SC Diversity with Equal and Un-equal Gain Branches in Correlated-Weibull Fading Environments

Joy Iong-Zong Chen, Ying Chuan-Chen
Department of Communication Engineering, Da Yeh University
112 Shan-Jiau, Rd., Da-Tsuen, Chang-Hua 51505 Taiwan (R.O.C.)
Tel: +886-4-851-1888 ex:2523
Fax: +886-4-851-1245

Abstract: - The system performance of average LCR (level crossing rate) and AFD (average fade duration) versus normalized envelope level for a dual-branch SC (selection combining) receiver are discussed in this paper. Both of the equal and un-equal gain branches of the diversity paths are assumed in the evaluation of system performance, and the scenario of the suffered fading channel is considered characterized as correlated-Weibull fading environments. We derived new formulas and the numerical results are conduct for the purpose of accuracy validation. The different fading parameter of the Weibull distributed are compared each other in the numerical evaluation section. Besides, from the illustrated results it is worthwhile noting that the un-equal path gain, which definitely affects the system performance of the SC diversity, and should be taken into account for the analyzing or designing of the SC diversity system in wireless communication systems.

Keywords: AFD (average fade duration), average LCR (level crossing rate), SC (selection combining) diversity, Weibull fading, un-equal gain branch

I. Introduction

The multipath is caused by many physical phenomena such as reflection, refraction and scatter between the transmitter and the receiver station over wireless communication systems. The major reasons of linear distortion and ISI (inter-symbol interference) are not only generated by the multipath, but the power of received signal will also be destabilized. Therefore, there exist many well-known diversity techniques applied in wireless communication systems in order to reduce and mitigate the detrimental effects of channel fading. Such as the MRC (maximal-ratio combining) has the best diversity performance, EGC (equal-gain combining) with the equal weighting diversity gain, the sub-optimal one SC (selection combining), and GSC (generalized-selection combining) etc., are four the most popularly employed diversity techniques [1]. Among these techniques, SC scheme requires the lowest implementation complexity at the expense of performance providing, and in several versatile fading statistical models such as Rayleigh, Nakagami-*m* and Weibull distributions are for both independent and correlative fading [2-3].

In reviewing the previous researches about the issue of evaluating the LCR and AFD of SC diversity, Patzold analyzed and simulated the average LCR and AFD with the aid of computer over independent Rice fading channels [4]. The performance evaluation of LCR and AFD for time diversity working in independent Rayleigh fading channel is presented in [5]. In [6] where Yang studied the LCR and AFD performance for SC diversity scheme and the correlated Rayleigh fading environments were assumed. The authors Dong et. al. in [7] evaluated the LCR and AFD for SC diversity and considered both of the independent of branch when it is working in Rayleigh, Rice and Nakagami-*m* fading channel models. In paper [8] Tjhung assumed that the transmitted signal is operating over Nakagami-lognormal fading channels and analyzed the LCR and AFD for SC receipt system. The dual-branch correlated Weibull statistics is considered as the model of fading channel for the average BER (bit error probability) and outage probability analysis of SC diversity in [9]. Besides, Chen investigated the average LCR and AFD performance for dual-branch SC diversity over correlated-Nakagami-*m* and correlated-Weibull statistics channels are presented in [10], and [11], respectively. Recently, the authors Sagias et. al. in [12] presented the results from analyzing the channel capacity and second order statistic of a wireless communications with Weibull fading, and in [13] Alouini and Simon shown the performance of generalized selection combining over Weibull fading channels. In the wireless radio systems where is usually assumed that the fading path is relative long term mean signal level and will be equal in diversity branches of the receiver. However it is known that the predetection receiver branch gains and noise figures are not always equal for all branches in the real world. In practical implementation of the diversity combiner techniques it is usually assumed that the relative long term mean signal level will be equal in all diversity branches of the receiver. While as, it is known that the predetection receiver branch gains and noise figures are not always equal for all branches. On the other hand, the unequal branch gain will be induced into the diversity circuit wherever the unequal noise figure for the front end giving unequal mean carrier-to-noise ratios [14].

Based on the motivation described above, thus it is not only the equal branch gain of the SC diversity considered, but the un-equal branches are to be involved as an parameter in the analysis of the LCR and AFD performance of SC diversity in this paper. On the other hand, the discussion of un-equal branches is inspected with LCR and AFD criterions for SC scheme operating in Webull fading environments. The paper is organized as follows, following the introduction section system models and definitions of the LCR and AFD is presented in the section II, and in section III the analysis of LCR and AFD of un-equal dual-branch gain of SC diversity is held. The numerical results are shown in section IV for validation of the accuracy. The simply conclusion is drawn in section V.

