# Wireless-Based Identification of Patient Dynamics: Analysis and **Quantization Design**

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Abstract: This paper studies identification of systems in which the system output is quantized, transmitted through a communication channel, and observed afterwards. The problem is motivated by real-time remote monitoring and control of anesthesia patients in which patient models must be established in real time. When system resources are limited such as the transmission bandwidth of a wireless communication channel, appropriate utility of available resources to enhance information accuracy becomes imperative. This paper analyzes the impact of quantization on identification accuracy and communication errors. A complexity relationship is established that allows optimal selection of quantization to minimize the overall errors of system identification.

Key-Words: Anesthesia, Wiener models, identification, quantization, transmission channels, efficient estimation, space and time complexity.

#### Introduction 1

Real-time anesthesia decisions are exemplified by general anesthesia for attaining an adequate anesthetic depth (consciousness level of a patient), ventilation control, etc. One of the most critical requirements in this decision process is to predict the impact of the inputs (such as drug infusion rates) on the outcomes (such as consciousness levels). This prediction capability can be used for control, display, warning, predictive diagnosis, decision analysis, outcome comparison, etc. The core function of this prediction capability is embedded in establishing a reliable model that relates the drug inputs to the outcomes in real-time and in individual patients. The underlying problem is a real-time identification problem.

Rapid development in telecommunication technologies, especially wireless communications, has made remote monitoring and control of anesthesia and surgical procedures feasible. This leads to an identification problem of systems in which the system output must be quantized, transmitted through a digital communication channel, and observed afterwards. Communication errors introduce additional uncertainty that influences identification accuracy. This paper aims to characterize communication channels, establish impact of quantization, and derive identification algorithms. The problem of fundamental tradeoff of space and time complexities in identification problems with constrained communication resources are

investigated. Optimal resource allocation problems are discussed. The main findings of this paper indicate that effective utility of communication resources is essential when communication bandwidth is shared by many users, and hence is very limited for each connection.

### 2 Wiener Models and Anesthesia Patient Modeling

A basic information-oriented model structure for patient responses to drug infusion was introduced in [12]. Propofol (a common anesthesia drug) titration is administered by an infusion pump. The patient's anesthesia depth is measured by a BIS (bi-spectrum) monitor (Aspect Medical Systems, Inc.). The monitor provides continuously an index in the range of [0, 100] such that the lower the index value, the deeper the anesthesia state. The response from the titration command to the drug infusion at the needle point is the infusion pump dynamics and can be represented by a transfer function  $G_i(s)$ . Similarly, the BIS monitor dynamics can be represented by a transfer function  $G_m(s)$ . The patient dynamics is a nonlinear system. We characterize the patient response to propofol titration with three basic components: (1) Initial time delay  $\tau_p$  after drug infusion. (2) Dynamic reaction: This reflects how fast the BIS value will change once it starts to respond, and is modeled by a transfer function  $G_p(s)$ . (3) A nonlinear static function for sensitivity of the patient to a drug dosage at steady state. This is represented by a function or a look-up table f. Consequently, a model structure for titration response is shown in Figure 1, which is called Wiener models in the control community. To verify the utility of



Figure 1: Titration Model Structure

the Wiener model structure in anesthesiology, clinical data were collected. The actual BIS response is compared to the model response over the entire surgical procedure, shown in Figure 2. The model captures the key trends and magnitudes of the BIS variations in the surgical procedure. This indicates that the model structure contains sufficient freedom in representing the main features of the patient response.



