# Designing and Evaluation of an MC-DS-CDMA System with Dual-dimension Rake Receiver

Joy Iong-Zong Chen, Yen-Jung Su Department of Communication Engineering, Da Yeh University 112 Shan-Jiau, Rd., Da-Tsuen, Chang-Hua 51505 Taiwan (R.O.C.) Tel: +886-4-851-1888 ex:2523 Fax: +886-4-851-1245

## Abstract

The evaluation of system performance of an MC-DS-CDMA (multicarrier direct-sequence coded-division multiple-access) system with dual-dimension (antenna diversity) Rake receiver is proposed in this investigation. The working environment for the scenario is considered as situating in frequency selective fading. Furthermore, some of the system parameters, e.g., the resolvable multipath number, the number of a Rake receiver, the fading parameter the power decay factor MIP (multipath intensity profile), and the correlation characteristic between the antennas, are adopted for analyzing. It is the original proposal of such scenario with dual-dimension Rake receiver for the MC-DS-CDMA system. In order to validate the accuracy of the derivative, a lot of numerical results are conduct in this paper.

Key Words: MC-DS-CDMA system, dual-dimension Rake, MIP, antenna diversity

## **I. Introduction**

Recently, the wideband radio systems combining with the applications of multicarrier modulating and CDMA (coded-division multiple-access) schemes has been considerable interesting in the cellular wireless communications. It is the reason that DS (direct sequence) waveform exist the broader bandwidth for combating the ISI (inter-symbol interference) caused by the multipath fading during the transmission. Generally, a Rake receiver can be adopted to avoid the multipath fading and promote the system performance whenever the bandwidth of the propagated signal exceeds the coherence bandwidth of the channel [1, 2]. However, in order to create broader bandwidth and suppress the interference effect for a wireless radio system the multicarrier scheme has been applied. Based on the motivation, the 4G (fourth generation) system, MC-CDMA (multi-carrier CDMA), which based on the OFDM (orthogonal frequency division multiplexing) signaling techniques, are now engaged in exploring [4]. One of the most important types of multi-carrier CDMA system is called MC-DS-CDMA (multi-carrier direct-sequence CDMA) system. Generally, multi-carrier DS systems can be categorized into two types: (1). a combination of OFDM and CDMA, and (2). a parallel transmission-scheme of narrowband DS waveform in the frequency domain [6]. However, empirical results as well as physical reasoning have shown the fact that the Nakagami-m distribution is not only a more versatile model for a variety of fading environment such as urban, suburban radio for multipath channels and even indoor propagation, but also including Rayleigh distribution as a special case [6]. In this paper we propose a multiple-dimension Rake receiver, called as M-D Rake receiver, that adopts both multipath and antenna diversity at the output end. The system performance with BER (bit-error rate) of the MC-DS-CDMA system with M-D Rake receiver is evaluated under the assumption both of the independent and correlated characteristics between the paths.

## **II. System Models**

### A. Transmitter Model

The transmitter of an MC-DS-CDMA system block diagram proposed in [5] is adopted in this paper. The transmitted signal of a MC-DS-CDMA system of the *k*-th user is given as [2]

$$s^{(k)}(t) = \sqrt{2E_c} \sum_{n=-\infty}^{\infty} c_n^{(k)} d_{\lfloor n/N \rfloor}^{(k)} h(t - nMT_c - \tau^{(k)})$$

$$\times \sum_{i=1}^{N} \operatorname{Re}\left[e^{j(2\pi f_i t + \theta_i^{(k)})}\right]$$
(1)

where  $E_c$  is the chip energy,  $c_n^{(k)}$  is the pseudo-random spreading sequence,  $d_{\lfloor n/N \rfloor}^{(k)} \in \{+1, -1\}$  denotes the data bit of the *k*-th user,  $\lfloor \cdot \rfloor$  is the floor function extracts the integer value, where *N* indicates the length of *PN* (pseudo noise) sequence, h(t) is the impulse response of the chip wave shaping filter,  $\tau^{(k)}$  is an arbitrary time delay uniformly distributed over  $[0, NMT_c]$ , Re[·] denotes the real part,  $\theta_i^{(k)}$ and  $f_i$ 's, i = 1, 2, ..., M are a random carrier phase uniformly distributed over  $(0, 2\pi]$  and the carrier frequencies, respectively.