II. System Models and Definitions of LCR and AFD

The average LCR is defined as the average number of times per unit duration that the fading envelope crosses a given value in the negative or positive direction [1]. Let γ be the sampled received envelope and $\dot{\gamma} = d\gamma/dt$ represents the derivative of γ with respect to time. The average LCR is defined as

$$N(\gamma) = \int_0^\infty \dot{\gamma} \cdot P_{\dot{R},R}(\dot{\gamma},\gamma)\, d\dot{\gamma} \quad (1)$$

where $P_{\dot{R},R}(\dot{\gamma},\gamma)$ denotes the jpdf (joint probability density function) of $\dot{\gamma}$ and γ.

The AFD corresponds to the average length of time the envelope remains under a certain value once it crosses it in the negative or positive direction. The average AFD is then defined as

$$\mathcal{T}(\gamma) = \frac{F_R(\gamma)}{N(\gamma)} \quad (2)$$

where $F_R(\gamma)$ is the CDF (cumulative distribution function) of γ.

Now, suppose that the received power at the output of the SC diversity, with a nonlinearity, is characterized as Weibull distributed, so that the resulting envelope is observed as the modulus of the multi-path Rayleigh component $\{X_l\}_{l=1}^{L}$ to the power of $2/\beta_l$. Hence, the received envelope in the lth diversity branch is expressed as

$$R_l = X_l^{2/\beta_l},\ l = 1,2 \quad (3)$$

where R_l is assumed as an RV modeled as Weibull distributed. By taking the time derivative to previous equation for X_l and the results can be written as

$$\dot{R}_l = \frac{2}{\beta_l} R_l^{1-\beta_l/2} \dot{X}_l \quad (4)$$

where $\dot{X}_l$ is the time derivative of X_l. While the propagation environments are considered as with the phenomena of isotropic scattering, the $\dot{X}_l$ can be modeled as a Gaussian distributed RV (random variable) with zero-mean and variance $\hat{\sigma}_l^2 = \sigma_l^2 2\pi^2 f_m^2$ [1], where σ_l is the standard deviation of X_l and f_m is maximum Doppler frequency shift, and σ_l^2 indicates the variance of Guassian scatter components. Consider that the envelope occurs at the output of an dual-branch SC diversity is expressed as $R_i = \max\{R_l,\ l = 1,\ 2\}$, where the equivalent envelope are adopted, i.e., $R_i = R_j = R,\ i \neq j,\ i, j = 1,\ 2$. Then the conditional pdf of $\dot{R}$ is given as [15]

$$p_{\dot{R}}(\dot{\gamma}\,|\,\gamma) = \frac{1}{\sqrt{2\pi}\hat{\sigma}_R} \exp(-\frac{\dot{\gamma}^2}{2\hat{\sigma}_R^2}) \quad (5)$$

where the $\hat{\sigma}_R$ is a discrete RV with pdf given as

$$f_{\hat{\sigma}_R}(\hat{\sigma}_\gamma) = \sum_{i=1}^{L} P(\sigma_\gamma = \hat{\sigma}_i) \cdot \delta(\sigma_\gamma - \sigma_i)$$

$$= \sum_{i=1}^{L} \text{Prob}[R_i = \max\{R_l,\ l=1,\ 2\} \cdot \delta(\hat{\sigma}_r - \sigma_i)] \quad (6)$$

where $\Pr ob[x]$ is the probability value of the RV x, and $\delta(\cdot)$ expresses the Kronecker Delta function defined as $\delta(0)=1$, and zero otherwise. By substituting (5) and the jpdf $P_{\dot{R},R}(\dot{\gamma},\gamma) = P_{\dot{R}|R}(\dot{\gamma}\,|\,\gamma)P_R(\gamma)$ into the definition of average LCR shown in (1), then the average LCR conditioned on $\hat{\sigma}_R$ is given by $N(\gamma\,|\,\hat{\sigma}_R) = p_R(\gamma)\sigma_R/\sqrt{2\pi}$. Next by averaging $N(\gamma\,|\,\hat{\sigma}_R)$ over the pdf of $\hat{\sigma}_R$ as determined in (6), i.e., the LCR can be computed as $N(\gamma) = \varepsilon(N(\gamma\,|\,\hat{\sigma}_R))$, and by putting together with the equivalent $\hat{\sigma}_{R_l} = 2/\beta_l\, R_l^{1-\beta_l/2}\sigma_l$, the LCR yields as

$$N(\gamma) = \sum_{i=1}^{L} p_{R_i}(\gamma)\frac{2}{\sqrt{2\pi}\beta_l}\gamma^{1-\beta_l/2}\hat{\sigma}_l \times \Pr ob(R_i = \max\{R_l\}\,|\,R_i = R) \quad (7)$$