Figure 2: Patient Model Responses

# **3** Identification of Wiener Systems with Binary-Valued Observations

Consider a Wiener system

$$\begin{cases} x(k) = \sum_{i=0}^{n-1} a_i u(k-i), \\ y(k) = H(x(k), \eta) + d(k), \end{cases}$$
(1)

where u(k) is the input, x(k) the intermediate variable, and d(k) the measurement noise.  $H(\cdot, \eta)$ :  $\mathcal{D}_H \subseteq \mathbb{R} \to \mathbb{R}$ , is a parameterized static nonlinear function with domain  $\mathcal{D}_H$  and vector-valued parameter  $\eta \in \Omega_\eta \subseteq \mathbb{R}^m$ . Both n and m are known. By

defining  $\phi(k) = [u(k), \dots, u(k - n + 1)]^T$  and  $\theta = [a_0, \dots, a_{n-1}]^T$ , the linear dynamics can be expressed compactly as  $x(k) = \phi(k)^T \theta$ .

# Assumption A.

- 1. The noise  $\{d(k)\}$  is a sequence of independent and identically distributed (i.i.d.) random variables with finite variance. The distribution function  $F(\cdot)$  of d(1) is known, which is continuously differentiable together with a continuously differentiable inverse  $F^{-1}(\cdot)$  and a bounded density  $f(\cdot)$  with  $f(x) \neq 0$  for  $x \neq 0$ .
- 2. For any given  $\eta \in \Omega_{\eta}$ ,  $H(x, \eta)$  is bounded for any finite x, continuous and invertible in x.

The output y(k) is measured by a binary sensor with threshold C. That is, the sensor output s(k) = S(y(k)) is a function of y(k) indicating only whether  $y(k) \leq C$ , where C is known. We use the indicator function

$$s(k) = \mathcal{S}(y(k)) = I_{\{y(k) \le C\}} = \begin{cases} 1, & \text{if } y(k) \le C, \\ 0, & \text{otherwise} \end{cases}$$
(2)

to represent the sensor.

Under appropriate input design, identification of a Wiener system can be reduced to a set of much simplified core identification problems. The input signal, that will be used to identify the system, is a 2n(m+1)-periodic signal u whose one-period values are  $(\rho_0 v, \rho_0 v, \rho_1 v, \rho_1 v, \dots, \rho_m v, \rho_m v)$ , where v = $(v_1, \dots, v_n)$  is to be specified. The scaling factors  $\{\rho_0, \rho_1, \dots, \rho_m\}$  are assumed to be nonzero and distinct.

If under the 2n input values u = (v, v), the linear subsystem has the following n consecutive output values at  $n, \ldots, 2n - 1, \delta_i = a_0 u(n+i) + \cdots + a_{n-1}u(1+i), \quad i = 0, \ldots, n-1$ , then the output under the scaled input (qv, qv) is  $x(n+i) = q\delta_i, \quad i = 0, \ldots, n-1$ . Without loss of generality, assume  $\delta_0 \neq 0$ . The output of the linear subsystem contains the following (m+1)-periodic subsequence with its single period values  $\{\rho_0\delta_0, \rho_1\delta_0, \ldots, \rho_m\delta_0\}$ :

$$x(n) = \rho_0 \delta_0, \dots x((2m+1)n) = \rho_m \delta_0, \dots$$

By concentrating on this subsequence of x(k), under a new index l with l = 1, 2, ..., the corresponding output of the nonlinear part may be rewritten as

$$\widetilde{y}(l(m+1)) = H(\rho_0 \delta_0, \eta) + d(l(m+1)), 
\widetilde{y}(l(m+1)+1) = H(\rho_1 \delta_0, \eta) + \widetilde{d}(l(m+1)+1), 
\vdots 
\widetilde{y}(l(m+1)+m) = H(\rho_m \delta_0, \eta) + \widetilde{d}(l(m+1)+m) 
(3)$$

The equations in (3) form the basic observation relationship for identifying  $\eta$  and  $\delta_0$ .

