On the Localization of the Prime Mover Torque Perturbations in Synchronous Electric Machines via Multirate Digital Controllers

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Abstract: - In this paper, a new H^∞-control technique is proposed, in order to attenuate changes in the prime mover torque which degrade the performance of synchronous electric machines. The proposed technique relies on multirate-output controllers. Its main feature consists in reducing the original problem, to an associate discrete H^∞-control problem for which a fictitious static state feedback controller is to be designed. The effectiveness of the method is demonstrated by several simulation results.

Key-Words: - H^∞-control, disturbance attenuation, multirate digital control, electric machines

1 Introduction

After its original formulation in [1], the H^∞-optimization problem has drawn great attention [2]-[13]. Several approaches have been reported in order to solve this important design problem for a variety of systems types. In particular, the H^∞-control problem for discrete-time and sampled-data singlerate and multirate systems has successfully been treated in the past in [7]-[13]. Generally speaking, when the state vector is not available for feedback, the H^∞-control problem is usually solved, in both the continuous and the discrete-time cases, by the use of dynamic measurement feedback. This approach, however, requires the solution of two coupled algebraic Riccati equations, which is, in general, a hard task. On the basis of these two Riccati equations, it is plausible to compute a dynamic controller that achieves the desired design requirments [10]. Nevertheless, a complete characterization of all controllers satisfying the design requirements is not as yet available.

Recently, a new technique is presented, for the solution of the H^∞-disturbance attenuation problem in [13]. This technique is based on multirate-output controllers (MROCs). MROCs contain a multirate sampling mechanism with different sampling period to each system measured output. The technique proposed in [13], in order to solve the sampled-data H^∞-disturbance attenuation problem relies mainly on the reduction, under appropriate conditions, of the original H^∞-disturbance attenuation problem, to an associated discrete H^∞-control problem for which a fictitious static state feedback controller is to be designed, eventhough state variables are not available for feedback. This fact has beneficial impact on the theoretical and the numerical complexity of the problem, since using the technique reported in [13], only one algebraic Riccati equation is to be solved, as compared to two algebraic Riccati equations needed by other well known H^∞-control techniques. Another key feature of the approach proposed in [13] is the ability of choosing, under appropriate conditions, the dynamics of the MROC arbitrarily. Thus, here, strong stabilization is assured without imposing the parity interlacing property requirement in the system under control.

In this respect, in the present paper, the technique reported in [13], is used to treat the H^∞-disturbance attenuation problem in synchronous electric machines, whose application in many engineering areas, such as energy production, robotics, etc., is well known. Our design objective here, is to reduce the effect of the load disturbance (change in the prime mover torque) on the controlled outputs of the machine, which are the torque angle, the machine speed, the excitation voltage, the prime mover torque, the prime mover valve setting and the generated field voltage, to an acceptable level. In particular, in the present paper, we consider both unsaturated and saturated synchronous machines [14]. Various simulations of the proposed technique are performed and their results show the effectiveness of the proposed method. In particular, it is shown in the paper that the less disturbance attenuation level we want to achieve (and thus the less the effect of the disturbances on the controlled system outputs), the larger must be the value of the control law. Moreover, it is shown that the minimum achievable disturbance attenuation level is increased whereas the control effort is decreased, if the fundamental sampling period is increased.

2 H^{∞}-Control using MROCs

Consider the controllable and observable linear state-space system of the form

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)+\mathbf{D}\mathbf{q}(t),\ \mathbf{x}(0)=\mathbf{0} \tag{1a}$$

$$y_m(t)=\mathbf{C}\mathbf{x}(t)+\mathbf{J}_1\mathbf{u}(t),\ y_c(t)=\mathbf{E}\mathbf{x}(t)+\mathbf{J}_2\mathbf{u}(t) \tag{1b}$$

where,

$\mathbf{x}(t)\in\mathbf{R}^n$, $\mathbf{u}(t)\in\mathbf{R}^m$, $\mathbf{q}(t)\in\mathbf{L}_2^d$, $\mathbf{y}_m(t)\in\mathbf{R}^{p_1}$, $\mathbf{y}_c(t)\in\mathbf{R}^{p_2}$ are the state, the input, the external disturbance, the measured output and the controlled output vectors, respectively. In (1), all matrices have real entries and appropriate dimensions.

The following definition is useful in the sequel.

