A Sliding Mode Control for Induction Motors Using Adaptive Switching Control Law

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Abstract: An adaptive sliding-mode control system, which is insensitive to uncertainties, is proposed to control the position of an induction motor drive. The designed sliding mode control presents an adaptive switching gain to relax the requirement for the bound of uncertainties. The switching gain is adapted using a simple algorithm which do not implies a high computational load. Stability analysis based on Lyapunov theory is also performed in order to guarantee the closed loop stability. Finally simulation results show, on the one hand that the proposed controller provides high-performance dynamic characteristics, and on the other hand that this scheme is robust with respect to plant parameter variations and external load disturbances.

Key–Words: Sliding Mode Control, Induction Motor, Adaptive Control, Field Oriented Control.

1 Introduction

In recent years the induction motors have been increasingly taking place of the DC motors in high performance electrical motor drives [5]. The main advantage of the DC motors is that their speed control can be carried out in a simple way, since the torque and flux are decoupled. However, the technique of vectorial control [3] based on the rotor field orientation applied to the induction motors provides the decoupling between the torque and flux in a similar way to the DC machine. Therefore, with the progress of the power electronics and the appearance of low cost and very fast microprocessors, the induction motor drives have reached a competitive position compared to DC machines. However, the control performance of the resulting linear system is still influenced by uncertainties, which usually are composed of unpredictable parameter variations, external load disturbances and unmodelled and nonlinear dynamics [4].

In the past decade, the variable structure control strategy using the sliding-mode has been focussed on many studies and research for the control of the AC servo drive system [2], [1]. The sliding-mode control offers many good properties, such as good performance against unmodelled dynamics, insensitivity to parameter variations, external disturbance rejection and fast dynamic response [6]. These advantages of the sliding-mode control may be employed in the position and speed control of an AC servo system. However, the traditional sliding control schemes require the prior knowledge of an upper bound for the system uncertainties using this bound for the switching gain calculation. This upper bound should be determined as precisely as possible, because the higher is the upper bound, the higher value should be considered for the sliding gain, and therefore the control effort will also be high which is undesirable in practice. Then, to relax the requirement for the bound of uncertainties, an sliding mode control scheme with adaptive switching gain is proposed to control the induction motor drive. The switching gain is adapted using a simple algorithm which do not implies a high computational load.

2 Induction motor model

A dynamic model of an induction motor in a synchronously rotating d-q reference frame expressed in terms of state variables is given by the following equations [3]:

$$\begin{aligned}
\frac{di_{sd}}{dt} &= -\left(\frac{R_s}{\sigma L_s} + \frac{1-\sigma}{\sigma \tau_r}\right) i_{sd} + w_e i_{sq} \\
&\quad + \frac{L_m}{\sigma L_s L_r \tau_r}\psi_{rd} + \frac{L_m w_r}{\sigma L_s L_r}\psi_{rq} + \frac{1}{\sigma L_s} V_{sd} \\
\frac{di_{sq}}{dt} &= -w_e i_{sd} - \left(\frac{R_s}{\sigma L_s} + \frac{1-\sigma}{\sigma \tau_r}\right) i_{sq} \\
&\quad - \frac{L_m w_r}{\sigma L_s L_r}\psi_{rd} + \frac{L_m}{\sigma L_s L_r \tau_r}\psi_{rq} + \frac{1}{\sigma L_s} V_{sq}
\end{aligned}$$

$$\frac{d\psi_{rd}}{dt} = \frac{L_m}{\tau_r} i_{sd} - \frac{1}{\tau_r}\psi_{rd} + (w_e - w_r)\psi_{rq}$$

$$\frac{d\psi_{rq}}{dt} = \frac{L_m}{\tau_r} i_{sq} - (w_e - w_r)\psi_{rd} - \frac{1}{\tau_r}\psi_{rq}$$

