Statistical approach in complex circuits by a wavelet based Thevenin's theorem

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Abstract: - In the present work we study the treatment of parameters' uncertainties in complex circuits, i.e. composed by lumped and distributed elements. These problems are usually treated by the use of Monte Carlo techniques, which are extremely time-consuming.
A simple procedure for the calculation of the upper and lower limit of the response (i.e. the response bounds) is defined by using the wavelet expansion in time domain of the circuit variables.
When only a part of the circuit is affected by uncertainties we use the Thevenin's equivalent in the wavelet domain (straightforwardly evaluated) to further reduce the analysis complexity.
The proposed method allows us to directly evaluate the response bounds related to the parameters uncertainties without performing repeated simulations (Monte Carlo method), with a consequent CPU time saving.

Key-Words: - Wavelet analysis, parameters' uncertainties, Monte Carlo method, response bounds.

1 Introduction

increasing performances of integrated circuits have as a direct consequence the need of simulation tools, which at the same time guarantee accuracy of the results and low CPU time consumption.
It is common practice, especially in the industrial environment, to model complex microwave circuits by the use of equivalent circuits, composed by lumped elements and transmission lines, to be used in SPICE-like simulators.
As well known, there are different techniques for the extraction of the circuit parameters, each of them leading to different values of the parts [1]; in fact we can consider the different values of the elements as an uncertainty on the value of the element itself. Furthermore a different class of uncertainties can be identified: industrial processes cannot guarantee 100% accuracy in the construction, while the aging of the materials modify the performances of the circuits.
All these aspects lead to equivalent circuits characterized by uncertain elements; in this work we suppose to know the interval of variation of the parameters.
In this paper we focus our attention on the following situation: only a part of the whole model is affected by parameters uncertainty; we propose a simple and accurate procedure through which it is possible to define the bounds (upper and lower bound) of the response.
The time domain simulation of the complex circuit is performed in the wavelet domain. Indeed, this technique has proven to give accurate results at a low CPU time cost [2]; taking into account the characteristics of the wavelet expansion, we define a Thevenin's representation in the wavelet domain for the part of the circuit which is not characterized by parameters' uncertainties.
Usual treatment of circuits affected by uncertainties is by Monte Carlo techniques [3], by probabilistic approaches under some simplifying hypotheses [4], or by calculating a time domain sensitivity function (see for example [5]).
Based on the wavelet representation we define a set of equivalent source terms, related to the uncertainties of the parameters, and a simple algebraic operation on such equivalent source terms leads us to easily define the response bounds. The results are compared with a standard Monte Carlo procedure, characterized by long computational time.
In the following sections the procedure to obtain the Thevenin's equivalent in the wavelet domain and the definition of the response bounds are described. Then the method is applied to a complex circuit, and the results obtained show the accuracy and low computational effort of the method.

2 Thevenin's equivalent in the wavelet domain

A Thevenin's equivalent of a complex circuit (accessed by a port) can be represented in the Laplace Domain by the well known relation

$$e(s) = e_{th}(s) - Z(s)i(s) \quad (1)$$

where s is the Laplace variable. In the same way if an evaluation in the frequency domain is performed, the previous equation can be represented as

$$\dot{e}(\omega) = \dot{e}_{th}(\omega) - Z(\omega)\dot{i}(\omega) \quad (2)$$

In [6] it is shown that, for lumped parameters circuits, it is possible to obtain the equivalent representation in the wavelet domain (where wavelet expansion is performed in time) directly by using the symbolic differential and integral operators; for this reason eq. (1) can be represented in the wavelet domain as follows

$$\mathbf{e} = \mathbf{e}_{th} - \mathbf{Zi} \quad (3)$$

where $\mathbf{e}$, $\mathbf{e}_{th}$ and $\mathbf{i}$ are vectors of wavelet coefficients and $\mathbf{Z}$ is a matrix representing the impedance in the wavelet domain.

In [6] the calculation is performed analytically or by the use of the general methods for circuit analysis. When more complex circuits, involving lumped parameters are considered, such simple analysis cannot be performed. In [7] a procedure for the direct calculation of the matrix $\mathbf{Z}$ is shown: its columns can be calculated by performing a direct measure on the system or by a low number of simulations; in both cases the excitations are a subset of the wavelet basis function.

In case the frequency characterization of the impedance $Z(\omega)$ is known (by measurement or as a simulation result) the same procedure can be performed by the use of the Fast Fourier Transform, which allows us to calculate the response of the system to the wavelet basis functions.