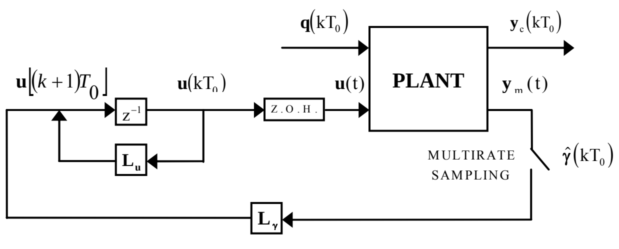

Fig. 1. Control of linear systems using MROCs

Definition. For an observable matrix pair $(\mathbf{A},\mathbf{C})$, with $\mathbf{C}^T=\begin{bmatrix}\mathbf{c}_1^T & \mathbf{c}_2^T & \cdots & \mathbf{c}_{p_1}^T\end{bmatrix}$ and $\mathbf{c}_i$, i=1, …,p_1, the *i*th row of the matrix $\mathbf{C}$, a collection of p_1 integers $\{n_1, n_2, \cdots, n_{p_1}\}$ is called an *observability index vector* of the pair $(\mathbf{A},\mathbf{C})$, if the following relationships simultaneously hold

$$\sum_{i=1}^{p_1} n_i = n\ ,$$

$$\mathrm{rank}\begin{bmatrix}\mathbf{c}_1^T & \cdots & (\mathbf{A}^T)^{n_1-1}\mathbf{c}_1^T & \cdots & \mathbf{c}_{p_1}^T & \cdots & (\mathbf{A}^T)^{n_{p_1}-1}\mathbf{c}_{p_1}^T\end{bmatrix}=n$$

To system (1) we next apply the multirate sampling mechanism depicted in Figure 1. Assuming that all samplers start simultaneously at t=0, a sampler and a zero-order hold with period T_0 is connected to each plant input $u_i(t)$, i=1,2,…,m, such that

$$\mathbf{u}(t)=\mathbf{u}(kT_0)\ ,\ \ t\in[kT_0,(k+1)T_0)$$

while the ith disturbance $q_i(t)$, i=1,…,d, and the ith controlled output $y_{c,i}(t)$, i=1,…,p_2, are detected at time kT_0, such that for $t\in[kT_0,(k+1)T_0)$

$$\mathbf{q}(t)=\mathbf{q}(kT_0)\ ,\ \mathbf{y}_c(kT_0)=\mathbf{E}\mathbf{x}(kT_0)+\mathbf{J}_2(kT_0)$$

The ith measured output $y_{m,i}(t)$, i=1,…,p_1, is detected at every T_i, such that for $\mu=0,...,N_i-1$

$$y_{m,i}(kT_0+\mu T_i)=\mathbf{c}_i\mathbf{x}(kT_0+\mu T_i)+(\mathbf{J}_1)_i\,\mathbf{u}(kT_0)$$

where, $(\mathbf{J}_2)_i$ is the ith row of the matrix $\mathbf{J}_2$. Here, $N_i\in\mathbf{Z}^+$ are the output multiplicities of the sampling and $T_i\in\mathbf{R}^+$ are the output sampling periods having rational ratio, i.e. $T_i=T_0/N_i$, i=1,…,p_1.

The sampled values of the plant measured outputs obtained over $[kT_0,(k+1)T_0)$, are stored in the N^*-dimensional column vector

$$\hat{\gamma}(kT_0)=\big[y_{m,1}(kT_0)\ \cdots\ y_{m,1}(kT_0+(N_1-1)T_1)\ \cdots\ y_{m,p_1}(kT_0)\ \cdots\ y_{m,p_1}[kT_0+(N_{p_1}-1)T_{p_1}]\big]^T$$

($N^*=\sum_{i=1}^{p_1}N_i$), that is used in the MROC of the form

$$\mathbf{u}[(k+1)T_0]=\mathbf{L}_\mathbf{u}\mathbf{u}(kT_0)-\mathbf{L}_\gamma\hat{\gamma}(kT_0) \tag{2}$$

where $\mathbf{L}_\mathbf{u}\in\mathbf{R}^{m\times m}$, $\mathbf{L}_\gamma\in\mathbf{R}^{m\times N^*}$. In the sequel, let

$$\mathbf{H}=\begin{bmatrix}\mathbf{H}_1\\ \vdots\\ \mathbf{H}_{p_1}\end{bmatrix},\ \mathbf{\Theta}_u=\begin{bmatrix}(\mathbf{\Theta}_\mathbf{u})_1\\ \vdots\\ (\mathbf{\Theta}_\mathbf{u})_{p_1}\end{bmatrix},\ \mathbf{\Theta}_\mathbf{q}=\begin{bmatrix}(\mathbf{\Theta}_\mathbf{q})_1\\ \vdots\\ (\mathbf{\Theta}_\mathbf{q})_{p_1}\end{bmatrix}$$

