Applying the Correlated Gamma Statistics in Channel Capacity Evaluation for Dual-branch MRC Diversity over Correlated-fading Channels

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Abstract: - The channel capacity of dual-branch MRC (maximal ratio combining) diversity system over correlated waveform intensity, which is characterized as correlated-Nakagami-*m* fading (the power is modeled as the correlated-Gamma statistics), is evaluated in this paper. The formulas of channel capacity performance are provided with a pdf (probability density function)-based approach. The pdf of sum of Gamma variates based on the representation of the Moschopoulos single gamma series is adopted in the report. The corresponding expressions for Rayleigh fading are obtained as a special case of Nakagami-*m* fading. Finally, the numerical examples are presented for illustrating the purpose of the validation of the channel capacity equations derived in this paper.
Key-Words: - MRC diversity, channel capacity, Gamma variates, Nakagami-*m* fading

I. Introduction

It is recognized that the mobile radio links are subject to severe multipath fading in the received signal envelope due to the combination of randomly delayed reflected, scattered, and diffracted signal components. Such fading will result in degradation in the capacity of diversity schemes. The channel capacity is one of the most important criterions for evaluating the system performance of the wireless radio systems. The channel capacity analysis of fading channel is increasingly growing a primary adopted in either wired or wireless communication systems. Especially, the important issues of evaluating channel capacity are focused on calculating the SNR (signal-to-noise ratio) and the bandwidth of the communications. Since Shanno proposed the channel capacity theorem in 1948 [1], there are several papers have been published dealing with the channel capacity characteristic of the diversity combining techniques under fading channels (e.g., [2-9]). Particularly, the capacity of Nakagami-*m* fading channels was investigated in [3, 5, 10, 11]. In [3], an expression for the channel capacity of Nakagami-*m* fading with MRC (maximal ratio combining) diversity was obtained with the assumption that the diversity branches are *i.i.d.* (independent and identically distributed). The expressions for the channel capacity of Nakagami-*m* fading channels with MRC were obtained in [4] under different adaptive transmission techniques for the *i.i.d.* case. The effect of branch correlation on the capacity of Nakagami-*m* fading channels for different diversity combining techniques was investigated. The diversity combining techniques are known to be a powerful technique that can be used to improve system performance under channel fading caused by the maneuver phenomena [12]. It is worthwhile noting that the pdf (probability density function) of the received SNR (signal-to-noise ratio) applied in this paper for deriving an expression of the channel capacity for MRC has been determined in [13] based on the work of Gurland [14], which is different but equivalent to the pdf used in [11]. The corresponding expressions for Rayleigh fading are obtained as a special case of Nakagami-*m* fading. We assume the channels are slowly varying flat correlated-fading channels. Several models have been proposed to explain the behavior of multipath fading of the received signal envelope. It has been a long time that the Rayleigh and Rice distributions are employed to characterize the envelope of faded signals over small geographical area and/or short term fading. Recently, Nakagami-*m* distribution has been thoroughly investigated due to the fact that it is verified to be a more versatile model for a variety of fading environments such as the urban/suburban radio multipath channel for wireless communication systems, even for indoor propagation, and it includes the Rayleigh fading as a special case where the fading factor *m*=1 [20]. The other property of a multipath-fading channel is in its MIP (multipath intensity profile), which depicts the average power at the output of the channel as a function of path delay [15, 16]. The multipath propagation can be resolved at the receiver by means of the wideband characteristics of the signal [17].

Based on the motivation mentioned above, the evaluation of channel capacity for dual-branch MRC schemes over correlated Nakagami-*m* fading channels with integer fading parameter, *m*, and the MIP factor, *u*, are conduct in this paper. The power of the fading signal is considered modeled as Gamma distributed with the characteristic of branch correlation. After the introduction section, the organization of this paper is constructed as: the receiver and channel models of the correlated-Gamma for the received signal at the output of the dual-branch MRC diversity are described in section II. In section III the evaluation of system performance of channel capacity is performed, and the results of the equation are numerical evaluated by the software package in section IV. Finally, in section V a simple conclusion are presented

II. Receiver and Channel Models

In this section, the receiver of an dual-branch MRC diversity and the correlated channels models are described. Consider that the waveform intensities at the output of the dual-branch MRC scheme are $r_i(t),\ \ i=1,\ 2$, and the signal appears at the output of the dual-branch MRC diversity is the summation of the two branch waveforms, given as $r(t)=\sum_{i=1}^{2} r_i(t)$.