### B. Channel Model

The low-pass equivalent impulse response of the bandpass channel from the transmitter to the j-th receive antenna for the k-th user can be written as

$$h_{j}(t) = \sum_{l=0}^{L^{(k)}-1} \alpha_{j,l}^{(k)} e^{j\varphi_{j,l}^{(k)}} \delta(\tau - \tau_{j,l}^{(k)})$$
(2)

where  $L^{(k)}$  is the number of resolvable propagation paths that reach the receive antenna. Each path is characterized by its instantaneous fading amplitude  $\alpha_{j,l}^{(k)}$ , its phase shift, and its propagation delay  $\tau_{j,l}^{(k)}$ . Assuming a Nakagami-*m* fading channel model, the instantaneous power of the *l*-th path,  $l = 0, 1, ..., L^{(k)} - 1$ , follows the gamma PDF

$$f_{\gamma_{j,l}}^{(k)}(\gamma_l) = \frac{(m_{j,l}^{(k)} / \Omega_{j,l}^{(k)})^{m_l^{(k)}}}{\Gamma(m_{j,l}^{(k)})} (\gamma_l)^{m_{j,l}^{(k)} - 1} \cdot \exp\{-\frac{m_{j,l}^{(k)}}{\Omega_{j,l}^{(k)}}\}, \ \gamma_l \ge 0$$
(3)

where  $\Omega_l^{(k)} = E[(\alpha_l^{(k)})^2]$  is its average channel power. In addition, the total time average channel gain per antenna for each user is normalized to one, i.e.,

$$\sum_{l=0}^{L^{(k)}-1} E[(\alpha_{j,l}^{(k)})^2] = \sum_{l=0}^{L^{(k)}-1} \Omega_{j,l}^{(k)} = 1$$
(4)

In order to consider the real world case of the wireless mobile channel, generally the MIP follows the exponential relationship and is given as

$$\Omega_{j,l}^{(k)} = \Omega_{j,0}^{(k)} e^{-l\delta}, \ l = 0, 1, ..., L^{(k)} - 1$$
(5)

where the parameter  $\delta$  indicates the rate of decay of the average path strength as a function of the path delay.

#### C. Receiver Model

The dual-dimension Rake receiver block diagram of a MC-DS-CDMA system with BPF (band-pass filter) and LPF (low pass filter) for the referenced user (the first subscriber) is illustrated in Fig. 1,  $r_t(t)$  can be obtained as

$$r_{i}(t) = \sum_{k=1}^{K} \sqrt{2E_{c}} \sum_{n=-\infty}^{\infty} d_{b}^{(k)} c_{n}^{(k)} h(t - nMT_{c} - \tau^{(k)}) \\ \times \sum_{i=1}^{M} \alpha_{i}^{(k)} e^{j(\theta_{i}^{(k)} + \alpha_{i}^{(k)})} + z(t) + z_{j}(t)$$
(6)

where the subscript *l* represents the low-pass, the symbol  $\otimes$  denotes the convolution operator,  $\delta(\tau)$  is the impulse function, z(t) is the equivalent low pass AWGN,  $z_J(t)$  indicates PBI after low pass filter. The complex low-pass equivalent impulse response of the *i*-th channel is  $\{c_i = \xi_i \cdot \delta(t), i = 1, ..., M\}$ , and  $\xi_i^{(k)} = \alpha_i^{(k)} \exp(j\varepsilon_i^{(k)})$ , where  $\alpha_i^{(k)}$  and  $\varepsilon_i^{(k)}$  are corresponding to represent the attenuation factor and phase-shift of the *i*-th channel of the *k*-th user. The complex equivalent impulse response of the channel is expressed as  $c_I(t) = \sum_{l=1}^{L} \alpha_l \delta(t - lT_c)$ . Hence the received signal at the output of receiver is given as [5]

$$r(t) = \sum_{k=1}^{K} \left\{ \sqrt{2E_c} \sum_{n=-\infty}^{\infty} d_h^{(k)} c_n^{(k)} h(t - nMT_c - \tau^{(k)}) \right.$$

$$\times \left. \sum_{i=1}^{M} \alpha_i^{(k)} \cos(2\pi f_i t + \psi_i^{(k)}) \right\} + N_w(t) + N_J(t)$$
(7)

