On the OFDM Systems Combining with Different Diversities over Selective Fading Channels

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Abstract: - In this paper the system performance of OFDM (orthogonal frequency division multiplexing) with the cases of MRC (maximal ratio combining) and dual branch SC (selective combining) diversities over correlated-Gamma distributed and correlated-Weibull fading, respectively, are investigated. Since the reason of an alternative expression of the Q-function is adopted for deriving the results of average BER (bit-error rate) of the OFDM system, the obtained formulas are not only calculated much simpler but the conducting of numerical analysis is also arrived at easily and accurate. It is valuable to claim that the system performance of the OFDM system is definitely dominated by the propagation environments, which is decided by the fading parameters both of the Nakagami-*m* and Weibull distributed, of the transmission of the radio systems.
Key Words: OFDM signaling, MRC, SC, Nakagami-*m* fading, Weibull fading

I. Introduction

It is known that the OFDM (orthogonal frequency division multiplexing) system is one of the important radio communication systems for wide-band transmission techniques. As usual, there are some of the statistical models utilized in the wireless communication fading channels, *i.g*. Rayeigh, Rice, Nakagami-*m* (Gamma) [1], and Weibull distributes. Furthermore, according to the results of experimental measurements, it has demonstrated that the characteristic of Nakagami-*m* distribution is versatile than Rayleigh and Rice distribution in the urban area and it can include the special case as of one-sided Gaussian and Rayleigh distributions [2]. In [3] Weibull proposed the Weibull distribution first for estimating machinery lifetime and become popularly used in several fields of science. The reasons of assuming that the fading channel characterized by the Weibull distributed in this paper is not only it can be regarded as an approximation to the generalized Nakagami-*m* distributed. Anyway, the issues of wireless communications over Weibull fading environment have begun to attract much interesting of the researchers. For example, the results presented in [4] evaluated the performance of linear diversity of GSC (generalized-selection combining) over independent Weibull fading channel. The authors Sagias *et al.* dealt with the performance of SC (selection combining) diversity by means of evaluating the average *SNR* (signal-to-noise ratio) with the parameters of AOF (amount of fading) and switching rate in [6].

Traditionally, the suboptimal one is SC diversity, which is operated by choosing the highest signal intensity branch at output of the combiner and the noise power among the different branches are assumed all equivalent [7, 8].

Recently, Kang, et. al. in [9] proposed the results from evaluating the system performance of OFDM system with receiver diversity over correlated Nakagami-*m* distribution. The investigation of OFDM system mixing with BFSK (binary frequency-shift keying) scheme, called as OFDM-BFSK system, over underwater radio environments modeled as Rayleigh and Rice fading is researched by the authors Glavieux et. al. in [10]. In [11] Lu et. al. evaluated the BER (bit-error rate) performance for so called OFDM-MDPSK (OFDM M-ary differential phase-shift keying) system with diversity reception under the consideration of the LOS (line-of-sight) and diffusion components exist in the fading channel which modeled as Rice and Rayleigh fading, respectively. The derivation of the optimal power loading in for a multicarrier transmission working over correlated Nakagami-*m* channel was proposed by Scaglione in [12], where an adaptive scheme can reduced the system complexity is implied for reducing the BER performance of OFDM modulation.

Based on the motivation mentioned above, in this paper the system performance of OFDM systems combining with dual-branch SC (selective combining) diversity and the channel fading scenarios are characterized as the Weibull statistics are evaluated. Besides, the OFDM system combining with MRC scheme working in correlated-Gamma fading is also analyzed with the BER criterion. The organization of the paper is as follows, after the introduction, the system models and channel fading models of the signals are presented in Section 2. The PDF (probability density function) of the transmission fading channels are described in section 3. In section 4 the system performance with error probability is analyzed. The numerical results for the derived formulas are presented in section 5. Finally, a simple conclusion is drawn in section 6.