$$T_e = \frac{3p}{4}\frac{L_m}{L_r}(\psi_{rd}\, i_{sq} - \psi_{rq}\, i_{sd}) \quad (1)$$

where i_{sd} and i_{sq} are d-q components of the stator current; V_{sd} and V_{sd} are d-q components of the the stator voltage; ψ_{rd} and ψ_{rq} are d-q components of the rotor flux linkage; $\sigma = 1 - \frac{L_m^2}{L_r L_s}$ is the leakage coefficient; L_s, L_r and L_m are stator, rotor and mutual inductances; R_s and R_r are stator and rotor resistances; w_e is the synchronous speed; and w_r is the stator electrical speed; $\tau_r = \frac{L_r}{R_r}$ is the rotor time constant; T_e is the induction motor torque and p is the is the pole numbers.

The relation between the synchronously rotating reference frame and the stationary reference frame is performed by the so-called reverse Park's transformation:

$$\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = \begin{bmatrix} \cos(\theta_e) & -\sin(\theta_e) \\ \cos(\theta_e - \frac{2\pi}{3}) & -\sin(\theta_e - \frac{2\pi}{3}) \\ \cos(\theta_e + \frac{2\pi}{3}) & -\sin(\theta_e + \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} x_d \\ x_q \end{bmatrix} \quad (2)$$

where θ_e is the angle position between the d-axis of the synchronously rotating reference frame and the a-axis of the stationary reference frame, and it is assumed that the quantities are balanced.

The main objective of the vector control of induction motor is, as in direct current (DC) drives, to control the torque and the flux independently. This DC machine-like performance is only possible if the current component i_{sd} is oriented (or aligned) in the direction of flux $\bar{\psi}_r$ and the other current component i_{sq} is established perpendicular to it (field orientation control principle). This means that we may control the current i_s by means of i_{sq} without affecting the flux $\bar{\psi}_r$, and similarly, when the flux $\bar{\psi}_r$ is controlled by means of i_{sd} the q-component of the current i_{sq} is not affected [3].

Under this condition of field orientation control it is satisfied that:

$$\psi_{rq} = 0, \qquad \psi_{rd} = |\bar{\psi}_r| \quad (3)$$

Then, the dynamic equations (1) may be simplified to:

$$\frac{di_{sd}}{dt} = -\left(\frac{R_s}{\sigma L_s} + \frac{1-\sigma}{\sigma\tau_r}\right) i_{sd} + w_e i_{sq}$$

$$+\frac{L_m}{\sigma L_s L_r \tau_r}\psi_{rd} + \frac{1}{\sigma L_s}V_{sd} \quad (4)$$

$$\frac{di_{sq}}{dt} = -w_e i_{sd} - \left(\frac{R_s}{\sigma L_s} + \frac{1-\sigma}{\sigma\tau_r}\right) i_{sq}$$

$$-\frac{L_m w_r}{\sigma L_s L_r}\psi_{rd} + \frac{1}{\sigma L_s}V_{sq} \quad (5)$$

$$\frac{d\psi_{rd}}{dt} = \frac{L_m}{\tau_r} i_{sd} - \frac{1}{\tau_r}\psi_{rd} \quad (6)$$

$$0 = \frac{L_m}{\tau_r} i_{sq} - (w_e - w_r)\psi_{rd} \quad (7)$$

$$T_e = \frac{3p}{4}\frac{L_m}{L_r}\psi_{rd}\, i_{sq} \quad (8)$$

Therefore, for the ideal decoupling, the torque equation (8) become analogous to the DC machine as follows:

$$T_e = \frac{3p}{4}\frac{L_m}{L_r}\psi_{rd} i_{sq} = K_T\, i_{sq} \quad (9)$$

where K_T is the torque constant, and is defined as follows:

$$K_T = \frac{3p}{4}\frac{L_m}{L_r}\psi_{rd}^* \quad (10)$$

where ψ_{rd}^* denotes the command rotor flux.

