23), we finally obtain

$$J = \frac{1}{2}\sum_{k=0}^{\infty}\left[\mathbf{x}^T\left(kT_0\right) \quad \hat{\mathbf{u}}^T\left(kT_0\right)\right]\begin{bmatrix}\widetilde{\mathbf{Q}}_N & \widetilde{\mathbf{G}}_N\\ \widetilde{\mathbf{G}}_N^T & \widetilde{\Gamma}_N\end{bmatrix}\begin{bmatrix}\mathbf{x}\left(kT_0\right)\\ \hat{\mathbf{u}}\left(kT_0\right)\end{bmatrix} \tag{24}$$

>From the previous analysis, it becomes clear that, the original optimal LQ regulation problem has been reduced to an associated LQ optimal regulation problem for the performance index with cross-product terms (abbreviated here as LQRCPT), namely, the problem of finding a TPMRC of the form (7), (9), which minimizes the performance index (24) subject to the dynamic constraints defined by equation (14).
In the sequel, the nature of the control law (9) will be explained. To this end, we establish the following fundamental Theorems.

**Theorem 1.** The following basic formula of the multirate sampling mechanism holds

$$\mathbf{H}\mathbf{x}\left[\left(k+1\right)T_0\right] = \hat{\gamma}\left(kT_0\right) - \mathbf{D}\hat{\mathbf{u}}\left(kT_0\right), \quad k \geq 0 \tag{25}$$

where, matrices

$$\mathbf{x}\left(kT_0+\rho T_i\right) = \hat{\mathbf{A}}_i^{\rho-M_i}\mathbf{x}\left[\left(k+1\right)T_0\right] + \hat{\mathbf{B}}_{i,\rho}\hat{\mathbf{u}}\left(kT_0\right)$$

are defined as follows

$$\mathbf{H} = \begin{bmatrix}\mathbf{c}_1^T\left(\hat{\mathbf{A}}_1^{M_1}\right)^{-1}\\ \vdots\\ \mathbf{c}_1^T\hat{\mathbf{A}}_1^{-1}\\ \vdots\\ \mathbf{c}_p^T\left(\hat{\mathbf{A}}_p^{M_p}\right)^{-1}\\ \vdots\\ \mathbf{c}_p^T\hat{\mathbf{A}}_p^{-1}\end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix}\mathbf{c}_1^T\hat{\mathbf{B}}_{1,0}\\ \vdots\\ \mathbf{c}_1^T\hat{\mathbf{B}}_{1,M_1-1}\\ \vdots\\ \mathbf{c}_p^T\hat{\mathbf{B}}_{p,0}\\ \vdots\\ \mathbf{c}_p^T\hat{\mathbf{B}}_{p,M_p-1}\end{bmatrix} \tag{26}$$

and where, in (26),

$$y_i\left(kT_0+\rho T_i\right) = \mathbf{c}_i^T\hat{\mathbf{A}}_i^{\rho-M_i}\mathbf{x}\left[\left(k+1\right)T_0\right] + \mathbf{c}_i^T\hat{\mathbf{B}}_{i,\rho}\hat{\mathbf{u}}\left(kT_0\right) \tag{27}$$

**Proof:** Solving (14) for $\mathbf{x}\left(kT_0\right)$ and substituting its solution in (15), we obtain

$$\mathbf{x}\left(kT_0+\rho T_i\right) = \hat{\mathbf{A}}_i^{\rho-M_i}\mathbf{x}\left[\left(k+1\right)T_0\right] + \hat{\mathbf{B}}_{i,\rho}\hat{\mathbf{u}}\left(kT_0\right) \tag{28}$$

where, in producing (28), use was made of the fact that $\mathbf{\Phi} \equiv \hat{\mathbf{A}}_i^{M_i}$. Introducing (28) in (8), yields

$$y_i\left(kT_0+\rho T_i\right) = \mathbf{c}_i^T\hat{\mathbf{A}}_i^{\rho-M_i}\mathbf{x}\left[\left(k+1\right)T_0\right] + \mathbf{c}_i^T\hat{\mathbf{B}}_{i,\rho}\hat{\mathbf{u}}\left(kT_0\right) \tag{29}$$

for $\rho = 0,1,\ldots,M_i-1$. Moving the terms containing $\mathbf{x}\left[\left(k+1\right)T_0\right]$ to the left hand side, moving the terms of $y_i\left(kT_0+\rho T_i\right)$ to the right and expressing the equations for $\rho = 0,1,\ldots,M_i-1$ by a vector matrix form, we finally obtain (25).