II. System Models

Generally speaking, the IFFT (inverse fast Fourier transform) and FFT (fast Fourier transform) processing are adopted as the modulation and demodulation of the propagation waveform, respectively. The most important feature of the OFDM system is to share the fading components for many conveyed bits. Hence, there is almost few fading going to take place in the propagation of the signals. Consider that there are N subcarriers applied in an OFDM system. By taking the result of IFFT in time-domain for OFDM signals and for every one OFDM symbol, which can be expressed as $I(0), I(1),..., I(N-1)$, thus the modulated data sequence can be written as [12]

$$s(n) = \frac{1}{N}\sum_{k=0}^{N-1} I(k)\exp\left(\frac{j2\pi nk}{N}\right) \quad (1)$$

where $n = 0, 1, ..., N-1$, and $j = \sqrt{-1}$. If one FIR (finite impulse response) filter is characterized as multipath fading channel with taps $h(n)$, $n = 0, 1, ..., N-1$, and assume that L is the maximum delay components of the multipath fading channel with the fact of the number L is less than the total number of the subcarrier, $L << N$, i.e., $h(n) = 0$ for $n = L, L+1, ..., N-1$. Therefore the assumed channel impulse response under frequency-domain is given as

$$H(k) = \sum_{n=0}^{N-1} h(n)\exp\left(-j2\pi nk/N\right) \quad (2)$$

where $k = 0,1,...,N-1$. Assume that the fading paths are considered as mutually independent, the channel impulse response of the channel tap coefficient is able to be written as $h(n) = |h(n)|\exp\left[j\phi(n)\right]$, where $n = 0, 1, ..., L-1$ (the channel paths is less than the subcarrier number), and $|h(n)|$ is the amplitude intensity, which is characterized as the Nakagami-*m* distributed can be writtened as [1]

$$f_{|h(n)|}(c) = \frac{2}{\Gamma(i_n)}\left(\frac{i_n}{\Psi_n}\right)^{i_n} c^{2i_n-1} e^{-\frac{i_n}{\Psi_n}c^2} \quad , i_n > \frac{1}{2}, c > 0 \quad (3)$$

where $\Gamma(\cdot)$ is the Gamma function, $\Omega_n = E\left[|h(n)|^2\right]$ is either the expectation of $|h(n)|^2$ or the average power of the n-*th* tap, and i_n is the Nakagami-*m* fading parameter of the n-*th* tap. In addition, i_n is generally used to decide the degradation of fading channels. The value of i_n will influence the fading channel exist in two special cases described as follows. As regards of $i_n = 1$, the Nakagami-*m* fading is particularly referred to the Rayleigh fading, and the other acts as $i_n = \infty$, the Nakagami-*m* fading channel draw an static channel, i.e., the PDF of $h(n)$ is turned into $f_{|h(n)|}(c) = \delta\left(c - \sqrt{\Psi_n}\right)$, where the process of $\delta(\cdot)$ is the Dirac delta function. Moreover the parameters $\phi(n)$'s shown in the impulse response are the fading phases that assumed to be mutually independent and independent of the fading amplitudes $|h(n)|$'s, and the phases uniformly distribute over $[0,2\pi)$.

Furthermore, the signal at the output of the receiver of an OFDM system can be obtained as

$$r(k)=\sum_{n=0}^{N-1}R(n)\exp\left(-\frac{j2\pi nk}{N}\right)=H(k)\cdot D(k)+N(k) \quad (4)$$

where $N(k)$, k=0,1,..., $N-1$ is complex Gaussian noise with as zero mean and unit variance and *i.i.d.* (independent identically distributed) distribution model. From the frequency domain view point, the channel impulse response which can be expressed as

$$H(k)=\sum_{n=0}^{L-1}|h(n)|e^{j\theta(n)}\triangleq\sum_{n=0}^{L-1}(A_n+jB_n) \quad (5)$$

where the RV (random variable) $\theta(n)=\phi(n)-(2\pi nk/N)$ (mod 2π) that uniformly distribute over $[0,2\pi)$ for distinct n values. Suppose that the fading channel of the receiver has intact known message, then the phase of $H(k)$ can intact be offset and estimated at the receiver. Based on the conditions claimed above, to determine the statistical characteristics of $|H(k)|=\left|\sum_{n=0}^{L-1}(A_n+jB_n)\right|$, which is the modulus of a sum of L complex random vectors A_n+jB_n, $n=0,...,L-1$, is enough and efficient for analyzing the performance of the OFDM signaling over multipath Nakagami-*m* fading channels.