where *K* denotes the user number,  $\psi_i^{(k)} = \theta_i^{(k)} + \varepsilon_i^{(k)}$ ,  $N_w(t)$  is AWGN with a double-sided PSD (power spectral density) of  $\eta_0/2$ ,  $N_j(t)$  is partial band of Gaussian interference with a PSD of  $S_{n_j}(f)$ , which is written as

$$S_{n_{J}}(f) = \begin{cases} \frac{\eta_{J}}{2}, & f_{J} - \frac{W_{J}}{2} \le |f| \le f_{J} + \frac{W_{J}}{2} \\ 0, & otherwise \end{cases}$$
(8)

where  $f_J$  and  $W_J$  represent the bandwidth of the interference and the center frequency, respectively. Then the interference (Jamming)-to-signal ratio, JSR, is defined as the ratio of the interference power value to signal power, and can be written as

$$JSR = \frac{\eta_J W_J}{E_b/T} = (1+\mu) \frac{\eta_J}{E_b} \frac{N}{M}$$
(9)

The output signals after LPF,  $\zeta_i^{(1)}(t)$ , i = 1,..., M, of the chip-matched filter in the branch for the referenced user is given by

$$\zeta_{i}^{(1)}(t) = D_{\zeta_{i}}^{(1)}(t) + MAI_{\zeta_{i}}^{(1)}(t) + JSR_{\zeta_{i}}^{(1)}(t) + N_{\zeta_{i}}^{(1)}(t)$$
(10)

The first term of (10) indicates the desired signal of the referenced user can be written as

$$D_{\zeta_i}(t) = \sqrt{E_c} \alpha_i \sum_{n=-\infty}^{\infty} d_n c_n x(t - nMT_c)$$
(11)

where  $\alpha_i$ , i = 1,...,M, denotes as fading envelopes characterized as Nakagami-*m* RV's (random variables). The second term in (10) is the interference comes from the other users, called as MAI (multiple access interference), and can be determined as (the superscript of user will be omitted form here)

$$MAI_{\zeta_i}(t) = \sum_{k=2}^{K} \left\{ \sqrt{E_c} \xi_i \sum_{n=-\infty}^{\infty} d_n \cdot c_n \cdot x(t - nMT_c - \tau) \right\}$$
(12)

where  $\xi_i \equiv \alpha_i \cos \phi_i$  is the envelope of the MAI component, which is allowed to approximate Gaussian random variable under the assumption with large user number *K* [3]. The third term in (10) is the JSR defined in (9) and can be represented as

$$JSR_{\zeta_{i}}(t) = Lp \left\{ \sqrt{2n_{i,j}}(t) \cos(2\pi f_{i}t + \psi_{i}) \right\}$$
(13)

and the last term of (10) presents the output signal caused by the fact that the AWGN can be expressed as

$$N_{\zeta_i}(t) = Lp\left[\sqrt{2}n_{i,w}(t)\cos(2\pi f_i t + \psi_i)\right]$$
(14)

where the terms  $n_{i,j}(t)$  in (13) and  $n_{i,w}(t)$  in

(14) are results from passing  $N_J(t)$  and  $N_w(t)$  into (7), respectively, through the *i*-th bandpass filter. The SNR (signal-to-noise ratio) at the output are to be determined and expressed as

$$\chi_i = D_{\chi_i} + MAI_{\chi_i} + JSR_{\chi_i} + N_{\chi_i}$$
(15)

where the representation of each terms are adopted as the same results that evaluated and shown in [5]. We follow the analysis in [7], therefore, conditioned on the channel amplitudes  $\alpha_{j,n}$ , the noise variance of the *n*-th Rake finger of the *j*-th antenna due to all the multiple access users in the same cell is given by

$$\sigma_{mai,j,n}^{2} = \frac{E_{b}T}{6N} (\alpha_{n})^{2} \sum_{k=2}^{K} \sum_{l=0}^{L-1} \Omega_{j,l}$$
(16)

Similarly, the variance of the SI (self-interference) due to multipath is approximated by [7]

$$Var\{J_{Zi}\} = NR_{J_i}(0) + \sum_{a=1}^{N-1} R_{J_i}(aMT_c) \cdot 2\sum_{n=a}^{N-1} c_n c_{n-a} \quad (17)$$