**Theorem 2** [14]. Let $n_i, i = 1,2,..,p$, be positive integers which comprise an observability index vector of the observable pair $\left(\mathbf{A},\mathbf{C}\right)$. If $M_i, i = 1,2,\ldots,p$ are chosen such that $M_i \geq n_i$, then the matrix **H** has full column rank.

**Theorem 3.** Let $\left(\mathbf{A},\mathbf{C}\right)$ be an observable pair and suppose that $M_i \geq n_i, i = 1,2,\ldots,p$. Then, for almost every sampling period $T_0$, we can make the control law (9) equivalent to any static state feedback control law of the form

$$\hat{\mathbf{u}}\left(kT_0\right) = -\mathbf{F}\mathbf{x}\left(kT_0\right), \text{ for } k \geq 1 \tag{30}$$

by choosing suitably the controller pair $\left(\mathbf{K},\mathbf{L}_u\right)$, such that

$$\mathbf{K}\mathbf{H} = \mathbf{F}, \quad \mathbf{L}_u = \mathbf{K}\mathbf{D} \tag{31}$$

**Proof:** Pre-multiplying (25) by **K**, we obtain

$$\mathbf{K}\mathbf{H}\mathbf{x}\left[\left(k+1\right)T_0\right] = \mathbf{K}\hat{\gamma}\left(kT_0\right) - \mathbf{K}\mathbf{D}\hat{\mathbf{u}}\left(kT_0\right), \quad k \geq 0$$

Therefore, the control law (9) becomes equivalent to the state feedback law (30), if, for the matrix **K**, the first of equations (31) holds, and if we evaluate $\mathbf{L}_u$ by the second of (31). Since by Theorem 2, the matrix **H** has full column rank if we select $N_i \geq n_i, i = 1,2,...,p$, then for almost every $T_0$, there exists a matrix **K**, fulfilling (31).

**Theorem 4.** Let $(\mathbf{A}, \mathbf{C})$ be an observable pair and suppose that for some $M_i = n_i^*, i = 1,2,...,p$ such that $M^* \geq n + p_N$, the matrix $[\mathbf{H}\ \mathbf{D}]$ has full column rank. Then, for almost every sampling period $T_0$, there exists a matrix **K** such that

$$\mathbf{K}[\mathbf{H}\ \mathbf{D}] = [\mathbf{F}\ \mathbf{L}_u] \tag{32}$$

where **F** is an arbitrarily specified matrix corresponding to any desired state feedback and $\mathbf{L}_u$ is an arbitrarily specified matrix corresponding to the desired state transition matrix of the controller (9) itself.

**Proof:** From (31) and for $\mathbf{L}_u$ having a prespecified value we obtain (32). If for some $M_i = n_i^*$, i=1,2,…,p, such that $M^* \geq n + p_N$, the matrix $[\mathbf{H}\ \mathbf{D}]$ has full column rank, then (32) is solvable with respect to **K**, for almost every sampling period $T_0$.

**Remark 1.** In Theorems 3 and 4, the term "for almost every sampling period $T_0$" is used to shortly express the fact that the assertion fails only at isolated values of $T_0$.

>From the previous analysis, it becomes clear that, we can equivalently realize any desired static state feedback matrix **F** by a dynamic controller of the form (9), possessing any prescribed degree of stability, since we can choose the matrix $\mathbf{L}_u$ (which corresponds to the transition matrix of the controller itself) arbitrarily. The choice $\mathbf{L}_u = \mathbf{0}$ is of course permissible, leading to static TPMRCs of the form

$$\hat{\mathbf{u}}[(k+1)T_0] = -\mathbf{K}\hat{\gamma}(kT_0) \tag{33}$$

It becomes also clear that, in order to find a control law of the form (9) which minimizes the performance index (24), one has essentially to refer to an easier problem, i.e. to the design of a *fictitious* static state feedback law of the form (30), which has an equivalent action. The calculation of the matrix pair $(\mathbf{K}, \mathbf{L}_u)$ is then performed by using either (31) or (32), after choosing a desired (usually stable) matrix $\mathbf{L}_u$.

