End-to-End Loss Probabilities in Different Internet-like Networks with a Given Average Hop Count

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Abstract: - Realistic networks generators are necessary for simulation and performance evaluation of data communication systems. Such an aspect has driven the collection and the analysis of data on the Internet large-scale structure. The evidence of a *power-law* behavior of real networks has stimulated the introduction of new procedures to generate Internet-like topologies. Assuming a simple loss model for the links, this paper analyses how the prediction of the loss probabilities during communications obtained by simulation can be influenced by the adoption of a specific topological model for the Internet graph (here, the Waxman or the Barabási-Albert model), given a average node distance in terms of hops.

Key-Words: - Internet topology, Packet loss, Hop count distribution, Waxman model, Barabási-Albert model.

1 Introduction

The design of next generation networks, such as Next Generation Internet [1], requires a deep understanding of the behavior of large data networks, because their topologies have a great influence on protocol performance [2,3]. This need stimulated the study of the statistical aspects of the Internet topology and the birth of many research projects devoted to the mapping of nodes of the Internet and connections among them, such as the *Internet Mapping Project* [4], *Skitter* by Cooperative Association for Internet Data Analysis (CAIDA) [5] or *Rocketfuel* [6].

Graph-like representations of a large portion of the Internet are now available. Recent studies on such results, due to M. Faloutsos *et. al* in 1999 [7], have addressed some typical features of many real complex systems. Indeed, it has been realized that Internet is a *scale-free network* whose interconnection structure is governed by *power-law* distributions (as in the case of the *degrees* of the nodes, the eigenvalues distribution, etc.). This result is in contrast with the distributions of Internet-like networks produced by traditional generators, based on the Erdős-Rényi classical *random graph* (henceforth ER) model [8-10], including the Waxman generator [11] among others.

The discovery of the scale-free nature of the Internet stimulated the introduction of new mathematical models [12-14] reproducing such a scale-free behavior. The first and most popular model was proposed by A.-L. Barabási and R. Albert (henceforth BA) in late 1999 [12]. It is ruled by two simple concepts: *(i)* the graph grows as a consequence of the continuous addition of new nodes; *(ii)* each new node connects to the existing vertices with a probability proportional to their degree (*preferential attachment mechanism*). Since then, other models have been proposed in order to overcome some limitations of the ER-based and BA models. For further reading Ref. [15] is suggested.

In [16], we investigated the influence of different topological models on communications performance, in terms of end-to-end loss probabilities for a given link loss model, evaluated in Internet-like networks with a given average node degree. We showed that end-to-end loss probabilities are dominated by distance distribution between node pairs.

In this paper, we compare Internet-like topologies with a given average node distance in terms of hops.

The paper is organized as follows. In Sect. 2 we outline the relevant Internet topology models: ER, Waxman and BA models. In Sect. 3 we describe the loss model adopted for the links of the synthetic Internet-like graphs. In Sect. 4 numerical results are presented.

2 Internet topological models

Internet topologies are generally represented as graphs that mime the large-scale characteristics of the maps obtained by measurements of the real network. As it is well known, Internet is a world-wide network composed of computers (or *hosts*) communicating by means of intermediate nodes (or *routers*), responsible to forward properly the information flows, and of *links* physically interconnecting nodes. It is possible to represent the network as an undirected graph, whose vertices are the routers and whose edges stand for the physical connections between pairs of them: this is often referred as the *Internet Router* (IR) *level representation*. At an higher level, Internet can be partitioned into several autonomously administered routing domains, named *Autonomous Systems* (AS), that are groups of nodes managed by a common administration and sharing routing information. Then, another possible representation of Internet is an undirected graph in which vertices represent the ASs and edges are *peering relationships* between pairs of ASs: this is the so-called *AS level representation*.