$$\mathbf{H}_i=\begin{bmatrix}\mathbf{c}_i\left(\hat{\mathbf{A}}_i^{N_i}\right)^{-1}\\ \vdots\\ \mathbf{c}_i\hat{\mathbf{A}}_i^{-1}\end{bmatrix},\ (\mathbf{\Theta}_\mathbf{u})_i=\begin{bmatrix}\mathbf{c}_i\hat{\mathbf{B}}_{i,N_i}+(\mathbf{J}_1)_i\\ \vdots\\ \mathbf{c}_i\hat{\mathbf{B}}_{i,1}+(\mathbf{J}_1)_i\end{bmatrix},$$

$$\left(\mathbf{\Theta_q}\right)_i = \begin{bmatrix} \mathbf{c}_i\hat{\mathbf{D}}_{i,N_i} \\ \vdots \\ \mathbf{c}_i\hat{\mathbf{D}}_{i,1} \end{bmatrix}$$

$$\hat{\mathbf{A}}_i = \exp\left(\mathbf{A}T_i\right), \left(\hat{\mathbf{B}}_{i,\mu}, \hat{\mathbf{D}}_{i,\mu}\right) = \int_0^{-\mu T_i} \exp(\mathbf{A}\lambda)(\mathbf{B},\mathbf{D})d\lambda$$

for $i = 1,\dots,p_1$ and for $\mu = 1,\dots,N_i$. As it has been shown in [13], for an observable pair, under the assumption that

$$\text{rank}\begin{bmatrix} \mathbf{A} & \mathbf{D} \\ \mathbf{C} & \mathbf{0}_{p_1\times d} \end{bmatrix} = n + d \tag{3}$$

then matrix $\left[\mathbf{H} \quad \mathbf{\Theta_q}\right]$ has full column rank for almost every sampling period T_0, if

$$N_i \geq \sigma_i \quad , i=1,2,\dots,p_1$$

where, σ_i are positive integers which comprise an observability index vector of the matrix pair $\left(\mathbf{A}^*, \mathbf{C}^*\right)$, where

$$\mathbf{A}^* = \begin{bmatrix} \mathbf{A} & \mathbf{D} \\ \mathbf{0}_{d\times n} & \mathbf{0}_{d\times d} \end{bmatrix} \quad , \quad \mathbf{C}^* = \left[\mathbf{C} \quad \mathbf{0}_{p_1\times d}\right]$$

Moreover, if $\mathbf{L}_\gamma$ and $\mathbf{L_u}$ are chosen such that

$$\mathbf{L}_\gamma\left[\mathbf{H} \quad \mathbf{\Theta_q}\right] = \left[\mathbf{F} \quad \mathbf{0}_{m\times d}\right], \mathbf{L_u} = \mathbf{L}_\gamma \mathbf{\Theta}_\zeta \tag{4}$$

then, for almost every sampling period T_0, the control law of the form (2), is equivalent to a static state feedback control law of the form

$$\mathbf{u}\left(kT_0\right) = -\mathbf{F}\mathbf{x}\left(kT_0\right) \ , \ k \geq 1 \tag{5}$$

Finally, as it has also been shown in [13], when $\mathbf{L_u}$ is desired to take a prespecified value $\mathbf{L}_{\mathbf{u},sp}$, $\mathbf{J}_1 = \mathbf{0}$ and

$$\text{rank}\begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{D} \\ \mathbf{C} & \mathbf{0}_{p_1\times m} & \mathbf{0}_{p_1\times d} \end{bmatrix} = n + m + d \tag{6}$$

then, matrix $\left[\mathbf{H} \quad \mathbf{\Theta_u} \quad \mathbf{\Theta_q}\right]$ has full column rank if

$$N_i \geq \beta_i \quad , i=1,2,\dots,p_1$$

where β_i comprise an observability index vector of the observable matrix pair $\left(\widetilde{\mathbf{A}}, \widetilde{\mathbf{C}}\right)$, and where

$$\widetilde{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{D} \\ \mathbf{0}_{m\times n} & \mathbf{0}_{m\times m} & \mathbf{0}_{m\times d} \\ \mathbf{0}_{d\times n} & \mathbf{0}_{d\times m} & \mathbf{0}_{d\times d} \end{bmatrix}, \widetilde{\mathbf{C}} = \left[\mathbf{C} \quad \mathbf{0}_{p_1\times m} \quad \mathbf{0}_{p_1\times d}\right]$$

In this particular case, (2) is equivalent to (5), if $\mathbf{L}_\gamma$ is chosen such that

$$\mathbf{L}_\gamma\left[\mathbf{H} \quad \mathbf{\Theta_u} \quad \mathbf{\Theta_q}\right] = \left[\mathbf{F} \quad \mathbf{L}_{\mathbf{u},sp} \quad \mathbf{0}_{m\times d}\right] \tag{7}$$