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For  $\rho = [\rho_0, \dots, \rho_m]^T$  and a scalar  $\delta$ , we denote

$$\mathbf{H}(\rho\delta,\eta) = [H(\rho_0\delta,\eta),\dots,H(\rho_m\delta,\eta)]^T. \quad (4)$$

Then, (3) can be expressed as

$$\widetilde{Y}(l) = \mathbf{H}(\rho\delta,\eta) + \widetilde{D}(l), \quad l = 0, 1, \dots,$$
 (5)

where  $\delta \neq 0$ ,  $\tilde{Y}(l) = [\tilde{y}(l(m+1)), \ldots, \tilde{y}(l(m+1) + m)]^T$  and  $\tilde{D}(l) = [\tilde{d}(l(m+1)), \ldots, \tilde{d}(l(m+1) + m)]^T$ . Correspondingly, the outputs of the binary-valued sensor on  $\tilde{Y}(l)$  are  $\tilde{S}(l) = \mathcal{S}(\tilde{Y}(l)), l = 0, 1, \ldots$ . Let  $\tau = [\tau_0, \ldots, \tau_m]^T = [\delta, \eta^T]^T$ . We introduce the following identification problem.

**Core Identification Problem:** Estimate the parameter  $\tau$  from observations on  $\widetilde{S}(l)$ .

Denote  $\zeta_i = H(\rho_i \delta, \eta), i = 0, 1, ..., m$ . Then  $\zeta = [\zeta_0, ..., \zeta_m]^T = \mathbf{H}(\rho \delta, \eta)$  and (5) can be rewritten as  $\widetilde{Y}(l) = \zeta + \widetilde{D}(l), l = 0, 1, ...$  The main idea of solving the core identification problem is first to estimate  $\zeta$ , and then to solve the interpolation equations

$$\zeta_i = H(\rho_i \delta, \eta), \quad i = 0, 1, \dots, m \tag{6}$$

for  $\delta$  and  $\eta$ .

Since the equations (6) can be solved when  $\zeta_i$  are estimated, the remainder of the paper will concentrate on identification of  $\zeta_i$ , which is an unknown constant.

# 4 Identification with Quantized Observations

## 4.1 Identification Accuracy

Consider a constant system  $y_k = \theta + d_k$ , k = 1, 2, ..., where  $d_k$  is the disturbance and  $\theta$  is to be identified. The output  $y_k$  is measured by a sensor of m thresholds  $-\infty < C_1 < \cdots < C_m < \infty$ . The sensor is represented by a linear combination of indicator functions

$$s_k = \mathcal{S}(y_k) = \sum_{i=1}^{m+1} i s_k^i, \tag{7}$$

where  $s_k^i = I_{\{y_k \in (C_{i-1}, C_i]\}}$  with i = 1, ..., m+1 and

$$I_{\{y_k \in A\}} = \begin{cases} 1, & \text{if } y_k \in A, \\ 0, & \text{otherwise.} \end{cases}$$

Hence,  $s_k = i$ , for  $i = 1, \ldots, m + 1$ , implies that  $y_k \in (C_{i-1}, C_i]$  with  $C_0 = -\infty$  and  $C_{m+1} = \infty$ .

Under Assumption A,  $\{y_k\}$  is an i.i.d. sequence that has the accumulative distribution function  $F(\cdot - \theta)$ . A sensor of *m* thresholds  $C_1, \ldots, C_m$  divides the output range into m + 1 intervals  $(-\infty, C_1]$ ,

$$(C_1,C_2],\ldots,(C_m,\infty).$$
 The probability of  $\{s_k^i=1\}$  is 
$$p_i=P\{C_{i-1} < y_k \leq C_i\}$$

 $= F(C_i - \theta) - F(C_{i-1} - \theta) := \widetilde{F}_i(\theta).$ 

The premise of our approach is that although the sensor threshold  $C_i$  only indicates if the output is in  $(C_{i-1}, C_i]$ , the probability  $p_i$  may provide more information about the unknown parameter  $\theta$ . Let

$$h_i(\theta) = \frac{dp_i}{d\theta} = \frac{d\widetilde{F}_i(\theta)}{d\theta}$$
$$-f(C_i - \theta) + f(C_{i-1} - \theta), \ i = 1, \dots, m+1$$

where  $f(\cdot)$  is the density function. Then, the sensitivity of  $\theta$  with respect to  $p_i$  is  $d\theta/dp_i = 1/h_i(\theta)$ . Denote  $h(\theta) = [h_1(\theta), \dots, h_{m+1}(\theta)]^T$ .