The traditional approach to model data networks relied on the use of classical random graphs, introduced by P. Erdős and A. Rényi in 1959 [8,9] (ER model). Based on such a model, the computer science community developed some tools to reproduce Internet in order to test new protocols [11,17]. After the discovery of the scale-free nature of Internet [7], such models and generators became inadequate to describe or reproduce large Internet-like networks, so the rise of new models has been stimulated in order to catch its features and new paradigms have been proposed to generate representative synthetic networks [18].

2.1 Static random graphs and the Waxman model

Static random graph models entail a fixed number of nodes N throughout the generation process. Typical examples of such models are the Erdős-Rényi and the Waxman models.

Technically speaking, an undirected graph G is a pair of sets $G = \{V, E\}$ where V is the set of vertices and E is the set of edges connecting two vertices of V. Thus, the size of the graph is $|V| = N$.

An ER random graph $G_{N,p}$ can be defined [19] as a graph with N nodes where each of the $N(N-1)/2$ possible edges is present, independently from the others, with probability p, called *connection probability*, and absent with probability $1 - p$. In this ensemble, the number of edges M is a binomial random variable (r. v.), viz. $M \sim B\left(N(N-1)/2, p\right)$. Such a model, also named *binomial model*, has been widely adopted by Internet researchers.

Defining the *degree* of a node as the number of edges attached to it, the *average degree* can be easily computed as

$$\bar{k} = p(N-1) \cong pN, \qquad (1)$$

where the approximation holds for large N.

Since real networks (even evolving ones, such as the Internet) are characterized by an almost constant average degree [15], it is convenient to consider $p(N) = \bar{k}/N$, derived from eq. (1). If $\bar{k} < 1$, the network is composed by isolated subgraphs and hence it can be represented as a collection of *clusters*. On the other hand, when $\bar{k} > 1$ a giant cluster, called the *giant component*, emerges which, as $N \to \infty$, incorporates almost all nodes, As the average degree approaches the critical value $\bar{k} = 1$ an abrupt change in the cluster structure occurs. At the corresponding critical probability $p_c(N) = 1/N$ the random graph changes its topology abruptly from a collection of isolated components to a single giant cluster.

One of the main features of a random graph is its *degree distribution* $P(k)$. In an ER graph with N nodes and connection probability p it is

$$P(k) = \binom{N-1}{k} p^k (1-p)^{N-1-k} \cong e^{-\bar{k}} \frac{\bar{k}^k}{k!}, \qquad (2)$$

where the Poisson approximation holds for large N and for constant $\bar{k}$.

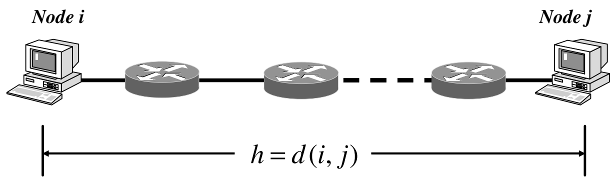

Figure 1 – Communication path between two generic nodes.

Another fundamental subject of investigation in graph theory for networks applications is the distribution of the distances among nodes, or *hop count distribution* for short,
expressed in terms of probability mass function (*pmf*) as $f(n) = \Pr\{h = n\}$, where $h = d(i, j)$ is a r. v. representing the length of the shortest path connecting a pair of randomly selected vertices i and j or, equivalently the number, plus 1, of hops to be traversed to reach node j starting from node i, as represented in Fig. 1. For ER graphs the average distance is [20]

$$\bar{h} \simeq \frac{\log N}{\log \bar{k}}. \qquad (3)$$

It is evident that $\bar{h}$ is much smaller than the size of the graph N, as a consequence of the *small-world effect* [21], exhibited by many real networks.

Despite reproducing the small-world behavior, the ER model fails to predict some features of Internet topology: for instance, it yields a binomial degree distribution which decreases exponentially for large N and, then, deviates from the heavy-tailed distribution observed in Internet.