The H^∞-disturbance attenuation problem treated in this paper, is as follows: Find a MROC of the form (2), which when applied to system (1), asymptotically stabilizes the closed-loop system and simultaneously achieves the following design requirement

$$\left\|\mathbf{T}_{\mathbf{qy}_c}(z)\right\|_\infty \leq \gamma \tag{8}$$

for a given $\gamma \in \mathbf{R}^+$, where $\left\|\mathbf{T}_{\mathbf{qy}_c}(z)\right\|_\infty$ is the H^∞-norm of the proper stable discrete transfer function $\mathbf{T}_{\mathbf{qy}_c}(z)$, from sampled-data external disturbances $\mathbf{q}\left(kT_0\right) \in \ell_2^d$ to sampled-data controlled outputs $\mathbf{y}_c\left(kT_0\right)$, defined by

$$\left\|\mathbf{T}_{\mathbf{qy}_c}(z)\right\|_\infty = \sup_{\mathbf{q}(kT_0)\in l_2} \frac{\left\|\mathbf{y}_c\left(kT_0\right)\right\|_2}{\left\|\mathbf{q}\left(kT_0\right)\right\|_2}$$

$$= \sup_{\theta\in[0,2\pi]} \sigma_{\max}\left[\mathbf{T}_{\mathbf{qy}_c}\left(e^{j\theta}\right)\right] = \sup_{|z|=1} \sigma_{\max}\left[\mathbf{T}_{\mathbf{qy}_c}(z)\right]$$

where, $\sigma_{\max}\left[\mathbf{T}_{\mathbf{qy}_c}(z)\right]$ is the maximum singular value of $\mathbf{T}_{\mathbf{qy}_c}(z)$, and where use was made of the standard definition of the ℓ_2-norm of a discrete signal $\mathbf{s}(kT_0)$

$$\left\|\mathbf{s}\left(kT_0\right)\right\|_2^2 = \sum_{k=0}^{\infty} \mathbf{s}^T\left(kT_0\right)\mathbf{s}\left(kT_0\right)$$

Our attention will now be focused on the solution of the above H^∞-control problem. To this end, the following assumptions on system (1) are made:

Assumptions:

a) The matrix triplets $\left(\mathbf{A}, \mathbf{B}, \mathbf{C}\right)$ and $\left(\mathbf{A}, \mathbf{D}, \mathbf{E}\right)$ are stabilizable and detectable.

b) Relation (3) and/or (6) hold.

c) $\mathbf{J}_2^T\left[\mathbf{E} \quad \mathbf{J}_2\right] = \left[\mathbf{0}_{m\times n} \quad \mathbf{I}_{m\times m}\right]$

d) There is a sampling period T_0, such that the discrete-time system

$$\mathbf{x}\left[(k+1)T_0\right] = \mathbf{\Phi}\mathbf{x}\left(kT_0\right) + \hat{\mathbf{B}}\mathbf{u}\left(kT_0\right) + \hat{\mathbf{D}}\mathbf{q}\left(kT_0\right)$$

$$\mathbf{y}_c\left(kT_0\right) = \mathbf{E}\mathbf{x}\left(kT_0\right) + \mathbf{J}_2\mathbf{u}\left(kT_0\right)$$

where

$$\mathbf{\Phi} = \exp\left(\mathbf{A}T_0\right), \left(\hat{\mathbf{B}}, \hat{\mathbf{D}}\right) = \int_0^{T_0} \exp(\mathbf{A}\lambda)(\mathbf{B},\mathbf{D})d\lambda$$

is stabilizable and observable and does not have invariant zeros on the unit circle.

>From the above analysis, it becomes clear that the procedure for H^∞-disturbance attenuation using MROCs, essentially consists in finding a fictitious state matrix **F**, which equivalently solves the problem and then, either determining the MROC pair $(\mathbf{L}_\gamma, \mathbf{L}_u)$ by (4) or choosing a desired $\mathbf{L}_\mathbf{u}$ and determining $\mathbf{L}_\gamma$ by (7). As it has been shown in [8], matrix **F** has the form

$$\mathbf{F} = \left(\mathbf{I} + \hat{\mathbf{B}}^T\mathbf{P}\hat{\mathbf{B}}\right)^{-1}\hat{\mathbf{B}}^T\mathbf{P}\boldsymbol{\Phi}$$

where **P** is an appropriate solution of the Riccati equation

$$\mathbf{P} = \mathbf{E}^T\mathbf{E} + \boldsymbol{\Phi}^T\mathbf{P}\boldsymbol{\Phi} - \boldsymbol{\Phi}^T\mathbf{P}\hat{\mathbf{B}}\left(\mathbf{I} + \hat{\mathbf{B}}^T\mathbf{P}\hat{\mathbf{B}}\right)^{-1}\hat{\mathbf{B}}\mathbf{P}\boldsymbol{\Phi}$$
$$+\mathbf{P}\hat{\mathbf{D}}_\gamma\left(\mathbf{I} + \hat{\mathbf{D}}_\gamma^T\mathbf{P}\hat{\mathbf{D}}_\gamma\right)\hat{\mathbf{D}}_\gamma^T\mathbf{P} \quad , \quad \hat{\mathbf{D}}_\gamma = \gamma^{-1}\hat{\mathbf{D}} \qquad (9)$$