And from eqn. (7) the slip frequency w_{sl} can be expressed as follows:

$$w_{sl} = w_e - w_r = \frac{L_m}{\tau_r}\frac{i_{sq}}{\psi_{rd}} \quad (11)$$

The dynamic equations (4) and (5) can be decoupled by means of voltage decoupled control, choosing the inverter output voltages such that:

$$V_{sd}^* = \left(K_p + K_i\frac{1}{s}\right)(i_{sd}^* - i_{sd}) - w_e\sigma L_s i_{sq}^*$$

$$V_{sq}^* = \left(K_p + K_i\frac{1}{s}\right)(i_{sq}^* - i_{sq}) + w_e\sigma L_s i_{sd}^*$$

$$+w_e\frac{L_m}{L_r}\psi_{rd} \quad (12)$$

where the symbol '*' denotes command magnitudes.

3 Variable structure robust speed control with adaptive sliding gain

In general, the mechanical equation of an induction motor can be written as:

$$J\dot{w}_m + Bw_m + T_L = T_e \quad (13)$$

where J and B are the inertia constant and the viscous friction coefficient of the induction motor system respectively; T_L is the external load; w_m is the rotor mechanical speed in angular frequency, which is related to the rotor electrical speed by $w_m = 2\, w_r/p$ where p is the pole numbers and T_e denotes the generated torque of an induction motor.

Substituting equation (9) in the equation (13) the mechanical equation becomes:

$$\dot{w}_m + a\, w_m + f = b\, i_{qs} \tag{14}$$

where the parameter are defined as:

$$a = \frac{B}{J}, \quad b = \frac{K_T}{J}, \quad f = \frac{T_L}{J}; \tag{15}$$

Now, we are going to consider the previous mechanical equation (14) with uncertainties as follows:

$$\dot{w}_m = -(a+\triangle a)w_m - (f+\triangle f) + (b+\triangle b)i_{qs} \tag{16}$$

where the terms $\triangle a$, $\triangle b$ and $\triangle f$ represents the uncertainties of the terms a, b and f respectively. It should be noted that these uncertainties are unknown, and that the precise calculation of its upper bound are, in general, rather difficult to achieve.

Let us define define the tracking speed error as follows:

$$e(t) = w_m(t) - w_m^*(t) \tag{17}$$

where w_m^* is the rotor speed command.

Taking the derivative of the previous equation with respect to time yields:

$$\dot{e}(t) = \dot{w}_m - \dot{w}_m^* = -a\, e(t) + u(t) + d(t) \tag{18}$$

where the following terms have been collected in the signal $u(t)$,

$$u(t) = b\, i_{qs}(t) - a\, w_m^*(t) - f(t) - \dot{w}_m^*(t) \tag{19}$$

and the uncertainty terms have been collected in the signal $d(t)$,

$$d(t) = -\triangle a\, w_m(t) - \triangle f(t) + \triangle b\; i_{qs}(t) \tag{20}$$

To compensate for the above described uncertainties that are presented in the system, it is proposed a sliding adaptive control scheme. In the sliding control theory, the switching gain must be constructed so as to attain the sliding condition [6]. In order to meet this condition a suitable choice of the sliding gain should be made to compensate for the uncertainties. For selecting the sliding gain vector, an upper bound of the parameter variations, unmodelled dynamics, noise magnitudes, etc. should be known, but in practical applications there are situations in which these bounds are unknown, or at least difficult to calculate. A solution could be to choose a sufficiently high value for the sliding gain, but this approach could cause a to high control signal, or at least more activity control than it is necessary in order to achieve the control objective. One possible way to overcome this difficulty is to estimate the gain and to update it by some adaptation law, so that the sliding condition is achieved.