2.1.1 The Waxman model

The ER model has inspired the first Internet topology generator used for protocol testing, proposed by Waxman [11]. According to the Waxman algorithm, nodes are (uniformly) randomly distributed on a rectangular coordinate grid and the probability of an edge between two vertices i and j is

$$p(i,j) = \alpha e^{-\frac{d_E(i,j)}{\beta D}} \qquad (4)$$

where $d_E(i,j)$ is the Euclidean distance from node i to j, α and β are parameters in the range (0,1] and D is the maximum distance between two vertices. On the other hand, there is no edge between i and j with probability $1-p(i,j)$. While the topological structure of the graph is not influenced by the value of D [17,22], it is highly dependent on the values of α and β: α controls directly the number of edges, while β rules the influence of the distance between nodes. In line with the approach described in [22], it is possible to derive in a closed form the average degree $\bar{k}$ as a function of N, α and β.

Without loss in generality, we assume that nodes are uniformly distributed in a square with size 1 and have coordinates (ξ, η). Noting that

$$\bar{k} = \frac{1}{N}\sum_{\substack{i,j=1 \\ i\neq j}}^{N} E\left[p(i,j)\right] \qquad (5)$$

and $E\left[p(i,j)\right]$ is constant $\forall i, j = 1,2,...,N$, eq. (5) can be cast as $\bar{k} = (N-1)E\left[p(i,j)\right]$ where

$$E\left[p(i,j)\right] = \alpha\int_0^1 d\xi_i \int_0^1 d\eta_i \int_0^1 d\xi_j \int_0^1 d\eta_j \, e^{-\frac{\sqrt{(\xi_i-\xi_j)^2+(\eta_i-\eta_j)^2}}{\beta\sqrt{2}}}.$$

Then, after [22], the average degree of Waxman graphs is

$$\bar{k} = \alpha(N-1)\zeta\left(\frac{1}{\beta\sqrt{2}}\right), \qquad (6)$$

where

$$\zeta(x) = \frac{2}{x^4}\Big[6\left(1-2e^{-x}+e^{-x\sqrt{2}}\right)+2x\left(-4-2e^{-x}+3\sqrt{2}e^{-x\sqrt{2}}\right)+x^2\left(4e^{-x\sqrt{2}}+\pi\right)+\frac{8}{x}\left(g_1(x)+\frac{g_2(x)}{x}\right)\Big]$$

and

$$g_1(y) = \int_1^{\sqrt{2}} dx e^{-yx}\sqrt{x^2-1}$$
$$g_2(y) = \int_1^{\sqrt{2}} dx e^{-yx}\sqrt{1-1/x^2}.$$

Like ER graphs, Waxman graphs yield values of $\bar{h}$ small with respect to the size of the network ($\bar{h} \sim \log N$) and hence consistent with the small-world effect, but fail to yield the heavy-tailed degree distributions observed in Internet.

2.2 Barabási-Albert model

Many complex systems, such as Internet, show degree distributions that are not peaked around a *typical* value, the average degree $\bar{k}$, but instead highly skewed (*scale-free* behavior). The first model for computer networks producing graphs with power-law degree distributions was proposed by A.-L. Barabási and R. Albert in 1999 [12], who claimed that the network is an open system growing with time, and that the probability that two nodes are connected depends on the degree of the nodes. It means that new edges are not placed at random but tend to connect to vertices that already have a large degree, respecting the paradigm *rich-get-richer* [23]. The algorithm inside Barabási-Albert (BA) models can be summarized as follows.

The network starts at time $t_0 = 0$ with a small number of nodes m_0. At every time unit a new vertex with m edges ($m < m_0$) is added and it is connected to m different nodes already present in the system. The edges of the new vertex are connected to the i-th already existing node with a probability $\Pi(k_i(t))$ proportional to its degree $k_i(t)$ at time t, such that

$$\Pi\left(k_i(t)\right) = \frac{k_i(t)}{\sum_j k_j(t)}. \qquad (7)$$

After t time units the BA procedure provides a graph $G_m^{(N)}$ with $N = t + m_0$ nodes and mt edges, whose average degree has a simple expression, i.e. it is

$$\bar{k} \cong 2m, \qquad (8)$$

where $N \gg m_0$ has been considered. Since m is an integer, eq. (8) implies that $\bar{k}$ can have integer values. BA graphs have a power-law degree distribution $P(k)$ [12]

$$P(k) \sim k^{-\gamma}, \qquad (9)$$

where γ = 3, similar to the degree distribution measured in Internet (in real networks $\gamma \cong 2.1$ [15]). The power-law distribution implies that the probability of finding vertices with a very large degree (hub) is not negligible in the BA graph.