Once matrix **F** is obtained, the MROC matrices $\mathbf{L}_\gamma$ and $\mathbf{L}_\mathbf{u}$ (in the case where $\mathbf{L}_\mathbf{u}$ is free), can be computed according to the following relations

$$\mathbf{L}_\gamma = [\mathbf{F} \quad \mathbf{0}_{m\times d}]\tilde{\mathbf{H}} + \boldsymbol{\Lambda}\left(\mathbf{I}_{N^*\times N^*} - [\mathbf{H} \quad \boldsymbol{\Theta}_\mathbf{q}]\tilde{\mathbf{H}}\right)$$
$$\mathbf{L}_\mathbf{u} = \left\{[\mathbf{F} \quad \mathbf{0}_{m\times d}]\tilde{\mathbf{H}} + \boldsymbol{\Lambda}\left(\mathbf{I}_{N^*\times N^*} - [\mathbf{H} \quad \boldsymbol{\Theta}_\mathbf{q}]\tilde{\mathbf{H}}\right)\right\}\boldsymbol{\Theta}_\mathbf{u}$$

where $\tilde{\mathbf{H}}[\mathbf{H} \quad \boldsymbol{\Theta}_\mathbf{q}] = \mathbf{I}$ and $\boldsymbol{\Lambda} \in \mathbf{R}^{m\times N^*}$ is an arbitrary specified matrix. In the case where $\mathbf{L}_u = \mathbf{L}_{u,sp}$, we have

$$\mathbf{L}_\gamma = [\mathbf{F} \quad \mathbf{L}_{\mathbf{u},sp} \quad \mathbf{0}_{m\times d}]\hat{\mathbf{H}} + \boldsymbol{\Sigma}\left(\mathbf{I}_{N^*\times N^*} - [\mathbf{H} \quad \boldsymbol{\Theta}_\mathbf{u} \quad \boldsymbol{\Theta}_\mathbf{q}]\hat{\mathbf{H}}\right)$$

where $\hat{\mathbf{H}}[\mathbf{H} \quad \boldsymbol{\Theta}_\mathbf{u} \quad \boldsymbol{\Theta}_\mathbf{q}] = \mathbf{I}$ and $\boldsymbol{\Sigma} \in \mathbf{R}^{m\times N^*}$ is arbitrary.

3 Synchronous Machine Model

In the present paper, we consider both unsaturated and saturated synchronous electric machines that supply power through a step-up transformer and a transmission line to an infinite grid. The model describing the system has been discussed before in [14], wherein a linearized state space model of the form (1) for the synchronous machine is obtained. In particular, assuming that the resistances of the system are neglected, we obtain

$$\mathbf{x} = [\delta \quad \omega \quad E_q \quad T_m \quad x_E \quad E_{fd}]^T ,$$
$$\mathbf{u} = [u_G \quad u_{VR}]^T \;,\; \mathbf{q} = T_L \;,\; \mathbf{y}_m = [\delta \quad \omega \quad E_{fd}]^T \;,\; \mathbf{y}_c = \mathbf{x}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 100\pi & 0 & 0 & 0 & 0 \\ -\frac{K_1}{M} & -\frac{\hat{D}}{M} & -\frac{K_2}{M} & \frac{1}{M} & 0 & 0 \\ -\frac{K_3K_4}{M} & 0 & -\frac{1}{T} & 0 & 0 & \frac{GK_3}{T} \\ 0 & 0 & 0 & -\frac{1}{T_\tau} & \frac{1}{T_\tau} & 0 \\ 0 & -\frac{1}{RT_G} & 0 & 0 & -\frac{1}{T_G} & 0 \\ -\frac{K_5}{T} & 0 & -\frac{K_6}{T} & 0 & 0 & -\frac{1}{T} \end{bmatrix}$$

$$, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0}_{2\times 4} & \begin{matrix} 0 & \frac{1}{T} \\ \frac{1}{T_G} & 0 \end{matrix} \end{bmatrix}^T ,$$

$$\mathbf{D} = \begin{bmatrix} 0 & -\frac{1}{M} & \mathbf{0}_{1\times 4} \end{bmatrix}^T$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_6 \end{bmatrix} \;,\; \mathbf{E} = \mathbf{I}_{6\times 6} \;,\; \mathbf{J}_1 = \mathbf{0}_{3\times 2} \;,\; \mathbf{J}_2 = \mathbf{0}_{6\times 2}$$