A state feedback law of the form (30) that minimizes the index (24) is well known to be [15]

$$\mathbf{F} = \left(\tilde{\boldsymbol{\Gamma}}_N + \mathbf{B}_N^T\mathbf{P}\mathbf{B}_N\right)^{-1}\left(\tilde{\mathbf{G}}_N + \mathbf{B}_N^T\mathbf{P}\boldsymbol{\Phi}\right) \tag{34}$$

where **P** is the symmetric positive definite solution of the following discrete algebraic Riccati equation

$$\mathbf{P} = \Phi^T\mathbf{P}\Phi + \tilde{\mathbf{Q}}_N - \left(\tilde{\mathbf{G}}_N + \Phi^T\mathbf{P}\mathbf{B}_N\right)\left(\tilde{\Gamma}_N + \mathbf{B}_N^T\mathbf{P}\mathbf{B}_N\right)^{-1}\left(\tilde{\mathbf{G}}_N^T + \mathbf{B}_N^T\mathbf{P}\Phi\right) \tag{35}$$

Note that the solvability of (35) and the asymptotic stability of the corresponding closed-loop system are assured by the following Lemmas, whose proofs can be found in [9].

**Lemma 1.** Matrix $\tilde{\Gamma}_N$ is positive definite if $\Gamma > \mathbf{0}$ or $\mathbf{C}^T\mathbf{Q}\mathbf{C} > \mathbf{0}$.

**Lemma 2.** Define

$$\hat{\mathbf{Q}}_N = \tilde{\mathbf{Q}}_N - \tilde{\mathbf{G}}_N\tilde{\Gamma}_N^{-1}\tilde{\mathbf{G}}_N^T$$

Then, matrix $\hat{\mathbf{Q}}_N$ is positive semidefinite.

**Lemma 3.** There exists a unique positive definite solution **P** of (35) and the corresponding closed-loop system with closed-loop system matrix

$$\boldsymbol{\Phi}_{cl} = \boldsymbol{\Phi} - \mathbf{B}_N\mathbf{F} \equiv \boldsymbol{\Phi} - \mathbf{B}_N\mathbf{K}\mathbf{H} \tag{36}$$

is asymptotically stable if (and only if) $(\mathbf{A}, \mathbf{B})$ is controllable (stabilizable) and $(\mathbf{A}, \mathbf{C}^T\mathbf{Q}\mathbf{C})$ is observable (detectable).

If a fictitious state feedback matrix **F** has been determined on the basis of (34), the TPMRC matrix pair $(\mathbf{K}, \mathbf{L}_u)$, in the case where $\mathbf{L}_u$ is not prespecified, can be obtained as follows:

<u>**Case I**</u> $(M_i = n_i)$: In this case, matrix **H** is nonsingular. Therefore,

$$\mathbf{K} = \left(\tilde{\Gamma}_N + \mathbf{B}_N^T\mathbf{P}\mathbf{B}_N\right)^{-1}\left(\tilde{\mathbf{G}}_N + \mathbf{B}_N^T\mathbf{P}\Phi\right)\mathbf{H}^{-1},$$

$$\mathbf{L}_u = \left(\tilde{\Gamma}_N + \mathbf{B}_N^T\mathbf{P}\mathbf{B}_N\right)^{-1}\left(\tilde{\mathbf{G}}_N + \mathbf{B}_N^T\mathbf{P}\Phi\right)\mathbf{H}^{-1}\mathbf{D} \tag{37}$$

<u>**Case II**</u> $(M_i > n_i)$: In this case,

$$\mathbf{K} = \left(\tilde{\boldsymbol{\Gamma}}_N + \mathbf{B}_N^T\mathbf{P}\mathbf{B}_N\right)^{-1}\left(\tilde{\mathbf{G}}_N + \mathbf{B}_N^T\mathbf{P}\boldsymbol{\Phi}\right)\mathbf{H}^{\ell} \tag{38a}$$

$$\mathbf{L}_u = \left(\tilde{\Gamma}_N + \mathbf{B}_N^T\mathbf{P}\mathbf{B}_N\right)^{-1}\left(\tilde{\mathbf{G}}_N + \mathbf{B}_N^T\mathbf{P}\Phi\right)\mathbf{H}^{\ell}\mathbf{D} \tag{38b}$$

where $\mathbf{H}^{\ell}$ is the left pseudoinverse of matrix **H** (i.e. the matrix fulfilling $\mathbf{H}^{\ell}\mathbf{H} = \mathbf{I}$).