**Lemma 1** [11] The Cramér-Rao Lower Bound for estimating  $\theta$  based on observations of  $\{s_k\}$  is  $\sigma_{CR}^2(N,m) = \left(N\sum_{i=1}^{m+1}\frac{h_i^2}{p_i}\right)^{-1}.$ 

# 4.2 Communication Channels

Two scenarios of system configuration shown in Figure 3 are considered. System identification with quantized observations is depicted in Figure 3(a) in which the observations on  $u_k$  and  $s_k$  are used. On the other hand, when the outputs of a system are transmitted through a communication channel and observed after transmission, the system parameters must be estimated by observing  $u_k$  and  $w_k$ , as shown in Figure 3(b).



### Figure 3: System Configurations

When the sensor output  $s_k \in \{1, \ldots, m+1\}$  is transmitted through a communication channel, the received sequence  $w_k \in \{1, \ldots, m+1\}$  is subject to channel noise and other uncertainties. When the communication channel is time invariant and memoryless, the relationship between  $s_k$  and  $w_k$  is characterized by the conditional probabilities  $\pi_{ij} = P\{w_k =$   $i|s_k = j\}, \quad i, j = 1, \dots, m + 1$ . It follows that with  $p_j := P\{s_k = j\}$  for  $j = 1, \dots, m+1$ ,

$$r_i = P\{w_k = i\} = \sum_{j=1}^{m+1} P\{w_k = i | s_k = j\} P\{s_k = j\}$$
$$= \sum_{j=1}^{m+1} \pi_{ij} p_j.$$

Let  $r = [r_1, \ldots, r_{m+1}]^T$ ,  $p = [p_1, \ldots, p_{m+1}]^T$ . Then

$$r = \begin{bmatrix} \pi_{11} & \cdots & \pi_{1,m+1} \\ \vdots & & \vdots \\ \pi_{m+1,1} & \cdots & \pi_{m+1,m+1} \end{bmatrix} p := \Pi p, \quad (8)$$

where

$$\Pi = \begin{bmatrix} \pi_{11} & \cdots & \pi_{1,m+1} \\ \vdots & \ddots & \vdots \\ \pi_{m+1,1} & \cdots & \pi_{m+1,m+1} \end{bmatrix}.$$
 (9)

**Assumption B.** (a)  $\Pi$  is invertible. (b) All  $p_i$ 's are strictly positive.

**Remark 1** Assumption B (a) ensures that probability information obtained at the receiver of the communication channel can be used to deduce the probabilities of the sensor thresholds, which are then used to estimate the system parameters. Since  $dp/dr = \Pi^{-1}$ , the variance of the estimation error depends proportionally on the operator norm of  $\Pi^{-1}$ . Under Assumption B, (8) yields that  $p = \Pi^{-1}r$ . If  $p_i = 0$ , the corresponding sensor threshold is not used. Such  $p_i$  can be eliminated from our consideration and the resulting pwill satisfy Assumption B (b).

Let  $\tilde{h}_i(\theta) = dr_i(\theta)/d\theta$  and  $\tilde{h}(\theta) = [\tilde{h}_1(\theta), \dots, \tilde{h}_{m+1}(\theta)]^T$ . Then  $\tilde{h} = \frac{dr}{d\theta} = \Pi \frac{dp}{d\theta} = \Pi h$ .