Straightforwardly the term $\mathbf{e}_{th}$ can be determined simply by performing the wavelet expansion of the open port voltage, known from a time domain simulation, a direct measurement or a frequency domain characterization.

In this way the part of the circuit not affected by uncertainty can be taken into account simply as a boundary condition for the wavelet model of the remaining part (affected by uncertainty). The result is a representation of the whole complex circuit by an algebraic system

$$\mathbf{Ax} = \mathbf{b} \quad (4)$$

where $\mathbf{A}$ is a sparse block matrix, analytically calculated, whose entries depend on the circuits parameters, $\mathbf{x}$ is the vector of unknown wavelet coefficients (of voltages and currents) and $\mathbf{b}$ is the known term (whose entries are the exciting generators).

3 Definition of the bounds

A variation of one (or several) parameter leads to a change in matrix $\mathbf{A}$, hence the new circuit equation can be written as

$$\tilde{\mathbf{A}}\tilde{\mathbf{x}} = \mathbf{b} \quad (5)$$

where $\tilde{\mathbf{A}}$ is the new matrix resulting from the variation; $\tilde{\mathbf{x}}$ is the new solution (coefficients of the new voltages and currents) and $\mathbf{b}$ remains unchanged since the energizing generators are the same.

Equation (5) can be more conveniently written as

$$(\mathbf{A} + \Delta\mathbf{A})(\mathbf{x} + \Delta\mathbf{x}) = \mathbf{b} \quad (6)$$

where the variation of the matrix $\mathbf{A}$ and of the vector $\mathbf{x}$ is now evidenced. Simple algebraic calculation performed on (6) lead to

$$(\mathbf{A} + \Delta\mathbf{A})\Delta\mathbf{x} = -\Delta\mathbf{A}\mathbf{A}^{-1}\mathbf{b} \quad (7)$$

The evaluation of $\Delta\mathbf{x}$ yields

$$\Delta\mathbf{x} = -(\mathbf{A} + \Delta\mathbf{A})^{-1}\Delta\mathbf{A}\mathbf{A}^{-1}\mathbf{b} \quad (8)$$

For uncertainties of the parameters such that $(\mathbf{A} + \Delta\mathbf{A})^{-1} \cong \mathbf{A}^{-1}$ we obtain the following expression of the variation

$$\Delta\mathbf{x} = -\mathbf{A}^{-1}\Delta\mathbf{A}\mathbf{A}^{-1}\mathbf{b} \quad (9)$$

This formula has the advantage of requiring practically no computation overhead with respect to the evaluation of the solution corresponding to the nominal values of the parameters. A better estimate of the variation $\Delta\mathbf{x}$ that also holds for greater variations of the matrix $\mathbf{A}$ (i. e. those that do not satisfy the condition $(\mathbf{A}+\Delta\mathbf{A})^{-1} \cong \mathbf{A}^{-1}$) may be obtained by the following series expansion:

$$(\mathbf{A}+\Delta\mathbf{A})^{-1} = \left[\sum_{k=0}^{\infty}\left(-\mathbf{A}^{-1}\Delta\mathbf{A}\right)^{k}\right]\mathbf{A}^{-1} \qquad (10)$$

Substituting equation (10) in equation (8) we obtain:

$$\Delta\mathbf{x} = \left[\sum_{k=1}^{\infty}\left(-\mathbf{A}^{-1}\Delta\mathbf{A}\right)^{k}\right]\mathbf{A}^{-1}\mathbf{b} \qquad (11)$$

Obviously the summation may be truncated to a proper order that can be estimated by evaluating the norm of $\mathbf{A}^{-1}\Delta\mathbf{A}$.

The evaluation of the summation in (10) can be quickly performed if we consider that both the matrices $\mathbf{A}$ and $\Delta\mathbf{A}$ are sparse. The matrix $\mathbf{A}^{-1}$ can be made sparse by a threshold procedure [8] and this implies a substantial degree of sparsity of the product $\mathbf{A}^{-1}\Delta\mathbf{A}$. As a consequence efficient algorithms for the treatment of sparse matrices can be used in the computation of (11) so reducing the computational overhead.