III. PDF of Fading Channels

In this section the model of the correlated-fading will be described corresponding to the correlated-Nakagami-*m* and the Weibull distributed.

3.1 Sum of Nakagami-*m* Distributed

If a sequence of the multipath $\{A_n\}_{n=1}^{N}$ indicate the power of the signal at the output of the MRC diversity, then the PDF of the sum has been proposed in [14], which is by using the Moschopoulos results and acquire an accuracy single gamma series indicating of the sum of arbitrarily correlated gamma variable. Some of the results described again as follows. Consider that an set of N correlated but not necessarily identically distributed Gamma variable which has the parameters are all with m , and η_n , comparatively, [that is, A_n~G(m,η_n) , where G(α,β) denotes the Gamma distributed with parameters α and β] and to enable ρ_{ij} indicate the correlation coefficient between A_i and A_j, that is,

$$\rho_{ij}=\rho_{ji}=\frac{\mathrm{Cov}(A_i,A_j)}{\sqrt{\mathrm{Var}(A_i)\mathrm{Var}(A_j)}},\ 0\le\rho_{ij}\le 1,\quad i,j=1,2,...,N \quad (6)$$

Since the MRC diversity scheme is adopted as the combiner, hence, the PDF of instantaneous *SNR* , $B=\sum_{n=1}^{N}A_n$, at the output of MRC is given as

$$f_B(R)=\prod_{n=1}^{N}\left(\frac{\lambda_1}{\lambda_n}\right)^m\sum_{k=0}^{\infty}\frac{\varepsilon_k R^{Nm+k-1}e^{-R/\lambda_1}}{\lambda_1^{N\alpha+k}\Gamma(Nm+k)}U(R) \quad (7)$$

where λ_1 is the minimum value of the eigenvalues, i.e., $\lambda_1=\min\{\lambda_n,\ n=1,\cdots,N\}$, the eigenvalues are come from the matrix $Z=XY$, where X is the $N\times N$ diagonal matrix with entries $\{\eta_n,\ n=1,\cdots,N\}$ and Y is the $N\times N$ positive definite matrix defined by

$$Y=\begin{bmatrix}1 & \sqrt{\rho_{12}} & \cdots & \sqrt{\rho_{1N}}\\ \sqrt{\rho_{21}} & 1 & \cdots & \sqrt{\rho_{2N}}\\ \vdots & \vdots & \vdots & \vdots\\ \sqrt{\rho_{N1}} & \cdots & \cdots & 1\end{bmatrix}_{N\times N} \quad (8)$$

and the coefficients ε_k is an iteration can be determined recursively by the formula written as

$$\begin{cases}\varepsilon_0=1\\ \varepsilon_{k+1}=\dfrac{m}{k+1}\displaystyle\sum_{i=1}^{k+1}\left[\sum_{j=1}^{N}\left(1-\frac{\lambda_1}{\lambda_j}\right)^i\right]\varepsilon_{k+1-i},\ k=0,\ 1,\ 2,\ ...\end{cases} \quad (9)$$