BA graphs also exhibit the small-world effect. Indeed, it has been proved rigorously [24] that its *diameter*, namely its *maximum* distance in the shortest path sense, shows different asymptotical behaviours, in the limit of large N, depending on the value of the parameter m. The average distance is supposed to behave in similar way. In particular, in BA graphs with $m = 1$, the average distance is, as $N \to \infty$,

$$\bar{h} \sim \log N$$

like ER graphs. Instead for $m \geq 2$ it is asymptotically

$$\bar{h} \sim \frac{\log N}{\log\log N}. \qquad (10)$$

3 Loss model

The hop count distribution of the Internet reflects the interconnection structure among routers and hence affects significantly the end-to-end communication performance. Thus, since every model proposed to represent Internet provides, in principle, a different distribution of the distance among nodes, the predicted performance may depend on the adopted topological model of the network. To shed some light on this subject, we carried on an evaluation of packet loss probabilities under different Internet-like topologies, given the loss model of single links.

Every communication between two end-points involves a *path* set up by the routing protocols, generally aiming to minimize the number of hops packets have to traverse to reach the destination. Although some intra-ASs routing policies might inflate the shortest paths, the distance, h, between source and destination can be assumed to be the length of one of the shortest paths between the two end-points [25] and can be considered fixed in time.

Further, we introduce a simple link loss model, in which every packet is lost on the n-th link, *independently* of the others, with probability

$$\lambda_n \sim U\left[0, \lambda_{max}\right]. \qquad (11)$$

Such a model introduces some relevant simplifications with respect to reality, since correlations are neglected both in space (from link to link at a given time) and in time (at different times on one link). The end-to-end packet loss probability L can thus be expressed as

$$L = 1 - \prod_{n=1}^{h}\left(1-\lambda_n\right) \qquad (12)$$

where λ_n is the loss probability of the n-th link in the path, and where h , as above, is a r. v. representing the shortest path length and conveying the influence of network topology on the packet loss rate. In the simulations, $\lambda_{max} = 10^{-2}$ in order to match to the order of magnitude of the maximum loss rate typically encountered inside Internet [26,27].

4 Numerical results

In order to assess the influence of network topology on performance prediction, representative numerical experiments have been carried by NS-2 on synthetic networks (according to different models) provided by BRITE (Boston university Representative Internet Topology gEnerator) [28]. In our study, network topologies have been produced by BRITE according to two algorithms outlined in Sect. 2: the BA model [12] and the Waxman model [11].

A Constant Bit Rate (CBR) traffic flow (representative of real-time applications, like VoIP) between two randomly selected nodes (connected by a communication path) is added to the network scenario, in order to compute the end-to-end packet loss probability L as the fraction of packets lost among those transmitted. The throughput of the CBR source is set to 100 kbit/sec, in order to match the order of magnitude of typical VoIP communications. The link loss model outlined in previous section has been implemented via the NS-2 *Error Model* class that allows to set the loss rate of every link according to eq. (11). In order to avoid other causes of packet loss, in every simulation no interfering traffic is present in the network and the capacity of every link is set to 10 Mbit/sec, a value much greater than that required by the CBR traffic flow. The duration of each NS-2 simulation has been established so that the CBR source send 10^5 packets to the destination. For each topological model, 5000 experiments have been performed via Monte Carlo techniques in graphs with the same size (N = 1000 vertices).