, and the variance due to AWGN is

$$Var\{N_{zi}\} = N\frac{\eta_0}{2} \tag{18}$$

where  $\Omega_l$  denotes the average power of the *l*-th path in the *j*-th antenna, and  $E_b = PT$  is the received signal energy per bit per antenna. Without loss of generality, we assume identical MIP among receive antennas (i.e.,  $\Omega_l = \Omega_l$ ,  $j = 1, 2, ..., M_r$ ). The desired signal is with mean

$$U_{s} = \sqrt{\frac{E_{b}T}{2}} \sum_{j=1}^{2} \sum_{n=0}^{L_{s}-1} (\alpha_{j,n})^{2}$$
(19)

, and variance given as

$$\sigma_{i}^{2} = Var\{MAI_{zi}\} + Var\{N_{zi}\} + Var\{J_{zi}\}$$

$$= \frac{(K-1)NE_{c}\Omega_{i}}{2} \left(1 - \frac{\mu}{4}\right) + \frac{N\eta_{0}}{2} + \frac{N\eta_{J}}{4}$$
(20)

The signal-to-interference and noise ratio (SINR) at the MRC output is

$$\gamma = \frac{U_s^2}{2\sigma_T^2} = \sigma_0 \sum_{j=1}^2 \sum_{n=0}^{L_r-1} (\alpha_{j,n})^2$$
(21)

Furthermore, with the help of (4), we obtain

$$\sigma_0 = \left\{ \frac{m(K-1)}{N} (1 - \frac{\mu}{4}) + \frac{1}{\eta_0 / E_b} mL \right\}^{-1}$$
(22)

## III. PDF of Dual Dimension Rake Decision Statistics

The instantaneous SINR  $\gamma$  at the output of the dual-dimension Rake receiver was shown in the previous section, and the expression can be written as

$$\gamma = \sum_{j=1}^{2} \sum_{n=0}^{L_r - 1} \gamma_{j,n}$$
(23)

where the antenna number at the receiver is preset as  $M_r = 2$  for the dual-dimension Rake receiver,  $L_r$  indicates the finger number of the Rake receiver, and  $\gamma_{i,n} = \sigma_0(\alpha_{i,n})^2$  is the instantaneous SINR of the *n*-th finger of he *j*-th antenna. On the other hand, when the fading figure are real and arbitrary, the PDF of  $\gamma$  is given as an approximate expression and shown [6], it can be expressed as an indefinite integral in [8] too. The other most common mean for determining the PDF of  $\gamma$  is the method of adopting the product of the MGF (moment generating function), and followed by the calculation of the inverse Laplace transform. Hence, by means of substituting (21) into the MGF formula, the MGF of  $\gamma_{i,n}$  can be expressed as

expressed a

$$M_{\gamma_{j,n}}(t) = \frac{(y_{j,n})^{m_{j,n}}}{\Gamma(m_{j,n})} \int_0^\infty e^{-xt} x^{m_{j,n}-1} \exp(-y_{j,n}x) dx$$
(24)

where

$$y_{j,n} = \frac{m_{j,n}}{\sigma_0 \Omega_{j,n}}, \ j = 1, 2, ..., M_r, \ n = 0, 1, ..., L_r - 1$$
 (25)

However, the integral in (24) will be determined by using of the approach proposed in [11]. Firstly, the exponential function may be expressed as a contour integral  $\exp(-x) = \int_{-i\infty}^{\infty} \Gamma(-s) x^s ds / 2\pi i$  [15, p. 43], where  $i = \sqrt{-1}$ . By interchanging the order of integration and substituting it into the MGF formula (24), which can be derived as

$$M_{\gamma_{j,n}}(t) = \frac{1}{\Gamma(m_{j,n})} \left(\frac{y_{j,n}}{t}\right)^{m_{j,n}} \frac{1}{2\pi i} \\ \times \int_{\mathbb{C}} \Gamma(-s) \Gamma(m_{j,n} + s) \left(\frac{y_{j,n}}{t}\right)^{s} ds$$
(27)

where the components of the multipath channel are assumed all identical and via the imaginary axis (in the complex s-plane), separating the poles of  $\Gamma(m_{j,n} + s)$ ,  $j=1, 2, ..., M_r$ , n=0,1, ...,  $L_r - 1$  from the poles of  $\Gamma(-s)$ . Thus following the inverse Laplace the PDF,  $f_{\gamma}(\gamma)$ , of the SINR can be calculated in terms of the confluent form of the multivariate Lauricella hyper-geometric function and obtained as [6]