Similarly, in the case where $\mathbf{L}_u$ is desired to have a prespecified value, one can easily obtain

$$\mathbf{K} = \left[\left(\widetilde{\boldsymbol{\Gamma}}_N + \mathbf{B}_N^T\mathbf{P}\mathbf{B}_N\right)^{-1}\left(\widetilde{\mathbf{G}}_N + \mathbf{B}_N^T\mathbf{P}\boldsymbol{\Phi}\right) \quad \mathbf{L}_u\right]\hat{\mathbf{H}}^{\ell} \quad (39)$$

where

$\hat{\mathbf{H}}^{\ell}$ is the left pseudoiverse of matrix $[\mathbf{H}\ \mathbf{D}]$.

## 3 Hydrogenerator system model and simulation results

In the present work, the aforementioned optimal control strategy is used to design a desirable excitation controller of a hydrogenerator system, for the purpose of enhancing its dynamic stability characteristics. The hydrogenerator system studied, is an 117 MVA hydrogenerator unit of the Greek Electric Utility Power System, which is installed in Sfikia, Himathia, Greece and which supplies power through a step-up transformer and a transmission line to an infinite grid. A linear model of the hydrogenerator can be obtained by linearizing its nonlinear Park's equations [11], [12] about various operating points. By mathematically eliminating the damper circuit currents $i_D$ and $i_Q$ and the field current $i_f$ from the standard Park's equations one obtains, after some algebraic manipulations, the following modified practical form of these equations in state variable form

$$\frac{d\delta}{dt} = \omega - \omega_0 \quad (40)$$

$$\frac{d\omega}{dt} = \frac{\omega_0}{2H}\left\{T_m - \left[\frac{x_{ad}(x_{ad}-x_D)}{x_{ad}^2 - x_D x_f}\Psi_f + \frac{x_{ad}(x_{ad}-x_f)}{x_{ad}^2 - x_D x_f}\Psi_D\right]i_q + \frac{x_{aq}}{x_Q}\Psi_Q i_d + \left(x_d - \frac{2x_{ad}^3 - x_{ad}^2 x_D - x_{ad}^2 x_f}{x_{ad}^2 - x_D x_f} - x_q - \frac{x_{aq}^2}{x_Q}\right)i_d i_q\right\} \quad (41)$$

$$\frac{d\Psi_f}{dt} = \left(\frac{\omega_0 R_f x_D}{x_{ad}^2 - x_D x_f}\right)\Psi_f - \left(\frac{\omega_0 R_f x_{ad}}{x_{ad}^2 - x_D x_f}\right)\Psi_D + \frac{\omega_0 R_f}{x_{ad}}E_{fd} - \left[\frac{\omega_0 R_f x_{ad}(x_{ad}-x_D)}{x_{ad}^2 - x_D x_f}\right]i_d \quad (42)$$

$$\frac{d\Psi_D}{dt} = \left(\frac{\omega_0 R_D x_{ad}}{x_f x_D - x_{ad}^2}\right)\Psi_f - \left(\frac{\omega_0 R_D x_f}{x_f x_D - x_{ad}^2}\right)\Psi_D + \left[\frac{\omega_0 R_D(x_{ad}-x_f)}{x_f x_D - x_{ad}^2}\right]i_d \quad (43)$$

$$\frac{d\Psi_Q}{dt} = -\frac{\omega_0 R_Q}{x_Q}\Psi_Q - \frac{\omega_0 R_Q x_{aq}}{x_Q}i_q \quad (44)$$

