# Left-LUD SC ladder filter with compensation for finite amplifier gain and offset voltage 

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#### Abstract

An approach for reducing the effects of operational amplifiers finite dc gain $A$ and offset voltage $V_{o s}$ in left-LUD SC ladder filters is proposed. The effectiveness of the proposed approach is demonstrated by designing a sixth-order bandpass elliptic SC filter. The variation of the finite gain $A$ from its nominal value $A_{0}$ is taken into account.


Key-Words: - Filters, gain- and offset- compensation, operational amplifiers, switched-capacitor integrators

## 1 Introduction

Switched-capacitor (SC) filter structures, which simulate passive RLC ladders are often employed where low sensitivity to element value deviation is important. Among various design approaches the leapfrog ladder and coupled biquad methods are the most popular [1, 2]. However, there is a major drawback for the elliptic leapfrog SC filters that there always exist delay-free loops formed by capacitors and operational amplifiers (op amps). This increases the op amps settling times.

One another alternative is the LUD ladder simulation method, which is based upon LU matrix decomposition technique and has the notable feature of being free from capacitor-coupled op amp loops [3, 4]. In [5] a large family of circuit structures is revealed depending on the choice of matrix decomposition, including the existing leapfrog, coupled biquad and LUD ones as specific cases. All circuits are insensitive to parasitic capacitances in SC implementation. A detailed comparison of various SC circuit structures has been undertaken and some notable conclusions are: the left-LUD method is the best choice for filters with very narrow and very wide passbands; the right-LUD method (related to the leapfrog structure) is the best choice for sharp transition lowpass filter design; the cascade biquads are the best choice for moderately selective lowpass and bandpass filter design.

SC circuits with low sensitivity to op amp characteristics such as finite dc gain $A$ and inputreferred offset voltage $V_{o s}$ are sometimes required, e. g. when op amps gain has to be sacrificed for bandwidth. For this reason several gain- and offset-
compensated (GOC) SC building blocks (integrators, gain stages, sample-and-hold circuits) have been reported in the literature.

In [6] a method of eliminating the influence of the finite op amps gain in a loop comprising two conventional integrators with opposite sign is presented. The price for the considerable improvement in circuit performance is a unity-gain buffer, 4 switches plus one additional capacitor per second-order stage. The method was applied to SC biquad and ladder structures comprising conventional integrators as basic building blocks.

In [7] a fifth-order elliptic lowpass SC filter with very low sensitivity to capacitance ratio errors and to finite op amps gains have been presented. The proposed structure comprises 9 op amps and 10 sample-and-hold circuits. The simulated frequency response of the filter, when op amps having gain equal to 1000 and infinite bandwidth are used, is very close to the ideal response (obtained with infinite op amps gains).

In this paper an approach for reducing the effects of op amps finite gain $A$ and offset voltage $V_{o s}$ in leftLUD SC ladder filters is proposed. It is based on the use of simple and fast amplifiers with low but precisely known and stable dc gain [8].

In a first step, the conventional integrators in the filter are replaced with GOC SC integrators and some appropriately chosen unswitched capacitors are split into two capacitors. The offset contribution to the output voltage of the filter is compensated by adding GOC sample-and-hold circuits.

In a second step, the nominal op amps gain value $A_{0}$ is taking into account in the capacitance sizing of the capacitors, connected to the uncompensated outputs of the integrators.

In a third step, the gain errors of the integrators are reduced by modifying the values of the integrating capacitances.

The effectiveness of the proposed approach is demonstrated by designing a sixth-order bandpass elliptic left-LUD filter. The variation of the op amps dc gain $A$ from its nominal value $A_{0}$ is taken into account. The GOC structure with changed topology
and modified capacitances values yields significant improvement in the passband response.

## 2 Conventional sixth-order bandpass elliptic left-LUD SC filter

The circuit schema of the sixth-order bandpass elliptic left-LUD SC filter being considered is shown in Fig. 1 [4].