$$f_{\gamma}(\gamma) = \frac{1}{\Gamma(\sum_{j=1}^{2} \sum_{n=0}^{L_{r}-1} m_{j,n})} \left[ \prod_{j=1}^{2} \prod_{n=0}^{L_{r}-1} (y_{j,n})^{m_{j,n}} \right] \\ \times \gamma^{\left(\sum_{j=1}^{2} \sum_{n=0}^{L_{r}-1} m_{j,n}\right)^{-1}} \Phi_{2}^{(2-L_{r})}(m_{1,0}, m_{1,1}, \dots, m_{2,L_{r}-1}; \\ \sum_{j=1}^{2} \sum_{n=0}^{L_{r}-1} m_{j,n}; -y_{1,0}\gamma, -y_{1,1}\gamma, \dots, -y_{2,L_{r}-1}\gamma)$$
(28)

where  $\Phi_2^{(n)}(b_1,...,b_n;c;x_1,...,x_n)$  is the confluent Lauricella hyper-geometric function define as[10]

$$\Phi_{2}^{(n)}(b_{1},...,b_{n};c;x_{1},...,x_{n}) = \sum_{i_{1},...,i_{n}=0}^{\infty} \frac{(b_{1})_{i_{1}}...(b_{n})_{i_{n}}}{(c)_{i_{1}+...+i_{n}}} \frac{x_{1}^{i_{1}}}{i_{1}!}...\frac{x_{n}^{i_{n}}}{i_{n}!}$$
(29)

The parameters  $y_{j,n}$  shown in (28) are defined in (25) equals to the ratio of the amount of fading  $m_{j,n}$  divided by the corresponding average SINR,  $\sigma_0 \Omega_{j,n}$ , of the *n*-th Rake finger of the *j*-th antenna. For the negative exponential MIP with power decay factor,  $\delta$ , from (4) and (5), the average power of the *n*-th path can be written as [1]

$$\Omega_{j,n} = \frac{e^{-n\sigma}}{q(L,\delta)}, \quad j = 1, 2, ..., M_r, \quad n = 0, 1, ..., L_r - 1$$
(30)

where the number of multipath L is considered for the desired user and the

$$q(L,\delta) = \sum_{l=0}^{L-1} e^{-l\delta} = \frac{1 - e^{-L\delta}}{1 - e^{-\delta}}$$
(31)

### **IV. Bit Error Probability**

4.1 System BER with un-correlated channels The coherent BPSK (binary phase shift keying) was applied as the modulator at the post-detector. It is known that the conditional BER of that in AWGN channel is given by [1]

$$P_{E}(\gamma) = Q(\sqrt{2\gamma}) = \frac{\Gamma(\frac{1}{2},\gamma)}{2\sqrt{\pi}}$$
(32)

where  $Q(\cdot)$  is the Q-function, and  $\Gamma(\alpha, x) = \int_x^{\infty} t^{\alpha-1} e^{-t} dt$  denotes the complementary incomplete gamma function [11, (8.350-2)]. By means of the random process the average BER of the dual-dimension Rake for MC-CDMA system working in Nakagami-*m* fading channel is then written as

$$\overline{P}_{E} = \int_{0}^{\infty} P_{E}(\gamma) f_{\gamma}(\gamma) d\gamma$$
(33)

Next, by the definition of the Lauricella multivariate hyper-geometric function  $F_D^{(n)}(...)$  shown as  $[19 \cdot 20]$ 

$$F_{D}^{(n)}(\alpha, b_{1}, ..., b_{n}; c; x_{1}, ..., x_{n}) = \frac{\Gamma(c)}{\Gamma(\alpha)\Gamma(c - \alpha)} \int_{0}^{1} t^{\alpha - 1} (1 - t)^{c - \alpha - 1}$$

$$\times \prod_{i=1}^{n} (1 - x_{i}t)^{-b_{i}} dt, \ \operatorname{Re}(c) > \operatorname{Re}(\alpha) > 0$$
(34)

where the Lauricella function  $F_D^{(n)}(...)$  for the order of n = 2 is able to provided with a library function in a common mathematical software package. The convergence of the Lauricella multivariate hyper-geometric function in (34) can be shown by using of the following transformation [11]

$$F_{D}^{(n)}(\alpha, b_{1}, ..., b_{n}; c; x_{1}, ..., x_{n}) = \left[\prod_{i=1}^{n} (1 - x_{i})^{-b_{i}}\right]$$
  