First, it is well known that the mobile radio channel is usually modeled as a discrete, slow-fading, and time-invariant multipath channel. The equivalent complex low-pass impulse response of the fading channel viewed from the *i*-th user is given by [21]

$$h_{(i)}(\tau)=\sum_{l=1}^{L}\beta_{(i)}^{l}e^{j\theta_{(i)}^{l}}\delta[\tau-\tau_{(i)}^{l}] \qquad (1)$$

where β^l, θ^l and τ^l are the path signal intensity, phase, and delay of the *l*-th multipath, respectively. Now recall that the fact of definition of a SNR variable is going to be a gamma variate and adopt the representation of two key results for the summation of correlated gamma variates. Assume that the random variable X follows a gamma distribution with parameter $m>0$ and $\beta>0$, the pdf of X is then given as

$$p_X(x)=\frac{x^{m-1}e^{-x/\beta}}{\beta^m\Gamma(m)}U(x) \qquad (2)$$

where $\Gamma(\cdot)$ is the gamma function [15], and $U(\cdot)$ denotes the unit step function. On the other hand, the shorthand notation of X can be represented as $\mathrm{X}\sim\omega(m,\beta)$. In order to determine the pdf of the sum of correlated Gamma variates, now extend the Moschopoulos result and an exact single gamma-series representation of the sum of arbitrarily correlated gamma variates can be obtained [10]. Next, express $X_n,\ n=1,\dots,L$ as a set of L correlated-gamma variates with parameters m and β_n, respectively, [i.e., $X_n\sim\omega(m,\beta)$] and let $\rho_{ij},\ i=j=1,2,\dots,L$ denotes the correlation coefficient between X_i and X_j, i.e.,

$$\rho_{ij}=\rho_{ji}=\frac{Cov(X_i,X_j)}{\sqrt{\mathrm{Var}(\mathrm{X}_i)\mathrm{Var}(\mathrm{X}_j)}},\quad 0\le\rho_{ij}\le 1$$
$$i,j=1,2,\dots,L \qquad (3)$$

then the pdf of $Y=\Sigma_{n=1}^{L}X_n$ can be expressed as

$$p_Y(y)=\prod_{n=1}^{L}\left(\frac{\lambda_1}{\lambda_n}\right)^m\sum_{k=0}^{\infty}\frac{\delta_k y^{Lm+k-1}e^{-y/\lambda_1}}{\lambda_1^{Lm+k}\Gamma(Lm+k)}U(y) \qquad (4)$$

where $\lambda_1=\min_n\{\lambda_n,\ n=1,\dots,L\}$, where $\min_n\{\cdot\}$ is a function of choosing the minimum one, $\{\lambda_n,\ n=1,2,\dots,L\}$ are the eigenvalues of the matrix *A*=*DC*, where *D* is the $L\times L$ diagonal matrix with the entries $\{\beta_n,\ n=1,2,\dots,L\}$ and *C* is the $L\times L$ positive definite matrix defined by

$$C=\begin{bmatrix}1 & \sqrt{\rho_{12}} & \cdots & \sqrt{\rho_{1L}}\\ \sqrt{\rho_{21}} & 1 & \cdots & \sqrt{\rho_{2L}}\\ . & . & . & .\\ . & . & . & .\\ \sqrt{\rho_{L1}} & \cdots & \cdots & 1\end{bmatrix}_{L\times L} \qquad (5)$$

and the coefficients δ_k can be obtained recursively by the formula

$$\begin{cases} \delta_0 = 1 \\ \delta_{k+1} = \dfrac{m}{k+1} \sum_{i}^{k+1} \left[\sum_{j=1}^{L} \left(1 - \dfrac{\lambda_1}{\lambda_j} \right)^i \right] \delta_{k+1-i} \\ k = 0,1,2,........ \end{cases} \quad (6)$$

For constant correlation, it can be show that the eigenvalues of the matrix A can be defined as [16, (2.8.3)]

$$\begin{cases} \lambda_1 = \cdots = \lambda_{L-1} = \dfrac{\bar{\gamma}}{m}\left(1-\sqrt{\rho}\right) \\ \lambda_L = \dfrac{\bar{\gamma}}{m}\left(1+\sqrt{\rho}\left(L-1\right)\right) \end{cases} \quad (7)$$