The sliding variable $S(t)$ is defined with an integral component as:

$$S(t) = e(t) + \int_0^t (a+k)e(\tau)\, d\tau \tag{21}$$

where k is a constant gain, and a is a parameter that was already defined in equation (15).

and the sliding surface is defined as:

$$S(t) = 0 \tag{22}$$

Now, we are going to design a variable structure speed controller, that incorporates an adaptive sliding gain, in order to control the AC motor drive.

$$u(t) = -k\, e(t) - \hat{\beta}(t)\gamma\, \mathrm{sgn}(S) \tag{23}$$

where the k is the gain defined previously, $\hat{\beta}$ is the estimated switching gain, γ is a positive constant, S is the sliding variable defined in eqn. (21) and $\mathrm{sgn}(\cdot)$ is the signum function.

The switching gain $\hat{\beta}$ is adapted according to the following updating law:

$$\dot{\hat{\beta}} = \gamma\, |S| \qquad \hat{\beta}(0) = 0 \tag{24}$$

where γ is a positive constant that let us choose the adaptation speed for the sliding gain.

In order to obtain the speed trajectory tracking, the following assumptions should be formulated:

($\mathcal{A}\,1$) The gain k must be chosen so that the term $(a+k)$ is strictly positive. Therefore the constant k should be $k > -a$.

($\mathcal{A}\,2$) There exits an unknown finite non-negative switching gain β such that

$$\beta > d_{max} + \eta \qquad \eta > 0$$

where $d_{max} \geq |d(t)| \quad \forall\ t$ and η is a positive constant.

Note that this condition only implies that the uncertainties of the system are bounded magnitudes.

$(\mathcal{A}\,3)$ The constant γ must be chosen so that $\gamma \geq 1$.

Theorem 1 *Consider the induction motor given by equation (16). Then, if assumptions* $(\mathcal{A}\,1)$, $(\mathcal{A}\,2)$ *and* $(\mathcal{A}\,3)$ *are verified, the control law (23) leads the rotor mechanical speed* $w_m(t)$ *so that the speed tracking error* $e(t) = w_m(t) - w_m^*(t)$ *tends to zero as the time tends to infinity.*

The proof of this theorem will be carried out using the Lyapunov stability theory.

Proof: Define the Lyapunov function candidate:

$$V(t) = \frac{1}{2}S(t)S(t) + \frac{1}{2}\tilde{\beta}(t)\tilde{\beta}(t) \tag{25}$$

where $S(t)$ is the sliding variable defined previously and $\tilde{\beta}(t) = \hat{\beta}(t) - \beta$

Its time derivative is calculated as:

$$\begin{aligned}
\dot{V}(t) = \;& S(t)\dot{S}(t) + \tilde{\beta}(t)\dot{\tilde{\beta}}(t) \\
&= S \cdot [\dot{e} + (a+k)e] + \tilde{\beta}(t)\dot{\hat{\beta}}(t) \\
&= S \cdot [(-a\,e + u + d) + (k\,e + a\,e)] + \tilde{\beta}\gamma|S| \\
&= S \cdot [u + d + k\,e] + (\hat{\beta} - \beta)\gamma|S| \\
&= S \cdot \left[-k\,e - \hat{\beta}\gamma\,\mathrm{sgn}(S) + d + k\,e\right] + (\hat{\beta} - \beta)\gamma|S| \\
&= S \cdot \left[d - \hat{\beta}\gamma\,\mathrm{sgn}(S)\right] + \hat{\beta}\gamma|S| - \beta\gamma|S| \\
&= d\,S - \hat{\beta}\gamma|S| + \hat{\beta}\gamma|S| - \beta\gamma|S| \qquad (26) \\
&\leq |d||S| - \beta\gamma|S| \\
&\leq |d||S| - (d_{max} + \eta)\gamma|S| \\
&= |d||S| - d_{max}\,\gamma|S| - \eta\,\gamma|S| \\
&\leq -\eta\,\gamma|S| \qquad (27)
\end{aligned}$$

then

$$\dot{V}(t) \leq 0 \tag{28}$$

It should be noted that in the proof the equations (21), (18), (23) and (24) have been used , and the assumptions $(\mathcal{A}\,2)$ and $(\mathcal{A}\,3)$.