In [16], we considered graphs with the same average degree ($\bar{k} = 4$), showing that the considered graphs, although with the same size N and the same average degree $\bar{k}$, yielded different packet loss probabilities. In particular, performances predicted on Waxman networks seem to be poorer than those predicted on BA networks in terms of average loss rates, probably due to a different average hop count induced by this two models. Then, it is meaningful to analyze the behavior of the graph models when they show the same $\bar{h}$. As in [16], the m parameter in (8) was set to the value m = 2, providing for a BA graph of N=1000 nodes an average hop count $\bar{h}_{BA} = 4.61$. For Waxman graphs, we selected the pair (α,β), by means of Fig. 2, providing Waxman graphs with $\bar{h}_W = 4.61 = \bar{h}_{BA}$. We recall that Fig. 2 (see [16]) represents the contour plots in (α,β) for different values of $\bar{k}$ and $\bar{h}$ obtained for Waxman graphs with N = 1000 nodes. In this case, a possible choice for the values of the parameters is α = 0.012 and β = 0.34, while $D = 1000\sqrt{2}$. The corresponding empirical pmf $f(n)$ is shown in Fig. 3, in which, for comparison, the hop count for the BA model with m = 2 is reported.

The measured average loss probability is $\bar{L} = 0.23$ for both models, since $\bar{L} = \lambda_{max}\bar{h}/2$, while the *Complementary Cumulative Distribution Function* (CCDF) $\bar{F}_L(x) = \Pr\{L > x\}$ of the loss probabilities under each model are reported in Fig. 4. The large overlap of the two CCDF's confirms that $\bar{h}$ plays a fundamental role in determining the loss probabilities.

As the *Quantile–Quantile Plot* (Fig. 5) qualitatively suggests, both empirical distributions are fitted by a Weibull CCDF [29]

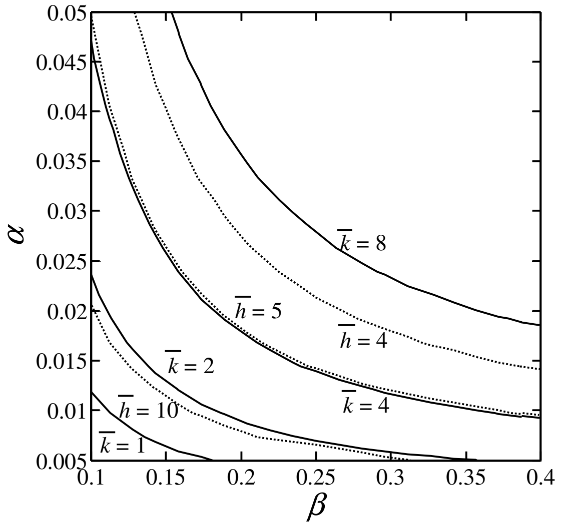

Figure 2 – Contour plots of different values of $\bar{k}$ (solid lines) and $\bar{h}$ (dotted lines) in Waxman graphs with N = 1000 nodes and $D = 1000\sqrt{2}$ in a fraction of the parameter space (α,β). $\bar{k} = 1, 2, 4, 8$, $\bar{h} = 4, 5, 10$ are reported [16].

$$\bar{F}(x;a,b) = e^{-ax^b}, \quad x \geq 0, \tag{13}$$

even if the maximum likelihood estimation from data of the parameters in eq. (13) provides slightly different values (a = 1.0·10^5 and b = 3.1 in Waxman graphs, while a = 2.2·10^5 and b = 3.4 in BA graphs).

5 Conclusion

The investigation was motivated by the fact that the various topological models of Internet yield different degree distributions, e.g. $P(k)$ decays exponentially for Waxman model while decays as a power-law for BA model. Under the assumed Bernoulli link loss model, our numerical experiments (performed by means of NS-2 network simulator on graphs generated by BRITE package with N = 1000) support the following conclusions: *(i)* despite the said discrepancy, both end-to-end loss probabilities are fitted by a Weibull distribution, even if graph models show different average distance [16]; *(ii)* when both models have the same average hop count Waxman and BA graphs have comparable behavior in terms of end-to-end loss probability. This outcome suggests that, in order to predict the end-to-end loss probability in real networks or else in generating synthetic networks, the average hop count among nodes plays a central role.