Once by substituting these eigenvalues in previous equation into (4), the statistical distribution of SNR at the combiner output can be obtained immediately. Thus, it is able easily to determine the channel capacity of the dual-branch MRC diversity over correlated-fading channels. Moreover, in order to take the MIP (multipath intensity profile) effect into account of the evaluation of channel capacity. Consider the condition of negative exponential MIP with power decay factor, μ, then the average power of the SNR can be computed as

$$\Omega_{j,n} = \frac{e^{-n\mu}}{q(L,\mu)} \quad j = 1,2,...,M_R \text{ , } n = 0,1,...,L_R - 1 \quad (8)$$

where assuming that L is the number of multipath of the desired signal, and the function of $q(L,\mu)$ is defined as

$$q(L,\mu) = \sum_{l=0}^{L-1} e^{-l\mu} = \frac{1-e^{-L\mu}}{1-e^{-\mu}} \quad (9)$$

After the definition is completed, the eigenvalues of λ_L shown in (7) is going to be replaced with the equivalent shown as

$$\lambda_L = \frac{1}{m}\left(1+\sqrt{\rho}\left(L-1\right)\right)\bar{\gamma}\frac{e^{-n\mu}}{q(L,\mu)} \quad (10)$$

where the parameters correlation coefficient and MIP are all included for consideration.

III. Channel Capacity Analysis

The channel capacity, which is also known as the ergodic capacity, of a radio system operating in flat fading channel can be obtained by averaging the capacity over an AWGN (additive white Gaussian noise) channel. The capacity which is defined as $C = B \log_2(1+\gamma)$, where γ represents the instantaneous SNR of the received signal at the output of a communication system, and B denotes the channel bandwidth (Hz) [1]. Thus, the average channel capacity, $\langle C \rangle$, of a radio system in flat fading channel is expressed as [2]

$$\langle C \rangle = B \cdot \int_0^\infty \log_2(1+\gamma) p_\gamma(\gamma) d\gamma \quad (11)$$

where the determination of pdf, $P_\gamma(\gamma)$, for the instantaneous SNR is necessary, for the random process reason. Moreover, because the evaluation of the channel capacity, η_{MRC}, is for a dual-branch MRC system over correlated Nakagami-*m* fading channels, the results can be calculated by substituting (4) into (11) and obtained as

$$\eta_{MRC} = \frac{\langle C \rangle}{B} = \prod_{n=0}^{L}\left(\frac{\lambda_1}{\lambda_n}\right)^m \sum_{k=0}^{\infty} \frac{\delta_k}{\lambda_1^{Lm+k-1}\Gamma(Nm+k)} \times \int_0^\infty \log_2\left(1+\gamma\right)\gamma^{Lm+k-1} e^{-\gamma/\lambda_1} d\gamma \quad (12)$$

where an integral formula with the logarithm function is meeting in the previous equation. The partial integration method may be applied to obtain the final closed-form of the formula, and after some algebra procedures, the result cab be expressed as

$$\eta_{MRC} = \frac{\langle C \rangle}{B} = \prod_{n=0}^{L}\left(\frac{\lambda_1}{\lambda_n}\right)^m \sum_{k=0}^{\infty} \frac{\delta_k}{\lambda_1^{Lm+k-1}\Gamma(Nm+k)} \times \frac{1}{\ln 2} \times \left(2m+k-1\right)! \times \exp\left(\frac{1}{\lambda_1}\right) \times \sum_{n=1}^{2m+k} \frac{\Gamma(-2m-k+n, \frac{1}{\lambda_1})}{\left(\frac{1}{\lambda_1}\right)^n} \quad (13)$$

where $\Gamma(\alpha, x) = \int_x^\infty e^{-t} t^{\alpha-1} dt$ is the upper incomplete gamma function.