where, δ is the torque angle, ω is the machine speed, E_q is the excitation voltage, T_m is the prime mover torque, x_E is the prime mover valve setting, E_{fd} is the generated field voltage, u_G is the governor input, u_{VR} is the voltage regulator input, T_L is the a perturbation of the prime mover torque (load disturbance), K_i, i=1,…,6, are constant coefficients, M is the inertia coefficient, $\hat{D}$ is the damping coefficient, T is the thyristor time constant, G is the thyristor gain, T_τ is the turbine time constant, R is the speed regulation due to governor action and T_G is the time constant of the speed governing mechanism. Note that K_i, i=1,2,4,5,6 have the same values for both saturated and unsaturated machine, while K_3 is given by

$$K_3 = \frac{x_e + x_d'}{x_e + x_d} \;,\; K_3 = \frac{x_e + x_d'}{m(x_e + x_d') + (x_d - x_d')}$$

for the cases of the unsaturated and the saturated machine, respectively, where, x_e is the series reactance of the transmission system, x_d is the d-axis synchronous reactance, x_d' is the d-axis transient reactance and m is the d-axis saturation factor.

4 Simulation Study

In this Section, the multirate H^∞-disturbance attenuation technique proposed in Section 2 is applied to several specific machine models.

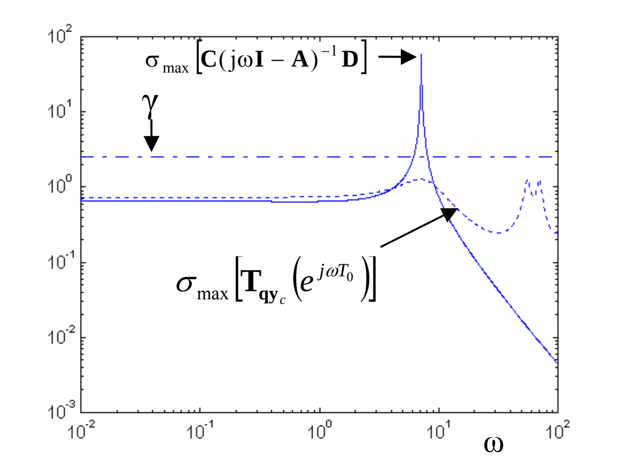

Fig.2. The maximum singular value of $\mathbf{T}_{\mathbf{qy}_c}(z)$ over ω, for the unsaturated machine and for γ=2.5.

As a first case, we next consider the unsaturated machine model with K_1=1.3307, K_2=1.3843, K_3= 0.4113, K_4= 1.0839, K_5=-0.0984, K_6=0.4724, M=10 sec, $\hat{D}$=3, T=0.05 sec, G=100, T_τ=1 sec, R= 0.04 p.u., T_G=0.1 sec. The proposed MROC based H^∞-control technique can be applied to the above model, since the conditions and assumptions listed in Section 2 are satisfied. In the sequel, the proposed technique is simulated for several values of γ and T_0. Consider, first, the case where γ=2.5 and T_0=0.1 sec. An observability index vector of ($\mathbf{A}^*$, $\mathbf{C}^*$), is $\{\sigma_1,\sigma_2,\sigma_3\}=\{4,1,2\}$. Setting $N_i=\sigma_i$, i=1,2,3, we obtain

$$\mathbf{L}_\gamma = 10^4\times\begin{bmatrix} -0.9758 & 1.6477 & -0.8834 & 0.2094 & -4.0333 & -0.0084 & 0.1504 \\ 2.0921 & -3.5069 & 1.8534 & -0.4339 & 8.7313 & 0.0154 & -0.3247 \end{bmatrix}$$

$$\mathbf{L}_u = \begin{bmatrix} 69.5047 & 0.1737 \\ -151.4299 & -0.4691 \end{bmatrix},$$

$$\sigma_{max}(\mathbf{L}_\gamma) = 1.0836\times 10^5$$

In Figure 2, the maximum singular value of $\mathbf{T}_{\mathbf{qy}_c}(z)$ is depicted, as a function of the frequency ω. Clearly, the design requirement $\left\|\mathbf{T}_{\mathbf{qy}_c}(z)\right\|_\infty \le 2.5$, is satisfied. Moreover, as it can be easily checked the poles of the closed loop system, lie inside the unit circle. Therefore, the requirement for the stability of the closed-loop system is also satisfied.