Lemma 2 [11] The Cramér-Rao Lower Bound for estimating  $\theta$  with observations on  $w_k$  is  $\widetilde{\sigma}_{CR}^2(N,m) = \left(N\sum_{i=1}^{m+1} \frac{\widetilde{h}_i^2}{r_i}\right)^{-1}.$ 

#### 4.3 **CR Ratio of Communication Channels**

Define  $D_p$ =  $\operatorname{diag}(p_1,\ldots,p_{m+1}),\quad D_r$ diag $(r_1, ..., r_{m+1}), \quad S_p = \sqrt{D_p}, \text{ and } S_r = \sqrt{D_r}.$  Then,  $\sum_{i=1}^{m+1} h_i^2/p_i = h^T D_p^{-1} h,$  $\sum_{i=1}^{m+1} \tilde{h}_i^2/r_i = \tilde{h}^T D_r^{-1} \tilde{h} = h^T \Pi^T D_r^{-1} \Pi h.$  It follows that

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$$\begin{split} &\sum_{i=1}^{m+1} \frac{h_i^2}{p_i} - \sum_{i=1}^{m+1} \frac{\widetilde{h}_i^2}{r_i} = h^T (D_p^{-1} - \Pi^T D_r^{-1} \Pi) h \\ &= h^T S_p^{-1} [I - (S_p \Pi^T S_r^{-1}) (S_r^{-1} \Pi S_p)] S_p^{-1} h \\ &= v^T (I - M^T M) v, \end{split}$$

 $\begin{array}{ll} \mbox{where } v = S_p^{-1}h, \; M = S_r^{-1}\Pi S_p. \\ \mbox{From} & \sigma_{CR}^2 & = & 1/(Nh^TD_p^{-1}h), \quad \widetilde{\sigma}_{CR}^2 \end{array}$ =  $1/(Nh^T\Pi^T D_p^{-1}\Pi h)$ , we define the error ratio of a communication channel by

$$\eta(p,h,\Pi) = \frac{\tilde{\sigma}_{CR}^2}{\sigma_{CR}^2} = \frac{h^T D_p^{-1} h}{h^T \Pi^T D_r^{-1} \Pi h}.$$
 (10)

h depends on actual function forms which satisfy only  $\mathbf{1}^T h = 0$ . Since h is not part of the communication channel, we introduce the following concept to characterize the worst-case impact of a communication channel on identification accuracy.

The CR ratio of a communication channel is defined as the worst-case error ratio

$$\eta(p,\Pi) = \max_{h \neq 0} \eta(p,h,\Pi) \text{ s.t. } \mathbf{1}^T h = 0.$$
 (11)

A communication channel is said to be *degenerate* if all singular values of M are equal to 1.

Theorem 1 Under Assumption B, if the channel is not degenerate, then  $\eta(p,\Pi) = \overline{\gamma}^2(M^{-1})$  where  $\overline{\gamma}$  is the largest singular value.

### Impact of Quantization on Com-5 **munication Channels**

The quantized signal will be transmitted through a WLAN (wireless local area network) channel. At a system level, the channel is modelled by (9)

$$\Pi = \begin{bmatrix} \pi_{11} & \cdots & \pi_{1,m+1} \\ \vdots & \ddots & \vdots \\ \pi_{m+1,1} & \cdots & \pi_{m+1,m+1} \end{bmatrix}.$$

The physical-level channels may vary. For instance, if the underlying modulation scheme is a BPSK (bi-phase shift keying) modulation, then a binary memoryless channel model may be used in representing the physical-level channel, with a probability matrix

$$\Pi_0 = \begin{bmatrix} \pi_{11}^0 & \pi_{12}^0 \\ \pi_{21}^0 & \pi_{22}^0 \end{bmatrix}.$$
 (12)

In this case,  $s_k$ , that takes m + 1 possible values, will be represented by a binary sequence of length  $l = \log_2(m+1)$ for transmission. The matrix  $\Pi$  can be derived from  $\Pi_0$ . For example, suppose  $m + 1 = 2^3$  and  $s_k$  takes values in  $\{1, 2, \dots, 8\}$ . Let 1 is coded by 000, 2 by 001, etc. If the binary sequences are independent, then  $\pi_{21} = P\{w_k =$  $1|s_k = 2\} = P\{000|001\} = \pi_{11}^0 \pi_{11}^0 \pi_{12}^0.$ 

In general, if  $l = \log_2(m+1)$  is an integer, the corresponding probability transition matrix  $\Pi$  in (9) under DBPSK modulation can be expressed as

$$\Pi = \underbrace{\Pi_0 \otimes \Pi_0 \otimes \cdots \otimes \Pi_0}_{l \text{ times}}$$

where  $\otimes$  is the Kronecker product.