3.2 Correlated-Weibull Distributed

The system performance of OFDM system combined with dual-branch SC diversity operating in dual-correlated Weibull fading, in this sub-section the PDF of the correlated-Weibull statistic is going to be described. While assume that a signal which is one of the multipath wave comes from a maneuver environment. Equal power is considered for all the components of the scattered waves, g_l, $l=1,\cdots,L$. Accordingly, the complex envelope g_l is able to be modeled as Weibull fading distributed, and can be written as a function of the multipath fading with the ingredients in-phase A_l and quadrature B_l , which is with the Gaussian random variable, and the equation can be expressed as

$$g_l = (A_l + jB_l)^{2/\beta_l} \tag{10}$$

where $j = \sqrt{-1}$, and the nonlinearity is proved corresponding to power parameter. Now, if the K_l is expressed as the magnitude of g_l, i.e., $K_l = |g_l|$, then K_l can be shown and transformed into the power of the Rayleigh distributed RV $r_l = |A_l + jB_l|$, which states that it can be implemented by means of the simple algebra transform, and the magnitude can be represented as $K_l = r_l^{2/\beta_l}$, in which the result of PDF of K_l is given as [5]

$$f_{K_l}(R) = \frac{\beta_l}{\Psi_l} R^{\beta_l - 1} \exp\left(-\frac{R^{\beta_l}}{\Psi_l}\right) \tag{11}$$

where $\Psi_l = E[K_l^{\beta_l}]$ denotes the average power of the component of the multipath. Furthermore, consider a SC receives, and given as

$$R_{sc} = \max\{R_1, R_2\} \tag{12}$$

where $\max\{\}$ indicates the function for choosing the maximum. The joint PDF of R_1 and R_2 can be derived by means of changing the RVs which has proposed in [14, p. 143], and rewritten as

$$f_{K_1,K_2}(R_1,R_2) = \frac{\beta_1\beta_2 R_1^{\beta_1-1} R_2^{\beta_2-1}}{\Psi_1\Psi_2(1-\rho)} \times\exp\left[-\frac{1}{1-\rho}\left(\frac{R_1^{\beta_1}}{\Psi_1}+\frac{R_2^{\beta_2}}{\Psi_2}\right)\right] I_0\left[\frac{2\sqrt{\rho}R_1^{\beta_1/2}R_2^{\beta_2/2}}{(1-\rho)\sqrt{\Psi_1\Psi_2}}\right] \tag{13}$$

where $I_0[\cdot]$ is the modified Bessel function with zero-*th* order, and by using of an alternative infinite series expression of Bessel function [14, (8.447/1)]

$$I_0(u) = \sum_{k=0}^{\infty} \frac{1}{(k!)^2}\left(\frac{u}{2}\right)^{2k} \tag{14}$$

By substituting then (14) into (13), thus, which becomes as

$$f_{K_1,K_2}(R_1,R_2) = \beta_1\beta_2 \exp\left[-\frac{1}{1-\rho}\left(\frac{R_1^{\beta_1}}{\Psi_1}+\frac{R_2^{\beta_2}}{\Psi_2}\right)\right] \times\sum_{k=0}^{\infty}\frac{1}{(k!)^2}\frac{\rho^k}{(1-\rho)^{2k+1}}\frac{R_1^{-1+(k+1)\beta_1}R_2^{-1+(k+1)\beta_2}}{(\Psi_1\Psi_2)^{k+1}} \tag{15}$$

In order to obtain the PDF of R_{sc} shown in (12), first to calculate the cdf (cumulative distribution function) of R_{sc} is necessary. Hence, through the formula of cdf and the equivalent shown in [14, (8.351/1)], then the cdf can be obtained as [5]

$$F_{R_{sc}}(R) = (1-\rho)\sum_{k=0}^{\infty}\frac{\rho^k}{(k!)^2}\prod_{i=1}^{2}\Upsilon\left[k+1, \frac{1}{1-\rho}\left(\frac{R}{a_i\overline{R_i}}\right)^{\beta_i/2}\right] \tag{16}$$

where $a_i = 1/\Gamma(L)$ is the branch number, $R/\overline{R_i}$ denotes the *SNR* of the signal received at the dual branch SC diversity output, i.e., $R/\overline{R_i} = SNR$, and

$$\Upsilon(\varpi, u) = \int_0^{\infty} x^{u-1}\exp\left(-x-\varpi x^{\beta_l}\right)dx \tag{17}$$

where β_l, $l = 1, 2$ indicates fading parameter. While the first argument of $\Upsilon(k, g)$ is an integer and the other one is an arbitrary positive number value, the previous function can be simplified to a standard function as [18, (8.352/1)]

$$\Upsilon(k+1, g) = k!\left[1-\exp(-k)\sum_{c=0}^{k}\frac{g^u}{u!}\right] \tag{18}$$