Fig. 1. Conventional sixth-order bandpass left-LUD SC filter

For the sampling frequency $f_{s}=128 \mathrm{kHz}$ the ideal filter requirements are: lower passband edge 3600 Hz ; upper passband edge 4000 Hz ; maximum passband ripple 0.1 dB ; minimum stopband attenuation 22 dB . The component values for the circuit are listed in Table 1.

Table 1: Component values for the SC filter of Fig. 1

$$
\begin{array}{llll}
C_{1}=1.2831 & C_{2}=1.0 & C_{3}=15.620 & C_{4}=2.3015 \\
C_{5}=1.0 & C_{6}=4.6369 & C_{7}=9.8960 & C_{8}=1.2620 \\
C_{9}=1.0623 & C_{10}=2.4729 & C_{11}=21.067 & C_{12}=4.1209 \\
C_{13}=1.0 & C_{14}=4.4922 & C_{15}=10.378 & C_{16}=2.0283 \\
C_{17}=1.0 & C_{18}=1.0910 & C_{19}=10.604 & C_{20}=2.4337 \\
C_{21}=1.0 & C_{22}=5.1269 & C_{23}=1.0 &
\end{array}
$$

Let us suppose that the capacitors and the switches are ideal. The op amps are assumed to have finite dc gain $A$ and infinite bandwidth. This
supposition is adequate for the analysis of SC circuits containing fast and relatively low-gain amplifiers.
The filter is simulated and analyzed with MATLAB 7.1. Fig. 2 gives a comparison in the passband between ideal performance and nonideal response for finite op amps dc gain $A=100$. It is observed that the filter with $A=100$ has a response that is clearly inadequate.

The influence of the input-referred op amps dc offset voltages $V_{o s_{q}}(q=1 \div 6)$, modeled as voltage sources at the noninverting input terminals, is evaluated by the corresponding output voltage $V_{\text {out }}(n)$ in steady state, for $V_{i n}=0$.

For the capacitance values from Table 1 and $A=100$ the output offset voltage of the SC filter in Fig. 1 is

$$
\begin{align*}
\lim V_{n \rightarrow \infty} V_{\text {out }}(n) & =0.5679 V_{o s_{1}}+0.0057 V_{o s_{2}}-1.9656 V_{o s_{3}}  \tag{1}\\
& -0.0197 V_{o s_{4}}+2.3976 V_{o s_{5}}+0.024 V_{o s_{6}} .
\end{align*}
$$



Fig. 2: Passband responses of the filter from Fig. 1

## 3 Gain- and offset- compensated sixthorder bandpass elliptic left-LUD SC filter

At first, according to the proposed approach all the integrators in the conventional SC filter from Fig. 1 are replaced with GOC integrators. The second integrator (around the op amp 2) is replaced with Nagaraj-86 integrator [9]. All the other integrators are replaced with Ki-89 integrators [10]. The two GOC integrators have one and the same topology, but the values of the holding capacitances $C_{h q}$ differ. For the Nagaraj-86 integrator the capacitance $C_{h}$ is equal to the smallest filter capacitance. For Ki-89 integrator the capacitance $C_{h}$ is equal to the integrating capacitance.

The unswitched capacitors $C_{7}, C_{10}, C_{15}$ and $C_{18}$ from Fig. 1 are split into two capacitors. Two GOC sample-and-hold-circuits (with the op amps 7 and 8) are added to the outputs of the first and the third integrators to reduce the influence of the offset voltages $V_{o s 1}$ and $V_{o s 3}$ [11]. The resulting GOC filter is shown in Fig. 3, where $C_{h 1}=C_{3}, C_{h 2}=1, C_{h 3}=C_{11}$, $C_{h 4}=C_{14}, C_{h 5}=C_{19}$ and $C_{h 6}=C_{22}$.