Using the Lyapunov's direct method, since $V(t)$ is clearly positive-definite, $\dot{V}(t)$ is negative semidefinite and $V(t)$ tends to infinity as $S(t)$ and $\tilde{\beta}(t)$ tends to infinity, then the equilibrium at the origin $[S(t), \tilde{\beta}(t)] = [0, 0]$ is globally stable, and therefore the variables $S(t)$ and $\tilde{\beta}(t)$ are bounded. Since $S(t)$ is also bounded then it is deduced that $e(t)$ is bounded.

On the other hand, making the derivative of equation (21) it is obtained that,

$$\dot{S}(t) = \dot{e}(t) + (a+k)e(t) \tag{29}$$

then, substituting the equation (18) in the equation (29),

$$\begin{aligned}
\dot{S}(t) &= -ae(t) + u(t) + d(t) + (a+k)e(t) \\
&= ke(t) + d(t) + u(t) \qquad (30)
\end{aligned}$$

From equation (30) we can conclude that $\dot{S}(t)$ is bounded because $e(t)$, $u(t)$ and $d(t)$ are bounded.

Now, from equation 26 it is deduced that

$$\ddot{V}(t) = d\,\dot{S} - \beta\,\gamma\frac{d}{dt}|S(t)| \tag{31}$$

which is a bounded quantity because $\dot{S}(t)$ is bounded.

Under these conditions, since $\ddot{V}$ is bounded, $\dot{V}$ is a uniformly continuous function, so Barbalat's lemma let us conclude that $\dot{V} \to 0$ as $t \to \infty$, which implies that $S(t) \to 0$ as $t \to \infty$.

Therefore $S(t)$ tends to zero as the time t tends to infinity. Moreover, all trajectories starting off the sliding surface $S = 0$ must reach it in finite time and then will remain on this surface. This system's behavior once on the sliding surface is usually called *sliding mode* [6].

When the sliding mode occurs on the sliding surface (22), then $S(t) = \dot{S}(t) = 0$, and therefore the dynamic behavior of the tracking problem (18) is equivalently governed by the following equation:

$$\dot{S}(t) = 0 \quad \Rightarrow \quad \dot{e}(t) = -(a+k)e(t) \tag{32}$$

Then, under assumption $(\mathcal{A}\,1)$, the tracking error $e(t)$ converges to zero exponentially.

It should be noted that, a typical motion under sliding mode control consists of a *reaching phase* during which trajectories starting off the sliding surface $S = 0$ move toward it and reach it in finite time, followed by *sliding phase* during which the motion will be confined to this surface and the system tracking error will be represented by the reduced-order model (32), where the tracking error tends to zero.

Finally, the torque current command, $i_{sq}^*(t)$, can be obtained directly substituting eqn. (23) in eqn. (19):

$$i_{sq}^*(t) = \frac{1}{b}\left[k\,e - \hat{\beta}\gamma\,\mathrm{sgn}(S) + a\,w_m^* + \dot{w}_m^* + f\right] \tag{33}$$

Therefore, the proposed variable structure speed control with adaptive sliding gain resolves the speed tracking problem for the induction motor, with some uncertainties in mechanical parameters and load torque.

4 Simulation Results

In this section we will study the speed regulation performance of the proposed adaptive sliding-mode field oriented control under reference and load torque variations by means of simulation examples.

The block diagram of the proposed robust control scheme is presented in figure 1.