$$\frac{dE_{fd}}{dt} = \frac{K_E}{\tau_E}V_{ref} - \frac{1}{\tau_e}E_{fd} - \frac{K_e}{\tau_e v_t}\times \left\{\left[-\frac{x_{aq}}{x_Q}\Psi_Q + \left(x_q - \frac{x_{aq}^2}{x_Q}\right)i_q - R_a i_d\right]v_d + \left[\frac{x_{ad}(x_{ad}-x_D)}{x_{ad}^2 - x_D x_f}\Psi_f + \frac{x_{ad}(x_{ad}-x_f)}{x_{ad}^2 - x_D x_f}\Psi_D + \left(-x_d + \frac{2x_{ad}^3 - x_{ad}^2 x_D - x_{ad}^2 x_f}{x_{ad}^2 - x_D x_f}\right)i_d - R_a i_q\right]v_q\right\} \quad (45)$$

where, $\delta$ is the torque angle, $\omega$ and $\omega_0$ are the machine and synchronous speed, respectively, H is an inertia constant, $T_m$ is the generator-shaft mechanical torque, $x_{ad}$ and $x_{aq}$ are the magnetizing reactances in d- and q-axis, $x_D$ and $x_Q$ are the damper circuit self-reactances in d- and q-axis, $x_f$ is the field winding self-reactance, $\Psi_f$ is the field flux linkage, $\Psi_D$ and $\Psi_Q$ are the damper circuit flux linkages in d- and q-axis, $i_d$ and $i_q$ are the stator currents in d- and q-axis circuits, $x_d$ and $x_q$ are the machine synchronous reactances in d- and q-axis, $R_f$ is the field resistance, $E_{fd}$ is the exciter output voltage, $R_D$ and $R_Q$ are the damper circuit resistances in d- and q-axis, $\tau_e$ is the exciter time constant, $K_e$ is the exciter gain, $v_t$ is the machine terminal voltage, $R_a$ is the phase stator resistance, $v_d$ and $v_q$ are the stator voltages in d- and q-axis and $V_{ref}$ is the voltage reference.

| | |
|---|---|
| MVA = 117 | $R_D$ = 0.014 |
| kV = 15.75 | $R_Q$ = 0.008 |
| RPM = 125 | $R_a$ = 0.002 |
| $x_d$ = 0.935 | $H$ = 3 |
| $x_q$ = 0.574 | $K_e$ = 50 |
| $x_{ad}$ = 0.827 | $\tau_e$ = 0.05 |
| $x_{aq}$ = 0.475 | $\omega_0$ =314.1593 |
| $x_f$ = 0.221 | $i_q$ = 0.6652 |
| $x_D$ = 0.992 | $i_d$ = 0.7467 |
| $x_Q$ = 0.551 | $v_q$ = 0.9242 |
| $R_f$ = 0.0006 | $v_d$ = 0.3819 |

**Table 1.** Principal hydrogenerator system data.

The principal data of the three phase hydrogenerator system under control is given in Table 1. Note that, in Table 1, all unspecified data is in p.u. on machine ratings, the time constants and the inertia constant of the generator and the prime-mover are in secs, while the synchronous speed is in rad/sec. Note also that the linkage reactances in d- and q-axis are given by $x_{ld}$=0.095 p.u. and $x_{lq}$=0.076 p.u.. The resistance and the reactance of the external system, consisting of the step-up transformer and the double-circuit transmission line are given by $R_e = 0.015$ p.u. and $X_e = 0.40$ p.u..

| | $v_t$ p.u. | $P_t$ p.u. | $Q_t$ p.u. | $\delta_{nom}$ rad | $\omega_{nom}$ rad/sec |
|---|---|---|---|---|---|
| **O.P. I** | 1.0 | 0.9 | 0.436 | 0.8024 | 100π |
| **O.P. II** | 1.0 | 1.1 | 0.5 | 0.9604 | 100π |
| **O.P. III** | 1.0 | 0.5 | 0.58 | 0.4592 | 100π |
| **O.P. IV** | 1.0 | 0.4 | -0.68 | 0.4914 | 100π |

| | $\Psi_{f,nom}$ p.u. | $\Psi_{D,nom}$ p.u. | $\Psi_{Q,nom}$ p.u. | $E_{fd,nom}$ p.u. |
|---|---|---|---|---|
| **O.P. I** | 1.44005 | 1.0062 | -0.3160 | 1.6123 |
| **O.P. II** | 1.4737 | 1.0001 | -0.3645 | 1.7720 |
| **O.P. III** | 1.4802 | 1.0508 | -0.1740 | 1.6069 |
| **O.P. IV** | 1.0107 | 0.8842 | -0.2911 | 0.4734 |