The relation between the output voltages of the fourth Ki-89 integrator (around the op amp 4) during the uncompensated phase 2 and during the compensated phase 1 is frequency independent and given by the expression

$$
\begin{equation*}
V_{o_{4}}^{2}(n)=\frac{1+1 / A_{0}}{1+2 / A_{0}} V_{o_{4}}^{1}\left(n-\frac{1}{2}\right) \tag{2}
\end{equation*}
$$

where $A_{0}$ is the nominal value of the op amp dc gain.
The uncompensated output voltage $V_{o_{4}}^{2}(n)$ is stored in the capacitors $C_{9}, C_{10}^{\prime}, C_{12}$ and $C_{16}$. By capacitance sizing of these capacitors the
corresponding charges transferred during the two phases of the clock period can be equated. From (2) one obtains

$$
\begin{align*}
& C_{9 p}=C_{9} \frac{1+2 / A_{0}}{1+1 / A_{0}}, \quad C_{10}^{\prime}=C_{10} \frac{1+2 / A_{0}}{1+1 / A_{0}} \\
& C_{12 p}=C_{12} \frac{1+2 / A_{0}}{1+1 / A_{0}}, C_{16 p}=C_{16} \frac{1+2 / A_{0}}{1+1 / A_{0}} \tag{3}
\end{align*}
$$

For reducing the influence of the offset voltage $V_{0 S_{6}}$ the output voltage of the filter is sampled in the uncompensated phase 2 of the sixth Ki-89 integrator.

The relation between the values of the output voltage $V_{\text {out }}$ during the phases 1 and 2 of the clock period is

$$
\begin{equation*}
V_{\text {out }}^{2}(n)=\frac{1+1 / A_{0}}{1+2 / A_{0}} V_{\text {out }}^{1}\left(n-\frac{1}{2}\right) . \tag{4}
\end{equation*}
$$

The effect due to the finite gain of op amp 6 is minimized by modifying the values of the capacitances $C_{21}$ and $C_{18}^{\prime}$, according to the expressions

$$
\begin{equation*}
C_{21 p}=C_{21} \frac{1+2 / A_{0}}{1+1 / A_{0}}, \quad C_{18}^{\prime}=C_{18} \frac{1+1 / A_{0}}{1+2 / A_{0}} \tag{5}
\end{equation*}
$$

Finally, the gain errors of the first, the second, the third and the fifth integrators during the phase 2, and the gain errors of the fourth and the sixth integrators during the phase 1 are reduced by modifying the integrating capacitances $C_{3}, C_{6}, C_{11}$, $C_{19}, C_{14}$ and $C_{22}$ on the basis of the relationships [12]

$$
\begin{align*}
& C_{3 p}=\left(C_{3}-\frac{C_{1}+C_{2}+C_{4}+C_{9 p}}{A_{0}}\right)\left(1+\frac{1}{A_{0}}\right)^{-1}, \\
& C_{6 p}=\left(C_{6}-\frac{C_{h 2}+C_{5}+C_{10}+C_{10}^{\prime}}{A_{0}}\right)\left(1+\frac{1}{A_{0}}\right)^{-1}, \\
& C_{11 p}=\left(C_{11}-\frac{2 C_{7}+C_{8}+C_{12 p}+C_{17}}{A_{0}}\right)\left(1+\frac{1}{A_{0}}\right)^{-1},  \tag{6}\\
& C_{19 p}=\left(C_{19}-\frac{2 C_{15}+C_{16 p}+C_{20}+C_{23}}{A_{0}}\right)\left(1+\frac{1}{A_{0}}\right)^{-1}, \\
& C_{14 p}=\left(C_{14}-\frac{C_{13}+C_{18}+C_{18}^{\prime}}{A_{0}}\right)\left(1+\frac{1}{A_{0}}\right)^{-1}, \\
& C_{22 p}=\left(C_{22}-\frac{C_{21 p}}{A_{0}}\right)\left(1+\frac{1}{A_{0}}\right)^{-1} .
\end{align*}
$$



Fig. 3. GOC sixth-order bandpass left-LUD SC filter

From the data in Table 1 and expressions (3), (5) and (6), for $A_{0}=100$, one finds

$$
\begin{aligned}
& C_{9 p}=1.0728, \quad C_{10}^{\prime}=2.4974, C_{12 p}=4.1617, \\
& C_{16 p}=2.0484, \quad C_{21 p}=1.0099, C_{18}^{\prime}=1.0803, \\
& C_{3 p}=15.409, \quad C_{6 p}=4.5220, C_{11 p}=20.599, \\
& C_{19 p}=10.239, C_{14 p}=4.4163, \\
& C_{22 p}=5.0653 .
\end{aligned}
$$

Then, the final capacitance values for the GOC filter of Fig. 3 are listed in Table 2.