The block 'VSC Controller' represents the proposed adaptive sliding-mode controller, and it is implemented by equations (21), (33) and (24). The blocks '$dq \rightarrow abc$' makes the conversion between the synchronously rotating and stationary reference frames, and is implemented by equation (2). The block '$i^*_{sdq} \rightarrow V^*_{sdq}$' represents the voltage decoupled control which transforms the stator current reference to the stator voltage reference and it is implemented by equation (12). The block 'PWM Inverter' is a six IGBT-diode bridge inverter with 780 V DC voltage source. The block 'Field Weakening' gives the flux command based on rotor speed, so that the PWM controller does not saturate. The block 'i^*_{ds} Calculation' provides the current reference d-component from the rotor flux reference through equation (6). The block 'w_{sl} calculation' calculates the slip frequency and it is implemented by the equation (11). The block 'IM' represents the induction motor.

The induction motor used in this case study is a 50 HP, 460 V, four pole, 60 Hz motor having the following parameters: $R_s = 0.087\,\Omega$, $R_r = 0.228\,\Omega$, $L_s = 35.5\,mH$, $L_r = 35.5\,mH$, and $L_m = 34.7\,mH$.

The system has the following mechanical parameters: $J = 1.662\,kg.m^2$ and $B = 0.12\,N.m.s$. It is assumed that there are an uncertainty around 20 % in the system parameters, that will be overcome by the proposed adaptive sliding control.

The following values have been chosen for the controller parameters: $k = 25$ and $\gamma = 15$.

In the example the motor starts from a standstill state and we want the rotor speed to follow a speed command that starts from zero and accelerates until the rotor speed is $120\,rad/s$. The system starts with an initial load torque $T_L = 0\,N.m$, and at time $t = 1\,s$ the load torque steps from $T_L = 0\,N.m$ to $T_L = 250\,N.m$ and it is assumed that there is an uncertainty around 70 % in the load torque.

Figure 2 shows the desired rotor speed (dashed line) and the real rotor speed (solid line). As it may be observed, after a transitory time in which the sliding gain is adapted, the rotor speed tracks the desired speed in spite of system uncertainties. However, at time $t = 1\,s$ a little speed error can be observed. This error appears because of the torque increment at this time, and then the control system lost the so called 'sliding mode' because the actual sliding gain is too small to overcome the new uncertainty introduced in the system due to the new torque. But then, after a small time the sliding gain is adapted so that this gain can compensate the system uncertainties and so the rotor speed error is eliminated.

Figure 3 presents the time evolution of the estimated sliding gain. The sliding gain starts from zero and then it is increased until its value is high enough to compensate for the system uncertainties. Then at time $0.21\,s$ the sliding gain is remained constant because the system uncertainties remain constant as well. Later at time $1\,s$, there is an increment in the system uncertainties caused by the rise in the load torque. Therefore the sliding gain is adapted once again in order to overcome the new system uncertainties. As it can be seen in the figure, after the sliding gain is adapted it remains constant again, since the system uncertainties remains constant as well.

Figure 4 shows the motor torque. This figure shows that in the initial state, the motor torque has a high initial value in the speed acceleration zone because it is necessary a high torque to increment the rotor speed owing to the rotor inertia, then the value decreases in a constant region and finally increases due to the load torque increment.

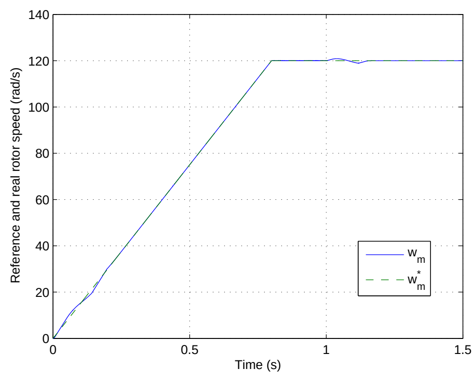

Figure 2: Reference and real rotor speed signals (rad/s)

5 Conclusion

In this paper a new adaptive sliding mode vector control has been presented. Due to the nature of the sliding control this control scheme is robust under uncertainties caused by parameter error or by changes in the load torque. Moreover, the proposed variable structure control incorporates an adaptive algorithm to calculate the sliding gain value. The adaptation of the sliding gain, on the one hand avoids the necessity of