**Table 2.** Some operating points of the hydrogenerator system

Defining the following vectors

$$\mathbf{x} = \begin{bmatrix} \Delta\delta & \Delta\omega & \Delta\Psi_f & \Delta\Psi_D & \Delta\Psi_Q & \Delta E_{fd} \end{bmatrix}^T$$

$$\mathbf{u} = \Delta V_{ref} \quad , \quad \mathbf{y} = \begin{bmatrix} \Delta\delta & \Delta\omega \end{bmatrix}^T$$

after linearization of the nonlinear equations (40)-(45), with respect to a nominal operating point of the system, we obtain a linear state space model for the hy-drogenerator. Note that, $\Delta$ defines incremental changes of the variables, involved in the description, around the particular operating point chosen for the linearization procedure. Some of the operating points of the hydrogenerator unit are given in Table 2, wherein $P_t$ and $Q_t$ denote the active and the reactive generator power.

In this study we consider linearization about operating point II. After linearization we obtain a linear state-space model of the hydrogenerator, of the form (1), with

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -72.654 & 0 & -19.926 & -46.354 & -41.242 & 0 \\ -0.066 & 0 & -0.641 & 0.493 & 0.003 & 0.228 \\ -3.586 & 0 & 11.494 & -19.56 & 0.13 & 0 \\ -2.051 & 0 & -0.032 & -0.074 & -7.867 & 0 \\ 125.886 & 0 & -176.154 & -409.789 & 264.883 & -20 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1000 \end{bmatrix}^T,$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

As it can be easily checked the above linear state space model is unstable, since matrix **A** has two unstable complex eigenvalues at $\lambda_{1,2} = 0.0931 \pm j7.7898$. Note also that the states $\Psi_f$, $\Psi_D$ and $\Psi_Q$ are not measurable quantities. Therefore, the traditional LQ optimal control strategies of [1]-[8] are not applicable in the present case.

In the sequel, a simulation study of the proposed multirate LQ optimal regulation technique on the basis of the above model is performed. The control objective is to minimize the performance index (10) with $\mathbf{Q} = \mathrm{diag}\{0.01, 0.001\}$ and $\mathbf{\Gamma}$ = 1, using a TPMRC of the form (7), (9). To this end, we discretize the open loop system with sampling period $T_0$=1 sec. As it can be easily checked, a set of observability indices of the pair (**A**,**C**) is given by $(n_1, n_2) = (1, 5)$. Then, we can choose the output multiplicities of the sampling as $M_1 = 2$ and $M_2 = 6$. The input multiplicity of the sampling is selected as $N_0$=6. Application of the proposed technique yields

$$\mathbf{K} = 10^{-4} \times \begin{bmatrix} -112 & -7 & -4 & -38 & 24 & -34 & -156 & 59 \\ -36 & 16 & -1 & -9 & -12 & 53 & -92 & -136 \\ -42 & -4 & -1 & -15 & -517 & -11 & -19 & -56 \\ -10 & -2 & 0 & -4 & 3 & -7 & -11 & 16 \\ -6 & -3 & 0 & -3 & 4 & -11 & -3 & 25 \\ -7 & 1 & 0 & -2 & 0 & 1 & -11 & -5 \end{bmatrix}$$

$$\mathbf{L}_u = 10^{-4} \times \begin{bmatrix} -282 & -2560 & 3474 & 235 & -1605 & 139 \\ 2104 & -436 & 2788 & -2847 & -887 & -423 \\ -294 & -999 & 1148 & 352 & -578 & 93 \\ -172 & -255 & 211 & 208 & -116 & 44 \\ -339 & -212 & 7 & 414 & -33 & 76 \\ 104 & -121 & 259 & -129 & -115 & -17 \end{bmatrix}$$

The closed-loop system has $N_0 = 6$ eigenvalues at the origin and the eigenvalues of $\mathbf{\Phi} - \mathbf{B}_{N_0}\mathbf{KH}$, which have the following values

$$\lambda_{1,2} = -0.0438 \pm j0.6693 \ ,$$

$$\lambda_{3,} = 0 \ , \ \lambda_4 = 0.0005 \ ,$$

$$\lambda_5 = 0.0011, \lambda_6 = 0.3882$$

and obviously lie inside the unit circle. Moreover, matrix $\mathbf{L}_u$ has the eigenvalues $\lambda_{1,2} = 0.0295 \pm j0.2636$, $\lambda_3 = 0.0001$ and $\lambda_{4,5,6} = 0$. That is strong stabilization is achieved. Finally, the optimal average cost is $J_{opt} = 60.8365$. Note that, the optimal average cost in the case where a continuous-time static state feedback LQ optimal regulator is to be designed has the value $\hat{J} = 29.7225$.