While taking the conditions of correlation between the input branches into account, the probability value becomes as

$$\Pr(R_i = \max\{R_l\}\,|\,R_i = R) = \prod_{k=1}^{L} F_{R_k}(\gamma) \quad (8)$$

where R_k is characterized as Weibull distributed RV with CDF given as

$$F_{R_k}(\gamma) = 1 - \exp(-\frac{\gamma^{\beta_k}}{\Omega_k}) \quad (9)$$

where $\Omega_k = E\left(R_k^{\,2}\right)$ is average fading power, it is easily recognized that the pdf of γ follows the Weibull distribution [16, Ch.17] with fading parameter β, It is known that as the values of β increase, the severity of fading decreases, and while for the special case of $\beta = 2$, the formula (9) reduces to the well-known Rayleigh pdf. The average LCR of the SC operating in Weibull fading can be computed in a closed-form as

$$N(\gamma) = \sqrt{2\pi}\, f_m \sum_{i=1}^{L}\frac{\gamma^{\beta_i/2}}{\Omega_i} \times \prod_{k=1}^{L}[1-\exp(-\frac{\gamma^{\beta_k}}{\Omega_k})] \quad (10)$$

where f_m denotes the Doppler frequency shift. For the purpose of discussion the case of i.i.d. (identical independent distributed) for input branches, after normalizing the signal level to its rms (root mean square) value, $\rho = \gamma/\gamma_{rms}$, with $\gamma_{rms} = \sqrt{E(\gamma^2)} = \Omega^{1/\beta}/\sqrt{a}$ and $a = 1/\Gamma(1+2/\beta)$, where $\Gamma(\cdot)$ is the Gamma function [17, (8.310/1)], and the average LCR for the Weibull channel can be obtained in a simple closed-form as

$$N(\rho) = Lf_m\sqrt{2\pi}(\frac{\rho}{\sqrt{a}})^{\beta/2}\exp[-(\frac{\rho}{\sqrt{a}})^{\beta}] \times \{1-\exp[-(\frac{\rho}{\sqrt{a}})^{\beta}]\}^{L-1} \quad (11)$$

The case of single branch , *L=1,* has been addressed in [18], and rewritten as

$$N(\rho) = \sqrt{2\pi}\, f_m(\frac{\rho}{\sqrt{a}})^{\beta/2}\exp[-(\frac{\rho}{\sqrt{a}})^{\beta}]\} \quad (12)$$

Moreover, when *L>1*, the normalized average LCR, $N(\rho)$, in this paper for the dual branch SC receiver, *L=2*, as defined

$$N(\rho) = 2\, f_m\sqrt{2\pi}(\frac{\rho}{\sqrt{a}})^{\beta/2}\exp[-(\frac{\rho}{\sqrt{a}})^{\beta}] \times \{1-\exp[-(\frac{\rho}{\sqrt{a}})^{\beta}]\} \quad (13)$$

The expression for the AFD, $\mathcal{T}(\rho)$, is obtained by normalizing the signal level to its rms value in (8) and replacing then, together with (13), into (2) the result becomes as

$$\mathcal{T}(\rho) = \frac{1}{2\sqrt{\pi}\, f_m(\frac{\rho}{\sqrt{a}})^{\beta/2}\exp[-(\frac{\rho}{\sqrt{a}})^{\beta}]} \quad (14)$$

Furthermore, consider the case of fading parameter with $\beta = 2$, the formulas in (13) and (14) reduce to previous published expressions for the well-known Rayleigh model [1, (1.3-35) and (1.3-43)]. The maximum value of the average LCR can be derived solving the equation which is obtained by differentiating (13) with respect to ρ, as setting the result equal to zero, i.e., taking the derivative equals zero, $dN(\rho)/d\rho\,|_{\rho=\rho_{\max}} = 0$, then replacing $\rho_{\max}$ with the value shown in (13). It can be easily shown that the average LCR is maximized at $\rho_{\max} = 2^{(-1/\beta)}\sqrt{a}$ as set the value of $N(\rho_{\max}) = f_m\sqrt{\pi/e}$. It is worthwhile noting that the severity of fading is not affected by the factor of $N(\rho_{\max})$.