Similar discussions can be made for DBPSK (differential bi-phase shift keying), DQPSK (differential quandary phase shift keying) modulation, or other modulation schemes which are used in IEEE 802.11b WLAN.

Communication errors increase when the signal/noise ratio decreases, or the transmission rate increases, or the assigned bandwidth decreases. The impact of signal power and bandwidth on the transmission channels is typically summarized in the normalized signal-to-noise ratio  $E_s/N_0$ , where  $E_s$  is energy per symbol and  $N_0$  is average noise power per unit bandwidth. This parameter defines the resource requirements since signal power and bandwidth are the key resources in a communication system. For a given physical level modulation, the transmission matrix  $\Pi$  defined in (9) depends on  $E_s/N_0$  and may be expressed as  $\Pi(E_s/N_0)$ . Roughly speaking, the larger the signal-tonoise ratio  $E_s/N_0$  is, the closer the matrix  $\Pi$  is to the identity matrix.

# 6 Quantization Design

Impact of quantization on identification can be explained in terms of identification accuracy and channels uncertainty.

# 6.1 Identification Accuracy

When one increases the number of quantization levels, m increases. The following result claims that the identification error will decrease.

Assume  $[y_{\min}, y_{\max}]$  is the range of values of the sequence  $\{y_k\}$ .

**Definition 1.** A placement of m sensors is a partition  $S_m = (y_{\min}, C_1, \ldots, C_m, y_{\max})$  with  $y_{\min} < C_1 < \cdots < C_m < y_{\max}$  of the interval  $[y_{\min}, y_{\max}]$  by m division points  $C_i$ , where  $C_i$  for  $i = 1, \ldots, m$  are the corresponding threshold values. In what follows, we also use the notation  $S_m = \{C_1, \ldots, C_m\}$  to denote the set of points of the threshold values.

**Definition 2.** Suppose  $m_1$  and  $m_2$  are two positive integers, and  $S_{m_1} = (y_{\min}, C_1^1, \ldots, C_{m_1}^1, y_{\max})$  and  $S_{m_2} = (y_{\min}, C_1^2, \ldots, C_{m_2}^2, y_{\max})$  are two placements of sensors. We say that  $S_{m_2}$  is a refinement of  $S_{m_1}$ , if  $\{y_{\min}, C_1^1, \ldots, C_{m_1}^1, y_{\max}\}$  is a subset of  $\{y_{\min}, C_1^2, \ldots, C_{m_2}^2, y_{\max}\}$ .

**Remark 1.** In the definition of placement of sensors,  $[y_{\min}, y_{\max}]$  can be either finite or infinite. In case one of these values is  $\infty$  or  $-\infty$ , it is understood that we work with the extended real number system. For practical utility, we have assumed that no sensor is placed at either  $y_{\min}$  or  $y_{\max}$ , otherwise they do not provide any useful information for system identification.

Note that the statement " $S_{m_2}$  is a refinement of  $S_{m_1}$ " means that  $S_{m_2}$  can be obtained by starting with the threshold points  $C_1^1 < \cdots < C_{m_1}^1$  and interposing  $m_2 - m_1$  points between them to form a finer subdivision.

**Theorem 2.** Suppose that  $S_{m_1}$  and  $S_{m_2}$  are two placements of sensor thresholds such that  $S_{m_2}$  is a refinement of  $S_{m_1}$ . For  $\iota = 1, 2$ , denote the corresponding quantities by  $\eta_{m_\iota}$ ,  $\mathbb{I}_{m_\iota}$ ,  $\Lambda_{m_\iota}$ ,  $\gamma_{m_\iota}$ , and  $M_{m_\iota}$ , respectively. Then given in satisfies  $\eta_{m_2} \leq \eta_{m_1}$ , which implies a reduction of error variance by increasing space complexity.