Therefore, the PDF of R_{sc} can be determined by taking the first derivative of (16) and expressed as

$$f_{R_{sc}}(R) = \frac{1}{2}\sum_{k=0}^{\infty}\frac{\rho^k}{(k!)^2(1-\rho)^k} \times\left\{\frac{\beta_1 R^{-1+(k+1)\beta_1/2}}{\left(a_1\overline{R_1}\right)^{(k+1)\beta_1/2}}\exp\left[-\frac{1}{1-\rho}\left(\frac{R}{a_1\overline{R_1}}\right)^{\beta_1/2}\right]\times\Upsilon\left[k+1, \frac{1}{1-\rho}\left(\frac{R}{a_2\overline{R_2}}\right)^{\beta_2/2}\right] + \frac{\beta_2 R^{-1+(k+1)\beta_2/2}}{\left(a_2\overline{R_2}\right)^{(k+1)\beta_2/2}}\exp\left[-\frac{1}{1-\rho}\left(\frac{R}{a_2\overline{R_2}}\right)^{\beta_2/2}\right]\times\Upsilon\left[k+1, \frac{1}{1-\rho}\left(\frac{R}{a_1\overline{R_1}}\right)^{\beta_1/2}\right]\right\} \tag{19}$$

IV. Bit-Error Rate Analysis

4.1 OFDM Combining with SC Scheme

It is well known that the complementary error function, $errfc(\cdot)$, is a function utilized for expressing the results of conditional BER (bit-error rate) and SER (symbol-error rate) of the coherent modulation schemes in evaluating of wireless communication systems. Generally, for the purpose of simplifying the integral *Q*-function can be alternative expressed as [7]

$$Q(SNR\cdot R)=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\exp\left(-\frac{SNR^2\cdot R^2}{2\sin^2\phi}\right)d\phi \tag{20}$$

where the term *SNR* is corresponding to the definition shown in (7) and (16) for the *SNR*s' at the output of SC and MRC diversities. Thus, the average BER of an OFDM system combining with dual-branch SC diversity can be averaged over the conditional PDF of the *SNR*, i.e., by putting (19) together with (20) then the average BER can be obtained as

$$\begin{aligned}
P_e(SNR)&=\int_0^{\infty}Q(SNR\cdot R)\cdot f_{|H(k)|}(R)\,dR\\
&=\int_0^{\infty}\left[\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\exp\left(-\frac{SNR^2\cdot R^2}{2\sin^2\phi}\right)d\phi\right]\times\frac{1}{2}\sum_{k=0}^{\infty}\frac{\rho^k}{(k!)^2(1-\rho)^k}\\
&\times\left\{\frac{\beta_1 R^{-1+(k+1)\beta_1/2}}{\left(a_1\overline{R_1}\right)^{(k+1)\beta_1/2}}\exp\left[-\frac{1}{1-\rho}\left(\frac{R}{a_1\overline{R_1}}\right)^{\beta_1/2}\right]\right.\\
&\times\Upsilon\left[k+1,\frac{1}{1-\rho}\left(\frac{R}{a_2\overline{R_2}}\right)^{\beta_2/2}\right]\\
&+\frac{\beta_2 R^{-1+(k+1)\beta_2/2}}{\left(a_2\overline{R_2}\right)^{(k+1)\beta_2/2}}\exp\left[-\frac{1}{1-\rho}\left(\frac{R}{a_2\overline{R_2}}\right)^{\beta 2/2}\right]\\
&\left.\times\Upsilon\left[k+1,\frac{1}{1-\rho}\left(\frac{R}{a_1\overline{R_1}}\right)^{\beta_1/2}\right]\right\}
\end{aligned} \tag{21}$$

where $\Upsilon[.,.]$ is defined in (17).