=  $F_{D}^{(n)}\left(c - a, b_{1}, ..., b_{n}; c; \frac{x_{1}}{x_{1} - 1}, ..., \frac{x_{n}}{x_{n} - 1}\right)$  (35)

Therefore, the system BER in (33) is going to be determined as an equation function of Lauricella multivariate hyper-geometric and shown as

$$\overline{P}_{E} = \frac{\Gamma(\frac{1}{2} + \sum_{j=1}^{2} \sum_{n=0}^{L_{r}-1} m_{j,n})}{2\sqrt{\pi}\Gamma(1 + \sum_{j=1}^{2} \sum_{n=0}^{L_{r}-1} m_{j,n})} \times \left[\prod_{j=1}^{2} \prod_{n=0}^{L_{r}-1} (\frac{y_{j,n}}{y_{j,n}+1})^{m_{j,n}}\right] \cdot F_{D}^{(2:L_{r})}(\frac{1}{2} + m_{1,0}, m_{1,1}, ..., m_{2,L_{r}-1}; (36)) \\ 1 + \sum_{j=1}^{2} \sum_{n=0}^{L_{r}-1} m_{j,n}, \frac{y_{1,0}}{y_{1,0}+1}, \frac{y_{1,1}}{y_{1,1}+1}, ..., \frac{y_{2,L_{R}-1}}{y_{2,1n-1}+1})$$

The average BER presented in the previous equation converges for all practical values of the system parameters (fading parameters), and where the representation in (36) provides a convenient method for fast and accurate numerical computation of the multivatiate hyper-geometric function.

#### 4.2 System BER with correlated channels

The degree of correlation depends on the distance between the antennas and their configuration [8]. Let the sequence of branch SINRs,  $\{\gamma_j\}_{j=1}^{M_r}$ , be a set of correlated, but not necessarily identically distributed, gamma varieties with parameters  $m_n$  and  $\sigma_0 \Omega_{j,n}$ , respectively, and let  $\rho_{i,j}^{(n)}$ denote the correlation coefficient between  $\gamma_i^{(n)}$ and  $\gamma_i^{(n)}$ , i.e.

$$\rho_{i,j}^{(n)} = \rho_{j,i}^{(n)} = \frac{Cov(\gamma_i^{(n)}, \gamma_j^{(n)})}{\sqrt{\operatorname{var}(\gamma_i^{(n)}) \cdot \operatorname{var}(\gamma_j^{(n)})}}, \ 0 \le \rho_{i,j}^{(n)} \le 1$$
(37)

where *i*,  $j = 1, 2, ..., M_r$ , and  $n = 0, 1, ..., L_r - 1$ . From [16], we find that the CF (characteristic function) of the instantaneous SINR  $\gamma$  given in (23) is obtained as

$$M_{\gamma}(t) = \prod_{n=1}^{L_{\gamma}-1} \prod_{j=1}^{2} (1 + t \cdot (y_{j,n})^{-1} \lambda_{j}^{(n)})^{-m_{n}}$$
(38)

where  $|\cdot|$  is the determinant operator, and  $I_{M_r}$ is the  $M_r \times M_r$  identity matrix. The matrices  $A^{(n)}$ ,  $n = 0, 1, ..., L_r - 1$ , are  $M_r \times M_r$  diagonal matrices with entries  $\sigma_0 \Omega_{j,n} / m_n = (y_{i,n})^{-1}$ , and  $C^{(n)}$ ,  $n=0, 1, ..., L_r - 1$ , are  $M_r \times M_r$  positive definite matrices defined by

$$C^{(n)} = \begin{bmatrix} 1 & \sqrt{\rho_{12}^{(n)}} & \dots & \sqrt{\rho_{12}^{(n)}} \\ \sqrt{\rho_{21}^{(n)}} & 1 & \dots & \sqrt{\rho_{22}^{(n)}} \\ \vdots & \dots & \dots & \vdots \\ \sqrt{\rho_{21}^{(n)}} & \dots & \dots & 1 \end{bmatrix}$$
(39)

In (38),  $\lambda_j^{(n)}$ , are the  $M_r$  eigenvalues of matrix  $C^{(n)}$ . In the case of independent fading among the receive antenna, we have  $\lambda_j^{(n)}=1$ ,  $j=1, 2, ..., M_r$ ,  $n=0, 1, ..., L_r - 1$ . It can be shown that (38) reduces to [19, p. 44].