Consider now the case where γ=1 and all the other parameters are as above. Then, we obtain

$$\mathbf{L}_\gamma = 10^4\times\begin{bmatrix} -1.7974 & 3.0345 & -1.6264 & 03854 & -7.4302 & -0.0155 & 0.2771 \\ 2.2701 & -3.8072 & 2.0142 & -0.4720 & 9.4687 & 0.0169 & -0.3522 \end{bmatrix}$$

$$\mathbf{L}_u = \begin{bmatrix} 127.9928 & 0.3213 \\ -164.1466 & -0.5026 \end{bmatrix},$$

$$\sigma_{max}(\mathbf{L}_\gamma) = 1.3573\times 10^5$$

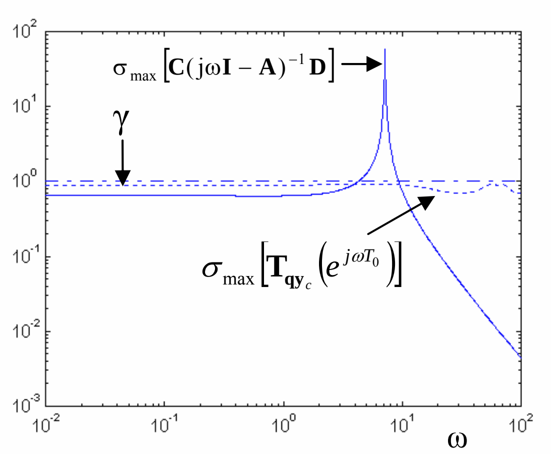

Fig.3. The maximum singular value of $\mathbf{T}_{\mathbf{qy}_c}(z)$ over ω, for the unsaturated machine and for γ=1.

In Figure 3, the maximum singular value of $\mathbf{T}_{\mathbf{qy}_c}(z)$ is given as a function of the frequency ω. Clearly, the design requirement $\left\|\mathbf{T}_{\mathbf{qy}_c}(z)\right\|_\infty \le 1$, is satisfied, and the closed-system is stable.

Note that, in the case of unsaturated machine the H^∞-norm of the open-loop system transfer function between disturbances and controlled outputs has the value $\left\|\mathbf{C}(j\omega\mathbf{I}-\mathbf{A})^{-1}\mathbf{D}\right\|_\infty = 58.4513$, while the minimum achievable disturbance attenuation level is $\gamma_{inf} = 0.9252$.

Consider now, as a second case, a saturated machine with d-axis saturation factor m=1.9. In this case, K_3=0.3002, and all the other parameters are as in the case of the unsaturated machine. Let γ=1 and T_0=0.1 sec. Since an observability index vector of ($\mathbf{A}^*,\mathbf{C}^*$), is $\{\sigma_1,\sigma_2,\sigma_3\}=\{4,1,2\}$, if we set $N_i=\sigma_i$, i=1,2,3, then, we obtain

$$\mathbf{L}_\gamma = 10^4\times\begin{bmatrix} -1.8072 & 3.0505 & -1.6361 & 03877 & -7.4715 & -0.0308 & 0.2768 \\ 2.2740 & -3.8128 & 2.0185 & -0.4730 & 9.4865 & 0.0427 & -0.3549 \end{bmatrix}$$

$$\mathbf{L}_u = \begin{bmatrix} 202.5799 & 0.3221 \\ -261.9697 & -0.5031 \end{bmatrix},$$
$$\sigma_{\max}(\mathbf{L}_\gamma) = 1.3616 \times 10^5$$

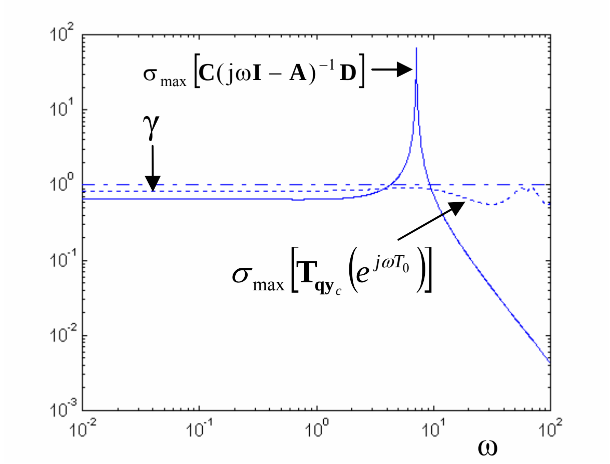

Fig.4. The maximum singular value of $\mathbf{T}_{\mathbf{qy}_c}(z)$ over ω, for a saturated machine with m=1.9 and for γ=1.