The passband responses of the GOC filter from Fig. 3 for the modified capacitance values (Table 2) and op amps gain variation $A=100 \pm 8$ [8] are shown in Fig. 4. It is obvious that these responses follow much more closely the ideal response than those of the conventional filter from Fig. 1.

Table 2. Component values for the GOC filter of Fig. 3

| $C_{1}=1.2831$ | $C_{2}=1.0$ | $C_{3}=15.409$ |
| :--- | :--- | :--- |
| $C_{4}=2.3015$ | $C_{5}=1.0$ | $C_{6}=4.5220$ |
| $C_{7}=9.8960$ | $C_{8}=1.2620$ | $C_{9}=1.0728$ |
| $C_{10}=2.4729$ | $C_{10}^{\prime}=2.4974$ | $C_{11}=20.599$ |
| $C_{12}=4.1617$ | $C_{13}=1.0$ | $C_{14}=4.4163$ |
| $C_{15}=10.378$ | $C_{16}=2.0484$ | $C_{17}=1.0$ |
| $C_{18}=1.0910$ | $C_{18}^{\prime}=1.0803$ | $C_{19}=10.239$ |
| $C_{20}=2.4337$ | $C_{21}=1.0099$ | $C_{22}=5.0653$ |
| $C_{23}=1$ | $C_{h 1}=15.409$ | $C_{h 2}=1$ |
| $C_{h 3}=20.599$ | $C_{h 4}=4.4163$ | $C_{h 5}=10.239$ |
| $C_{h 6}=5.0653$ | $C_{h}=1$ | $C_{H}=1$ |

The curves of Fig. 4 can be shifted near to the horizontal axis by modifying the input capacitance $C_{1}$ in accordance with the relationship

$$
\begin{equation*}
C_{1 p}=C_{1}\left(1-\Delta_{m}\right), \tag{7}
\end{equation*}
$$

where

$$
\Delta_{m}=\frac{H_{A_{0}}\left(f_{m}\right)-H_{i d}\left(f_{m}\right)}{H_{i d}\left(f_{m}\right)}
$$

is the relative error in the GOC filter response at the passband midpoint $f_{m}=3800 \mathrm{~Hz}$ for $A_{0}=100$.


Fig. 4. Passband responses of the GOC bandpass filter for $C_{1}=1.2831$

For $H_{A 0}\left(f_{m}\right)=0.9906181$ and $H_{i d}\left(f_{m}\right)=0.9997336$ one finds $C_{1 p}=1.2948$. The corresponding passband responses are shown in Fig. 5.

The output offset voltage of the GOC SC filter in Fig. 3 for $C_{1}=1.2948$ and $A=100$ is

$$
\begin{align*}
& \lim _{\substack{\text { out }}}(n)=0.0056 V_{\text {os }}-1.43610^{-5} V_{\text {os }}-0.0792 V_{\text {os }_{3}} \\
& \quad-6.027 .10^{-4} V_{\text {os }}+0.133 V_{\text {os }}-2.48910^{-4} V_{\text {os }}  \tag{8}\\
& \quad-0.0299 V_{\text {os }}+0.0547 V_{\text {os }} .
\end{align*}
$$



Fig. 5. Passband responses of the GOC bandpass filter for $C_{1}=1.2948$

## 4 Conclusion

A combined approach for reducing the effects of op amps finite gain and offset voltage in left-LUD switched-capacitor ladder filters has been presented. The effectiveness of the approach proposed has been demonstrated by an sixth-order bandpass filter. The passband response of the gain- and offsetcompensated filter with changed topology and modified capacitance values follows much more closely the ideal response than that of the conventional filter.

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