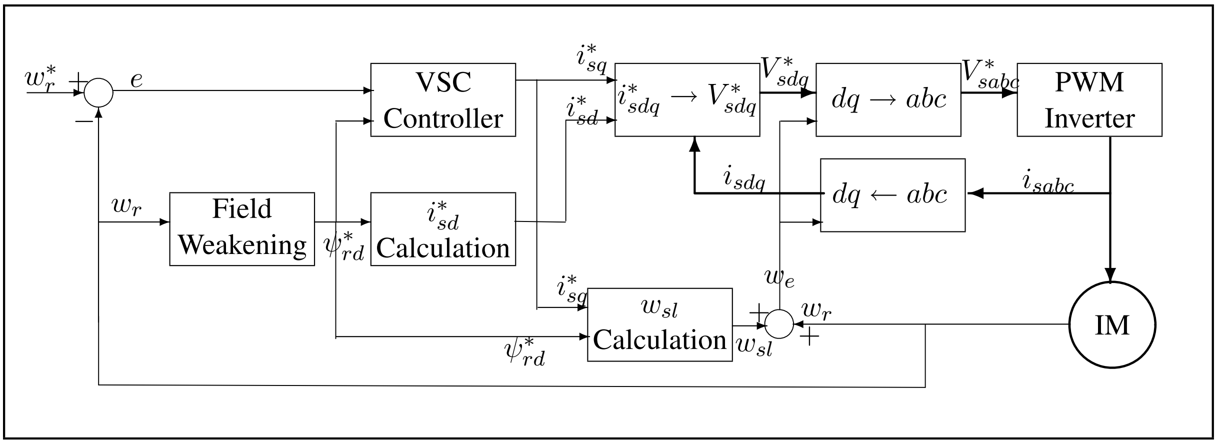

Figure 1: Block diagram of the proposed adaptive sliding-mode control

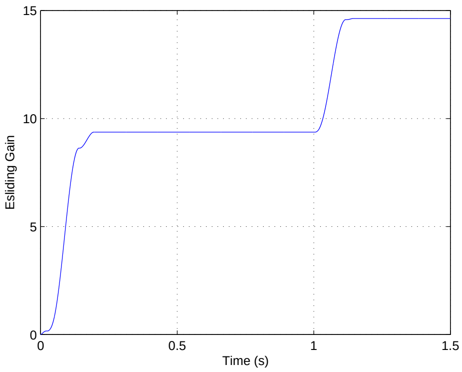

Figure 3: Estimated sliding Gain

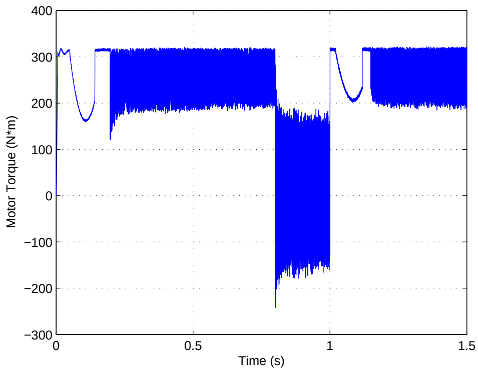

Figure 4: Motor torque (N.m)

computing the upper bound of the system uncertainties, and on the other hand allows to employ as smaller sliding gain as possible to overcome the actual system uncertainties. Then the control signal of our proposed variable structure control scheme will be smaller than the control signals of the traditional variable structure control schemes, because in the last one the sliding gain value should be chosen high enough to overcome all the possible uncertainties that could appear in the system along the time. Finally, by means of simulation examples, it has been shown that the proposed control scheme performs reasonably well in practice, and that the speed tracking objective is achieved under uncertainties in the parameters and load torque.

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