# 6.2 Quantization Design under Fixed Bandwidth

Bandwidth resources that limit data-flow rates will be denoted by R in bps. R is related to space and time complexities by  $R = N \log(m + 1)$ .

To express the dependence of  $\sigma_{CR}^2$  on the space complexity m, time complexity N, and the unknown parameter  $\theta$  we shall denote it by  $\sigma_{CR}^2(m, N, \theta)$ . The following two optimal resource allocation problems, which are dual problems in nature, are introduced:  $\mathbf{Z}_+$  will denote the set of positive integers. Suppose that the priori information on  $\theta$ is that  $\theta \in \Omega$ .

**1. Optimal Uncertainty Reduction:** This aims at reducing  $\sigma_{CR}^2(m, N, \theta)$  for a given resource *R*.

$$\varepsilon(R) = \min_{m \in \mathbf{Z}_+} \max_{\theta \in \Omega} \sigma_{CR}^2(m, N, \theta)$$

subject to  $N \log(m+1) \le R$ .

2. Optimal Resource Allocation: This aims at reducing R for a given error tolerance level  $\varepsilon$ , i.e.,  $\sigma_{CR}^2(m, N, \theta) \le \varepsilon$  for a given resource R.

$$R(\varepsilon) = \min_{m,N \in \mathbf{Z}_{+}} N \log(m+1)$$
  
subject to 
$$\max_{\theta \in \Omega} \sigma_{CR}^{2}(m,N,\theta) \le \varepsilon$$

One typical design result is shown in Figure 4. For this example, the optimal number of quantization thresholds is m = 3.



Figure 4: Space Complexity: Error variance vs.  $\log(m+1)$  (Top Plot); Error variance  $\times \log(m+1)$  vs.  $\log(m+1)$  (Bottom Plot)

# 6.3 Quantization Design without Bandwidth Limitations

Assume that the channel bandwidth is not limited (or the data rates are far below the assigned bandwidth limit). In this case, the data rates will be  $r = N \log(m + 1)$ . As a result, when one increases m, the rate r will increase.

We use IEEE 802.11b WLAN with DBPSK modulation as a typical communication environment for further development. The probability transition matrix  $\Pi_0$  in (12) under DBPSK modulation is

$$\Pi_0 = \left[ \begin{array}{cc} 1 - p_e & p_e \\ p_e & 1 - p_e \end{array} \right].$$

where by [6]

$$p_e = Q\left(\sqrt{\frac{2E_s}{N_0}}\right) \tag{13}$$

where Q is the complementary distribution function of a gaussian random variable. As a result,  $p_e$  will increase. Consequently,  $\eta(p, \Pi)$  will increase. The tradeoff between decreased identification errors  $\sigma_{CR}^2$  and increased  $\eta(p, \Pi)$  provide a performance index that can be used to select optimally the number of quantization levels.

A typical case of transmission errors as functions of signal-to-noise ratios with four WLAN transmission rates (1Mbps, 2Mbps, 5.5Mbps, 11Mbps) is shown in curve Figure 5.



Figure 5: PRD to Es/No using different WLAN transmission rates

# 7 Conclusions

Relationships between identification accuracy of patient dynamics and wireless communication channels are discussed using Wiener models as a typical model structure for methodology development. There is a fundamental trade-off between identification accuracy improvements and limitations in communication channels. For example, the more the quantization levels become, the higher the data rates will result in. Consequently, the data must be transmitted at a higher transmission speed, either consuming more bandwidth resources and/or increasing communication errors. For a given resource such as bandwidth and signal-to-noise ratio, there exists an optimal design on quantization that maximizes overall information accuracy after information processing and communications.

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