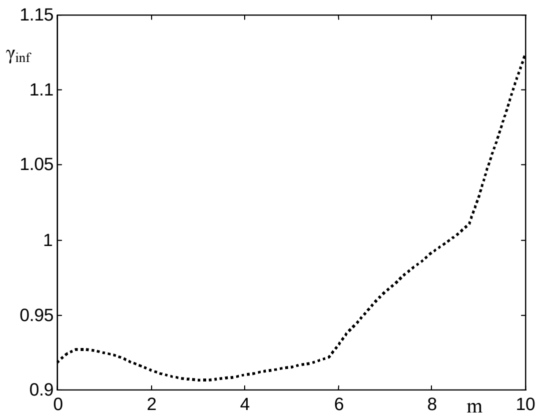

Fig. 5. Minimum achievable disturbance attenuation level $\gamma_{\inf}$ versus d-axis saturation factor m.

In Figure 4, the maximum singular value of $\mathbf{T}_{\mathbf{qy}_c}(z)$ is depicted, as a function of the frequency ω. Clearly, the design requirement $\left\|\mathbf{T}_{\mathbf{qy}_c}(z)\right\|_\infty \le 1$, is satisfied. Moreover, as it can be easily checked the closed loop system is once again stable. It is not difficult to check that, in this case, the H^∞-norm of the open-loop system transfer function between disturbances and controlled outputs has the value $\left\|\mathbf{C}(j\omega\mathbf{I}-\mathbf{A})^{-1}\mathbf{D}\right\|_\infty = 66.5855$, while the minimum achievable disturbance attenuation level is $\gamma_{\inf} =$ 0.9148, which, in the present case, is less than the minimum achievable disturbance attenuation level obtained in the case of the unsaturated machine. However, this is not true, in general, for all m, as it can be easily realized by Figure 5, where the variation of $\gamma_{\inf}$ with respect to the variation of the d-axis saturation factor m is depicted.

We repeat in the sequel the design of an H^∞ controller, for the case of the saturated machine with all the parameters as above, except for the sampling period T_0 which now has the value T_0=0.2 sec. In this case, as it can be easily checked the design requirement $\left\|\mathbf{T}_{\mathbf{qy}_c}(z)\right\|_\infty \le 1$ cannot be satisfied, since for the attenuation level γ=1, the Riccati equation (9) does not admit a positive definite solution. As it can be easily checked, in this case, the minimum achievable disturbance attenuation level is $\gamma_{\inf}$ =1.209 and the respective controller is given by

$$\mathbf{L}_\gamma = 10^3 \times \begin{bmatrix} -0.9752 & 1.6780 & -0.9316 & 0.2423 & -7.8200 & 0.1290 & -0.9269 \\ 0.7639 & -1.3110 & 0.7252 & -0.1885 & 6.1599 & -0.1015 & 0.7263 \end{bmatrix}$$
$$\mathbf{L}_u = \begin{bmatrix} -52.5846 & 0.3146 \\ 41.1423 & -0.3014 \end{bmatrix},$$
$$\sigma_{\max}(\mathbf{L}_\gamma) = 1.0396 \times 10^4$$

Finally, consider the case where T_0=0.02 sec. In this case, the minimum achievable disturbance attenuation level is $\gamma_{\inf}$ =0.8158. In this case, the controller achieving the disturbance attenuation level γ=1, has the value

$$\mathbf{L}_\gamma = 10^7 \times \begin{bmatrix} -0.9110 & 1.5008 & -0.7616 & 0.1717 & -0.7743 & 0.0002 & 0.0002 \\ 2.6999 & -4.4347 & 2.2358 & -0.5011 & 2.3030 & 0.0006 & -0.0005 \end{bmatrix}$$
$$\mathbf{L}_u = \begin{bmatrix} -21.8466 & 0.2441 \\ 87.0616 & -0.7777 \end{bmatrix},$$
$$\sigma_{\max}(\mathbf{L}_\gamma) = 6.4654 \times 10^7$$

>From the above two simulations, it can be easily recognized that the sampling period of the multirate mechanism is an important factor of the design procedure since both the achievable disturbance attenuation level and the control effort are crucially affected by T_0.

Remark 4.1. It can be easily checked that, in all simulations presented above, the eigenvalues of the respective matrices $\mathbf{L}_u$ lie outside the unit circle. Moreover, it is not difficult to check that, here, condition (6) cannot be satisfied for both cases of saturated and unsaturated machine models. That is, matrix $\mathbf{L}_u$ cannot be arbitrarily selected as a stable matrix and therefore, strong stabilization cannot be achieved, in our case, simultaneously to H^∞ disturbance attenuation.

5 Conclusion

An H^∞-control technique based on multirate-output controllers, has been proposed, in order to attenuate load disturbances which degrade the performance of synchronous electric machines. As it has been shown by various simulations, the less disturbance attenuation level we want to achieve, the larger must be the value of the control law. Moreover, it has been shown that the control effort in attenuating disturbances is decreased if the sampling period related to the multirate mechanism is increased and vice versa.

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