IV. Numerical Results

In this section the numerical results from the derived formula shown in previous section for dual-branch MRC diversity are illustrated in this section. The evaluation is implemented by the computer with the package software. In Fig. 1 the validation of the pdf of the output SNR obtained from the exact expression (4) with (5), and the gamma approximate expression offered in [19] for a constant correlation model with four paths, *L=4*, fading value, *m=1.8*, and the unit average SNR is applied, $\bar{\gamma}=1$, besides, the correlation is set as $\rho = 0.64$ is conduct . On the other hand, although the approximate solution matches quite well with the exact solutions in the high SNR region, it tends to deviate in the lower tail of the pdf. Hence, the approximate solution has to be used with caution as far as outage probability and average probability of errors calculations are concerned since the lower tail of the pdf is very critical for these calculations. The channel capacity per unit bandwidth of dual-branch MRC systems over correlated-Gamma fading channels as a function of average SNR, $\bar{\gamma}$, with different fading parameters, m = 1 and m = 2, respectively, and ρ = 0.49 are presented in Fig. 2. It is exact as expected the MRC diversity provides the largest improvement when the fading parameter is lager, that is, the capacity performance of the dual-branch MRC schemes becomes superior under the condition of much better of the selected propagation environments. Besides, the system capacity is also evaluated with different power decay factors, $\mu = 0.1$, 0.5, and, 0.9. It is easy to see that the channel capacity of dual-branches MRC diversity deteriorated as μ decreases. However, it is worthwhile note that the effect of power decay factor is much less than that of the fading parameter. This is the reason that the most important factors dominate the system performance of a wireless communication system is the searching of the transmission environments. Moreover, the results shown in Fig. 3 illustrates the capacity per unit bandwidth of a dual-branch MRC system over correlated-Gamma fading channels with different values, m = 1 and m = 2, distinctive values of the correlation coefficients, and ρ = 0.36, 0.49, and 0.64, as a function of the average SNR, $\bar{\gamma}$. For the purpose of comparison, the channel capacity of Nakagami fading channels without diversity, which was obtained in [6, Eq. (23)], is presented in Figs. 3. It is seen that MRC diversity improves the capacity of Nakagami-m fading channels. It is valuable to describe that the channel capacity of the dual-branches MRC diversity is deeply degraded by the higher value of the correlation coefficient value. Furthermore, the channel capacity definitely relates to the separation between the received paths. This degradation can be resolved by increasing the distance between the receiver antennas [13].

V. Conclusions

The closed-form expressions for the channel capacity of an dual-branch MRC systems over correlated-Gamma channels for integer m is computed in this paper. It was observed that the diversity combining techniques that are considered in this paper can provide with improvement to the capacity of Gamma-m fading channels. It was seen that channel capacity decreases as the correlation coefficient ρ increases. However, the decrease in capacity due to correlation diminishes as fading gets less severe. The derivative in this report relied on the Moschopoulos representation for the pdf of the sum of independent gamma variates and its extension to the pdf of the sum of correlated gamma variates to provide an complete pdf-based approach (including the parameters of fading parameter and correlation coefficients) for the capacity performance analysis of dual-branches MRC scheme over not necessarily independent nor identically distributed correlated-Gamma fading channels.

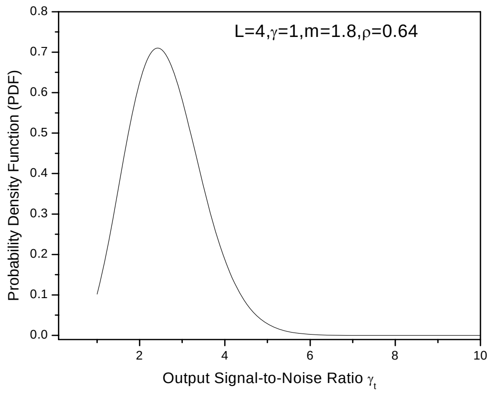

Fig. 1 The pdf plot of correlated-Gamma distributed with L = 4, $\bar{\gamma}$ = 1, m= 1.8, and constant power correlation $\rho = 0.64$

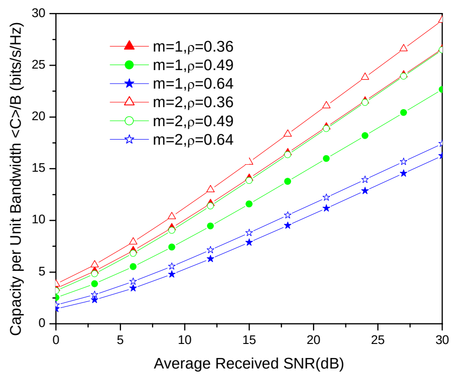

Fig. 2. The performance of channel capacity vs average received SNR with different fading parameter and fixed correlation value, $\rho = 0.49$

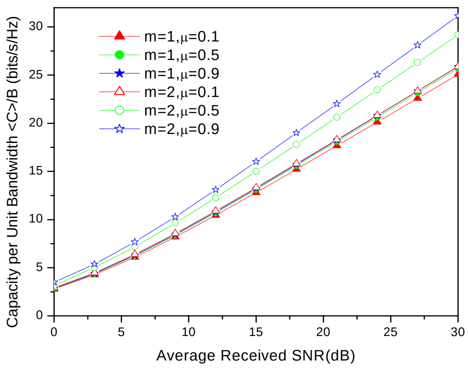

Fig. 3 Channel Capacity per unit bandwidth vs average received SNR with different fading parameters.

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