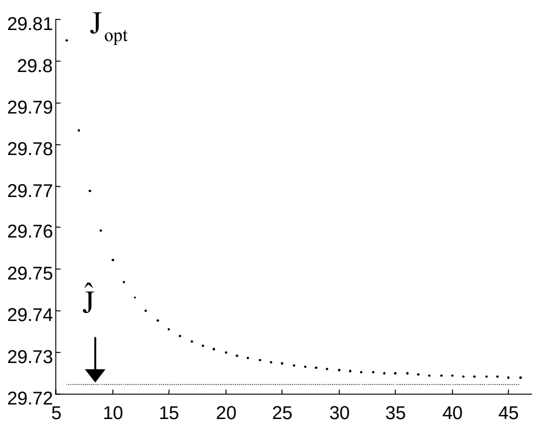


**Figure 2.** $J_{opt}$ and $\hat{J}$ versus $N_0$ for $T_0 = 0.2$ .

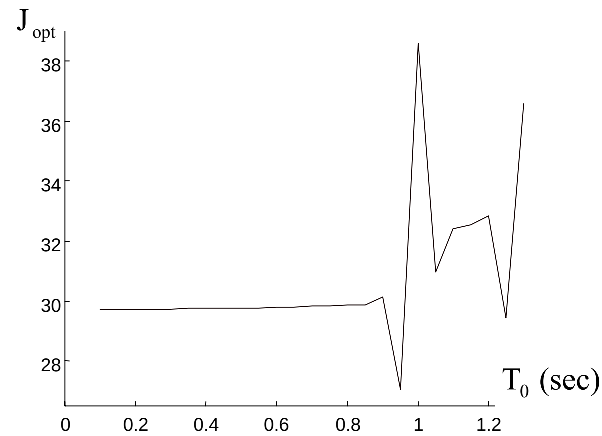


**Figure 3.** $J_{opt}$ versus $T_0$ .

Alternatively, with a sampling period $T_0 = 0.2$ sec and the same values for the other design parameters, we obtain

$$\mathbf{K} = 10^{-4} \times \begin{bmatrix} -40 & -54 & 500 & -2556 & 3748 & 153 & 4189 & 2361 \\ -118 & -126 & 253 & -1244 & 1677 & 198 & 1787 & 832 \\ 111 & 114 & -99 & 458 & -517 & -157 & 486 & -91 \\ -44 & -45 & 17 & -65 & 30 & 58 & 4 & -79 \\ 22 & 22 & -9 & 38 & -24 & -29 & 8 & 33 \\ 9 & 9 & -4 & 16 & -10 & -12 & 3 & 14 \end{bmatrix}$$

$$\mathbf{L}_u = 10^{-4} \times \begin{bmatrix} 382 & 1116 & 1023 & -1238 & -440 & 6 \\ 211 & 724 & 193 & -572 & -188 & -7 \\ -94 & -392 & 142 & 186 & 53 & 9 \\ 21 & 108 & -110 & -16 & 0 & -4 \\ -11 & -56 & 52 & 11 & 1 & 2 \\ -5 & -23 & 22 & 5 & 1 & 1 \end{bmatrix}$$

The eigenvalues of the closed-loop system are $N_0 = 6$ eigenvalues at the origin and

$$\lambda_{1,2} = -0.0002 \pm j0.7998 \ ,$$

$$\lambda_{3,4} = 0.0678 \pm j0.1877 \ \lambda_5 = 0.0060, \lambda_6 = 0.2797$$

which lie inside the unit circle. In this case, matrix $\mathbf{L}_u$ has the eigenvalues $\lambda_{1,2} = 0.0617 \pm j0.0249$ and $\lambda_{3,4,5,6} = 0$ and strong stabilization is once again achieved. Finally, the optimal average cost is $J_{opt} = 29.8050$.