Even if the assumed model for the link loss is far from representing the loss of the real Internet links, the methods presented in this paper could be applied under more realistic loss models, accounting for the burstiness and the long-range correlation of the loss process observed in real operation of Internet.

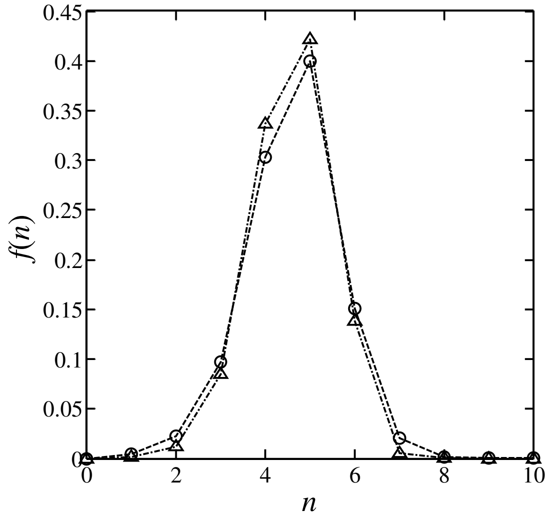

Figure 3 – Empirical pmfs of the hop count $f(n)$ in Waxman networks with α = 0.012, β = 0.34 and $D = 1000\sqrt{2}$ (circles and dashed lines) and BA networks with m = 2 (triangles and dot-dashed lines). All experiments are carried out in graphs with N = 1000 vertices.

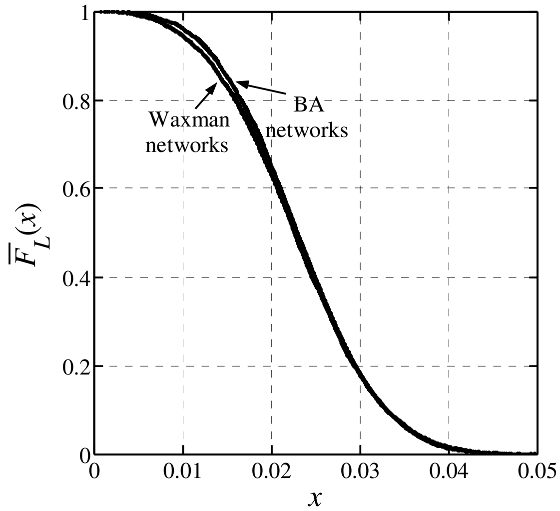

Figure 4 – Complementary Cumulative Distribution function $\bar{F}_L(x)$ of the end-to-end loss probability in BA and Waxman networks with the same $\bar{h} = 4.61$.

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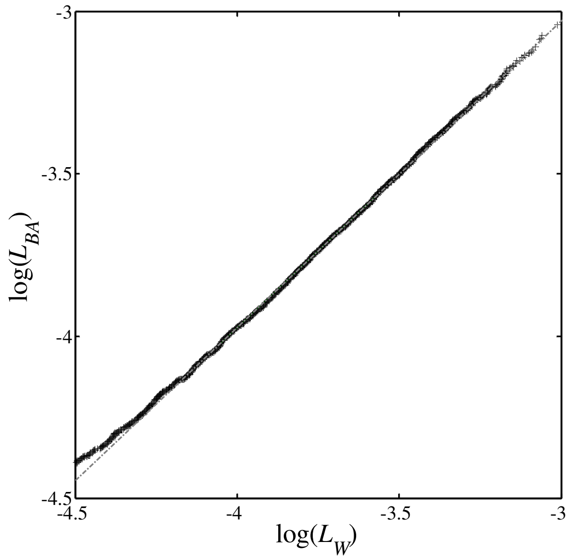

Figure 5 – Quantile–Quantile Plot of the logarithm of the end-to-end loss probability in Waxman L_W and BA graphs L_{BA} with the same $\overline{h} = 4.61$.

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