$$M_{\gamma}(t) = \prod_{n=0}^{L_{\gamma}-1} \prod_{j=1}^{2} ((y_{j,n})^{-1} \lambda_{j}^{(n)} t)^{-m_{n}} F_{0}(m_{n}; -; -\frac{y_{j,n}}{\lambda_{j}^{(n)} t})$$
(40)

The confluent hyper-geometric function may be written as a Barnes-Mellin contour-type integral [10, p. 43]:

$${}_{1}F_{0}(m_{n};-;-p\cdot t) = \frac{1}{2\pi i} \int_{\mathbb{C}} \frac{\Gamma(m+s)}{\Gamma(m)} \Gamma(-s(p\cdot t)^{s} ds$$
(41)

By substituting (40) into (41), the CF becomes as

$$M_{\gamma}(t) = \left(\frac{1}{2\pi i}\right)^{2L_{\gamma}} \int_{\mathbb{C}_{1}} \int_{\mathbb{C}_{2}} \dots \int_{\mathbb{C}_{2L_{R}}} \left\{ \prod_{j=1}^{2} \prod_{n=0}^{L_{r-1}} \frac{1}{\Gamma(m_{n})} \cdot \left(\frac{y_{j,n}}{\lambda_{j}^{(n)}t}\right)^{m_{n}} \right\}$$

$$\times \Gamma(-s_{j,n}) \Gamma(m_{n} + s_{j,n}) \cdot \left(\frac{a_{j,n}}{\lambda_{j}^{(n)}t}\right)^{s_{j,n}} ds_{1,0} ds_{1,1} \dots ds_{2L_{r-1}}$$

$$(42)$$

which is very similar to (30). It then follows that the PDF of  $\gamma$  in the case of correlated fading among Rake fingers with the same path delay in spatially separated antennas is given by

$$f_{\gamma}(\gamma) = \frac{1}{\Gamma(2 \cdot \sum_{n=0}^{L_{\gamma}-1} m_{n})} \left[ \prod_{j=1}^{2} \prod_{n=0}^{L_{\gamma}-1} \left( \frac{y_{j,n}}{\lambda_{j}^{(n)}} \right)^{m_{n}} \right] \times \Upsilon^{(2 \cdot \sum_{n=0}^{L_{\gamma}-1} m_{n})-1} \cdot \Phi_{2}^{2, L_{\gamma}}(m_{1,0}, m_{1,1}, ..., m_{1,1}, ..., m_{2,L_{\gamma}-1};; 2\sum_{n=0}^{L_{\gamma}-1} m_{n}; -\frac{y_{1,0}}{\lambda_{1}^{(0)}}\gamma, -\frac{y_{1,1}}{\lambda_{1}^{(1)}}\gamma, ..., -\frac{y_{2,L_{\gamma}-1}}{\lambda_{2}^{(L_{\gamma}-1)}}\gamma)$$
(43)

with the only restriction that  $m_{j,n} = m_n$  for  $j = 1, 2, ..., M_r$ . Similarly, the average BER in the spatially correlated Nakagami-*m* fading channel becomes

$$\overline{P}_{E} = \frac{\Gamma(\frac{1}{2} + \sum_{j=1}^{2} \sum_{n=0}^{L_{r}-1} m_{n})}{2\sqrt{\pi}\Gamma(1 + \sum_{j=1}^{2} \sum_{n=0}^{L_{r}-1} m_{n})} \left[ \prod_{j=1}^{2} \prod_{n=0}^{L_{r}-1} (\frac{y_{j,n}}{y_{j,n} + \lambda_{j}^{(n)}})^{m_{n}} \right] \times F_{D}^{(2,L_{r})}(\frac{1}{2} + m_{1,0}, m_{1,1}, ..., m_{M_{r},L_{r}-1};$$

$$1 + 2\sum_{n=0}^{L_{r}-1} m_{n}; \frac{y_{1,0}}{y_{1,0} + \lambda_{1}^{(0)}} \gamma_{th},$$

$$\frac{y_{1,1}}{y_{1,1} + \lambda_{1}^{(1)}} \gamma_{th}, ..., \frac{y_{2,L_{R}-1}}{y_{2,L_{r}-1} + \lambda_{2}^{(L_{r}-1)}} \gamma_{th})$$
(44)