In Figure 2, the variation of the optimal average cost with respect to $N_0$, is depicted for the case where $T_0 = 0.2$. Finally, in Figure 3, the variation of the optimal average cost with respect to $T_0$ is given.

>From the previous simulations, one can readily conclude that the proposed multirate method can be easily implemented in digital environment and that it is more effective in reconstructing the action of the sate feedback than estimator based techniques. The proposed technique provides a optimal average cost close to that provided by the continuous-time state feedback LQ regulator, particularly in cases where the input multiplicity of the sampling is large or the sampling period is fast enough. Finally, the proposed technique provides a smaller optimal average cost as compared to the singlerate control case (i.e. the case where $N_0$=1).

## 4 Conclusion

An optimal control strategy based on Two-Point-Multirate Controllers has been used in this paper in order to design a desirable excitation controller of a unstable hydrogenerator system, for the purpose of enhancing its dynamic stability characteristics. The proposed method offers acceptable closed loop response as well as more design flexibility (particularly in cases where the system states are not measurable), and its performance is at least comparable to known LQ optimal regulation methods.

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*References:*

[1]. M.Ramamoorty and M.Arumugam, "Design of optimal regulators for synchronous machines", *IEEE Trans. Power App. Syst.,* Vol. PAS-90, 1971, pp. 269-277.

[2]. M.Arumugam and M.Ramamoorty, "Optimal controllers for divided winding rotor synchronous machine", *Int. J. Control,* Vol. 16, 1972, pp. 437-450.

[3]. B.S.Habibullah and Y.N.Yu, "Phisically realizable wide power range optimal controllers for power systems", *IEEE Trans. Power App. Syst.,* Vol. PAS-93, 1974, pp. 1498-1506.

[4]. B.S.Habibullah, "Optimal governor control of synchronous machine", *IEEE Trans. Autom. Control,* Vol. AC-26, 1981, pp. 391-395.

[5]. A.B.R.Kumar and E.F.Richards, "An optimal control law by eigenvalue assignment for improved dynamic stability in power systems", *IEEE Trans. Power App. Syst.,* Vol. PAS-101, 1982, pp. 1570-1577.

[6]. I.A.Saleh, M.K.El-Sherbiny and A.A.M. El-Gaafary, "Optimal design of an overall controller of saturated synchronous machine", *IEEE Trans. Power App. Syst.,* Vol. PAS-102, 1983, pp. 1651-1657.

[7]. W.-C. Chan and Y.-Y. Hsu, "An optimal variable structure stabilizer for power system stabilization", *IEEE Trans. Power App. Syst.,* Vol. PAS-102, 1983, pp. 1738-1746.

[8]. Y.-C. Lee and C.-J. Wu, "Damping of power system oscillations with outut feedback and strip eigenvalue assignment", *IEEE Trans. Power Syst.,* Vol. PS-10, 1995, pp. 1620-1626.

[9]. H.M.Al-Rahmani and G.F.Franklin, "A new optimal multirate control of linear periodic and time-invariant systems", *IEEE Trans. Autom. Control,* Vol. AC-35, 1990, pp. 406-414.

[10]. K.G.Arvanitis, "An indirect model reference adaptive controller based on the multirate sampling of the plant output", *Int. J. Adapt. Control Sign. Proc.*, Vol. 10, 1996, pp. 673-705.

[11]. R.H.Park, "Two reaction theory of synchronous machines: General method of analysis", Part I, *Trans. AIEE,* Vol. 48, 1929, p. 716.

[12]. G.Shackshaft,"A general purpose turbo-alternator model", *Proc. IEE,* Vol. 110, 1963, pp. 703-713.

[13]. C.F.Van Loan, "Computing integrals involving the matrix exponential", *IEEE Trans. Autom. Control,* Vol. AC-23, 1978, pp. 395-404.

[14]. T.Hagiwara and M.Araki, "Design of a stable state feedback controller based on the multirate sampling of the plant output", *IEEE Trans. Autom. Control*, Vol. AC-33, 1988 pp. 812-819.

[15]. K.Ogata, *Discrete Time Control Systems,* Prentice Hall, Englewood Cliffs, New Jersey, 1987.