The most common way to define the bounds of the response in presence of parameter's uncertainty is to perform a Monte Carlo procedure, by repeating several simulations with a random variation of the parameters. Here we propose the following different approach, based on the evaluation of the variation $\Delta\mathbf{x}$ by (9) or (11)

Let us indicate the *k* varying parameters as $p1 = p1_n \pm x_1\%$, $p2 = p2_n \pm x_2\%$, ..., $pk = pk_n \pm x_k\%$, where the subscript n is related to the nominal value. From Eq. (9) it is possible to evaluate the $\Delta\mathbf{x}$ for the worst case condition for the general ith parameter, i.e. $pi = pi_n(1 + x_i/100)$ and $pi = pi_n(1 - x_r/100)$. In case we have *k* varying parameters, we determine *2k* values of $\Delta\mathbf{x}$. Among the two $\Delta\mathbf{x}$ ($\Delta\mathbf{x}_{p1+}$ and $\Delta\mathbf{x}_{pi-}$) calculated for each parameter we choose the maximum one (named $\Delta\mathbf{x}_{pim}$).

It is now possible to define the variation of the response as follows:

$$\Delta\mathbf{x} = \left|\Delta\mathbf{x}_{p1m}\right| + \left|\Delta\mathbf{x}_{p2m}\right| + \ldots + \left|\Delta\mathbf{x}_{pkm}\right| \qquad (12)$$

The upper and lower bounds of the response are straightforwardly defined as:

$$\begin{aligned}\mathbf{x}_{up} &= \mathbf{x} + \Delta\mathbf{x}\\ \mathbf{x}_{low} &= \mathbf{x} - \Delta\mathbf{x}\end{aligned} \qquad (13)$$

It is noteworthy that the bounds in (13) are obtained at the cost of an algebraic system solution of the nominal system (giving the solution $\mathbf{x}$) a matrix inversion and *2k* evaluation of the summation in (10) that usually requires a few sparse matrix products. This leads to a much lower CPU time consumption than the one characterizing Monte Carlo solutions.

Upper and lower bounds x_{up} and x_{low} can be also evaluated by using partial derivatives (sensitivities) of the network functions with respect to circuit parameters. To this aim the adjoint technique is widely used and formulas for first and second order sensitivity are available [9], [10].

In [10] the computation overhead of some methods for the sensitivity evaluation including the adjoint technique is reported. In both the proposed method and the adjoint technique based analysis, the CPU times required increase with the range of variation of the uncertain parameters. In the sensitivity analysis an increased number of partial derivatives has to be evaluated at the computational cost described in [10]. With the proposed approach we simply need to evaluate some additional sparse matrix products with a remarkable CPU time saving..

4 Numerical results

In this section we show the application of the method to a complex circuit characterized by transmission lines and lumped parameters. The circuit is shown in Figure 1, and the values of all the parameters can be found in [11].

The part of the circuit which is supposed to vary is the one included in the ellipse; for this reason the remaining part of the circuit has been represented by the Thevenin's equivalent in the wavelet domain, calculated at point A in figure 1. The topology of the system is shown in Figure 2, where the shaded box represents the part affected by uncertainty. The Thevenin's impedance has been evaluated by the use of FFT, starting from the impedance in the frequency domain calculated by a single SPICE simulation.

In particular the uncertain parameters are the per unit length parameter of the transmission line and the values of lumped inductance, capacitance and resistance in the ellipse. A variation of 10% of the values is considered, (satisfying eq. (9)) typically requiring 3 terms in (10) to obtain a good approximation of $(\mathbf{A}+\Delta\mathbf{A})^{-1}$..