### V. Numerical Results

Accordingly, the effect of the power decay factor,  $\delta$ , with different antenna numbers,  $M_r = 1$  (1D-Rake) and  $M_r = 2$  (2D-Rake), and different Rake finger numbers,  $L_r = 2$ , 4 and 6, are presented in Fig. 2. It is except reasonable to describe that the system performance become superior when the antenna number increases and the lever number of power decay factor will deteriorate the system performance. The fading MIP and the degradation of the number of Rake finger on the system BER performance is shown in Fig. 3, in which the finger number of Rake receiver are varied with  $L_r = 2$ , 4 and 6, the subscriber k=25, and power decay factor are varied with  $\delta = 0, 0.5, \text{ and } 1$ . It is valuable to describe that the higher values of power decay factor can supply better performance when the finger number of Rake is  $L_r = 2$ . The lower decay factor includes the higher power. In words, when the Rake finger is 4, the medium values of power decay factor can supply better performance inversely compared to the lower values. The reasons are that: (1) The balance of the combined Rake fingers can be reached, (2) The power of the combination of the Rake receiver is stronger, and thus it can obtain higher gain at the diversity receiver. In Fig. 4 the bit SNR versus BER curves are illustrated for comparing with the different number of subcarrier number. Three different subcarrier numbers are illustrated, the path number and the subscriber number are  $L = L_r = 4$ and k = 50, respectively. It is worthwhile noting that the larger number of subcarrier the superior performance of the MC-CDMA system is. This is accordance with the research report in [5].

### **VI.** Conclusions

The extension of system performance of MC-DS-CDMA system is investigated in this paper. The antenna diversity is adopted as a dual-dimension Rake receiver and the fading channel is characterized as the versatile Nakagami-*m* statistical distributed. The most important parameters are applied for the numerical analysis.



Fig.1. The receiver block diagram of dual-dimension Rake receiver.



Fig.2. Average BER versus  $E_b/\eta_0$  per antenna for 1-D and 2-D Rake receivers.



Fig.3. Average BER versus  $E_b/\eta_0$  for 1-D Rake receivers.



Fig.4. Average BER versus  $E_b/\eta_0$  per antenna for 1-D and 2-D Rake receivers.

References :

- [1] J. Proakis, *Digital communications*. New York: McGraw-Hill, 1989.
- [2] S. Kondo and L. B. Milstein, "On the use of multicarrier direct sequence spread spectrum systems," in *Proc. IEEE MILCOM* '93. Boston. MA, pp. 52-56, Oct. 1963.
- [3] N. Yee, J. P. Linnartz, and G. Fettweis, "Multi-carrier CDMA in indoor wireless radio," in Proc. PIMRC '93, Yokohama, Japan, pp. D1.3.-1.5, Dec. 1993.
- [4] L. -L. Yang, and L. Hanzo, "Multicarrier DS-CDMA: A multiple access scheme for ubiquitous broadband wireless communications," *IEEE Commun. Mag.*, pp. 116-124, Oct. 2003.
- [5] S. Kondo, and L. B. Milstein, "Performance of multicarrier DS-CDMA system," *IEEE Trans. on Commun.*, Vol. 44, pp. 238-246, Feb. 1996.
- [6] M. Nakagami, The m-distribution- A general formula of intensity distribution of rapid fading, in *statistical methods in radio wave propagation*. Oxford, U. K.: Pergamon, pp. 3-36, 1960
- T. Eng and L. Milstein, "Coherent DS-CDMA performance in Nakagami multipath fading," *IEEE Trans. Commun.*, vol. 43, no.3-4, pp.1134-1143,Feb./Apr.1995.
- [8]G. Efthymoglou and V. A. Aalo, "Performance of Rake receivers in Nakagami fading channel with arbitrary fading parameters," *Electron. Lett.*, vol.31 pp.1610-1612, Aug.1995.
- [9] J. Luo, J. Zeidler, and J. Proakis, "Error probability performance for W-CDMA systems with multiple transmit and receive antennas in correlated Nakagami fading channels," *IEEE Trans. Veh. Technol.*, vol.51, no. 6, pp.1502-1516, Dec. 2002.
- [10] H. M. Srivastava and H. L. Manocha, A treatise on generating functions. New York: Wiley, 1984.
- [11] V. A. Aalo, T. Piboongungon, and G.P. "Another look Efthymoglou, at the performance of MRC schemes in Nakagami-*m* fading channels with arbitrary parameters," IEEE Trans. Commun., Commun., in press.