4.2 OFDM Combining with MRC Scheme

By adopting the same procedures that is applied in the previous sub-section, the average BER of the OFDM system combining with the MRC scheme over correlated-Gamma fading can be computed by averaging (7) with (20), and the results can be represented as

$$\begin{aligned}
P_e(SNR)&=\int_0^{\infty}Q(SNR\cdot R)\cdot f_{|H(k)|}(R)\,dR\\
&=\int_0^{\infty}\left[\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\exp\left[-\left(\frac{SNR^2\cdot R^2}{2\sin^2\phi}+\frac{R}{\lambda_1}\right)\right]d\phi\right]\\
&\times\prod_{n=1}^{N}\left(\frac{\lambda_1}{\lambda_n}\right)^m\sum_{k=0}^{\infty}\frac{\varepsilon_k R^{Nm+k-1}}{\lambda_1^{N\alpha+k}\Gamma(Nm+k)}U(R)
\end{aligned} \tag{22}$$

V. Numerical Results and Discussion

The numerical results are implemented in this section. The results of the average *SNR* per bit versus BER of an OFDM system combining with dual correlated branch are shown in Fig.1, in which the varies fading parameters, $\beta=0.9$, 1.7 and 2 were adopted for comparison purpose. It is clear to note that the larger value of the fading parameter of Weibull fading, the better performance of OFDM system is. On the other hand, this is the reason that the larger fading parameter can provide with superior system performance. Then, how the system performance of OFDM system combining with SC diversity affected by the phenomena of branch correlation is illustrated in Fig. 2, in which the fading parameter is assumed as $\beta=1$, and the equal ($\overline{R_1}=\overline{R_2}$) and unequal ($\overline{R_1}=0.5\overline{R_2}$) branch are adopted. From the results shown in Fig. 2, it is worthwhile see that the branch correlation does degrade the system performance. It is reasonable to describe that the system performance of the OFDM system becomes deterioration when the number of the fading parameter decreases. Specifically speaking, the fading parameter of either the Weibull distributed or Nakagami-*m* distributed definitely dominates the system performance.

VI. Conclusion

The correlated-Gamma and dual correlated-Weibull fading statistics are considered as the fading models of the signal propagation environment for evaluating the system performance of OFDM system combining with MRC and dual-branch SC diversities, respectively, in this paper. For the purpose of comparison, the numerical results with different values of fading parameter of both fading models are illustrated. It is worthwhile note that the system performance of OFDM system is not only related to the fading parameters of the propagation channel model, but also has the relation of the equal and unequal of the SC diversities branches.

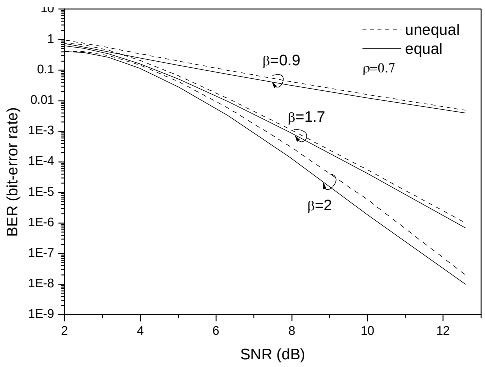

Fig. 1 The plots of BER vs bit SNR of OFDM with dual-branch SC diversity.

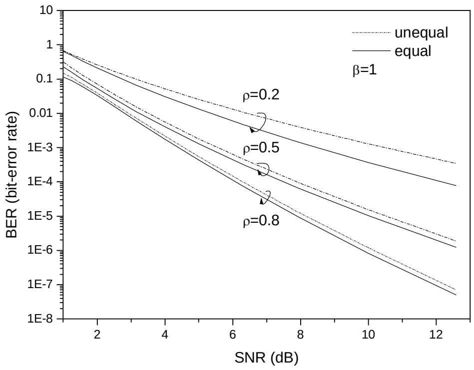

Fig. 2 The plots of BER vs bit SNR of OFDM with dual-branch SC diversity branch.

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