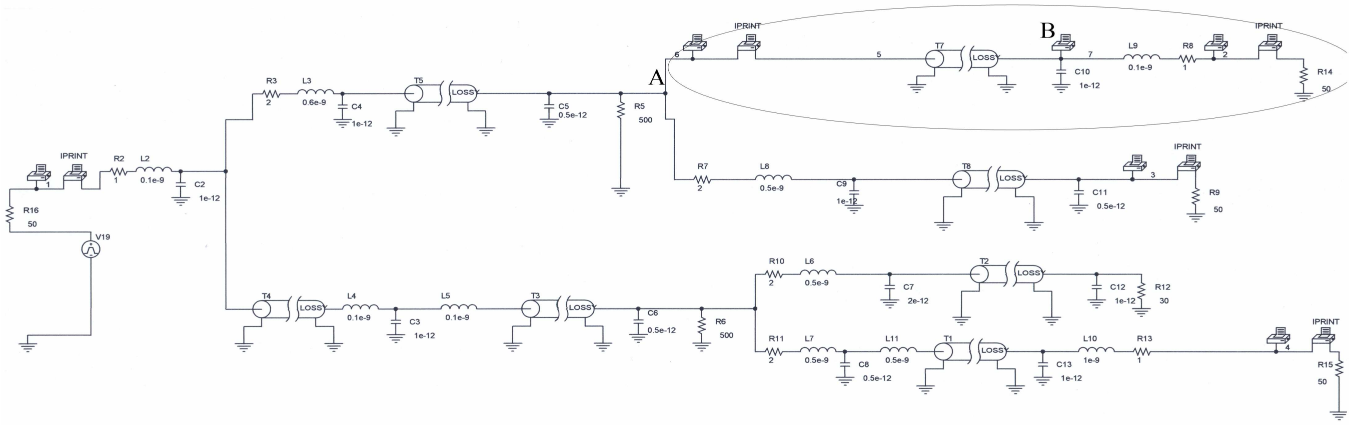

Figure 1. Selected circuit.

A second order sensitivity analysis via the adjoint technique has been also performed to obtain the response bounds. The two methods have produced practically coincident results. The proposed methods has allowed a CPU time saving of about 30% with respect to the adjoint technique.

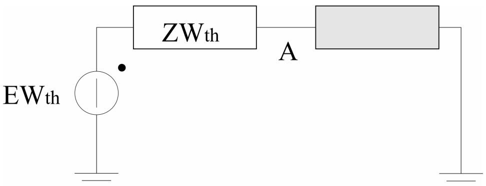

Figure 2. Examined circuit after the application of the Thevenin's equivalent.

The bounds obtained have been compared with standard Monte Carlo technique.

Figure 3 shows voltage at node B, where only the per unit length parameters of the line are uncertain; figure 4 shows the same results where also the lumped elements are uncertain. The Monte Carlo simulations are characterized by a number of 10000 runs, and the accuracy of the calculated bounds can be easily seen from the figures. The CPU time consumption for the calculation of the bounds is of the order of 3 minutes, 2 orders of magnitudes lower than the CPU time required for a consistent Monte Carlo simulation.

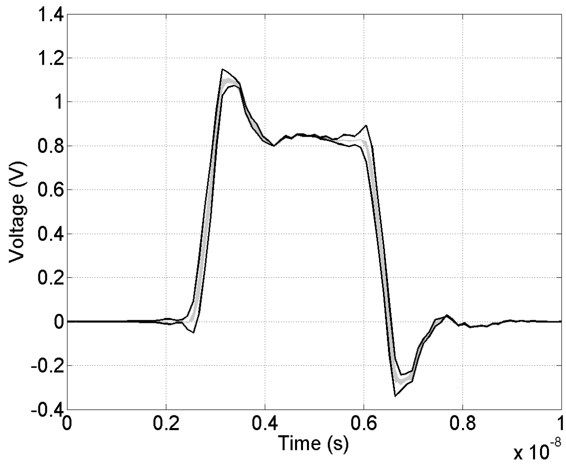

Figure 3. Comparison between the calculated bounds and Montecarlo simulations: only the p.u.l. parameters of the line are allowed to vary.

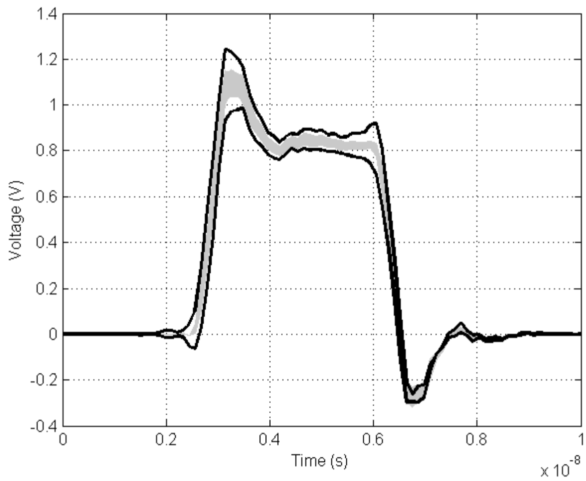

Figure 4. Comparison between the calculated bounds and Montecarlo simulations. All the parameters of the encircled portion of the circuit in figure 1 are uncertain.

5 Numerical results

A method for the evaluation of the effect of parameters' uncertainties in complex circuits has been presented. The method is based on an efficient wavelet representation of the entire circuit through the use of a Thevenin's equivalent in the wavelet domain; the resulting model is an algebraic system, whose matrix is characterized by uncertain coefficients. A simple procedure to determine the effects, in terms of response bounds, of such uncertainties has been defined, and the results are compared with standard Monte Carlo techniques.

The proposed method has proven to be accurate and characterized by low CPU time consumption.

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