III. Evaluation of LCR and AFD for Dual-branch SC Diversity

There are two conditions, the equal branch gain and un-equal branch gain, are adopted for the evaluation of average LCR and AFD for dual-branch SC schemes in this paper [19]. The LCR and AFD of equal gain dual-branch of SC diversity were calculated in previous section II. In this subsection the system of the un-equal gain branch of the diversity system is going to be established first. Generally speaking, it is well known that the equal branch gain will be induced into the diversity circuit wherever the equal mean carrier-to-noise ratios, the system performance is definitely impacted by the effect of unequal gain, which may be inspected by replacing the branch number, L, with the summation values $\sum_{i=1}^{L-1}\psi_i \times B_{in}$, where B_{in} is the LCR and AFD obtained from the results of (13) and (14), ψ_i, which is adopted as the expression of the coefficient of the un-equal gain caused by the factor exists in the branch of the diversity system, and the index i indicates the number of propagation subchannel is some factor less than the gain in the L-th branch. The value of ψ_i belongs to $(0,\ 1)$. Thus, the LCR of dual-branch SC diversity under the conditions of un-equal gain branch can be determined as

$$N(\rho)=\sum_{i=1}^{L-1}(\psi_i)\times Lf_m\sqrt{2\pi}(\frac{\rho}{\sqrt{a}})^{\beta/2}\exp[-(\frac{\rho}{\sqrt{a}})^{\beta}] \times\{1-\exp[-(\frac{\rho}{\sqrt{a}})^{\beta}]\}^{L-1} \tag{15}$$

where the previous formula is obtained by substituted the summation values $\sum_{i=1}^{L-1}\psi_i \times B_{in}$ into the average LCR obtained in (11). Thereafter, the coefficient, ψ_i, $i=1,\cdots,L$, of the un-equal gain branch, will dominate the system performance of the diversity.

Now, an example with the dual branch is going to be illustrated here and the numerical results conducting in the next section. By putting the branch number with $L=2$ into previously equation the average LCR yields as

$$N(\rho)=\sum_{i=1}^{2-1}(\psi_i)\times 2f_m\sqrt{2\pi}(\frac{\rho}{\sqrt{a}})^{\beta/2}\exp[-(\frac{\rho}{\sqrt{a}})^{\beta}] \times\{1-\exp[-(\frac{\rho}{\sqrt{a}})^{\beta}]\} \tag{16}$$

, and the AFD of SC diversity with un-equal dual-branch gain can be following computed as

$$\mathcal{T}(\rho)=\frac{1}{\sum_{i=1}^{1}(\psi_i)\times 2\sqrt{\pi}f_m(\frac{\rho}{\sqrt{a}})^{\beta/2}\exp[-(\frac{\rho}{\sqrt{a}})^{\beta}]} \tag{17}$$

IV. Numerical Results

The normalized average LCR and AFD derived from the mathematical analysis in previous section for SC diversity with equal and un-equal branch are discussed in this section. The results shown in Fig. 1 and Fig. 2 are corresponding to the average LCR and AFD normalized by the maximal Doppler frequency shift, f_m, for a dual-branch SC with i.i.d. input branches intensity as a function of the normalized envelope level, $20\log_{10}(\rho)$, with several different values of $\beta=1.4,\ 2,\ 2.5,\ 3$, and 3.3. As reasonable expectation, the fading severity becomes degradation causes the normalized average LCR promotes, which can be explained as that the signal envelope fluctuates more frequently under the case of degrading in the communication environments. Moreover, lower signal levels are crossed less frequency, whereas higher signal level is crossed more frequency. On the other hand, the normalized AFD is plotted as a function of the normalized envelope level, $20\log_{10}(\rho)$, with some different values of fading parameter in Fig 2. It is obviously to see that the average time of the value under the threshold becomes shorter such that the ratio of AFD disperses more quickly. The fact can also be described that the fading effect happens in the propagation channel is much lighter for SC diversity when Weibull fading parameter, β, increases. Furthermore, the reaction of the un-equal branch gain for the performance with LCR and AFD are illustrated in Fig. 3 and Fig. 4, respectively. In both figures the fading parameter is fixed as $\beta=1.4$, and the different values of branch gain, $\psi=0.1,\ 0.3,\ 0.5,\ 0.7,\ 0.9$, and *1*, where $\psi=1$ represents the situation of equal branch gain, are applied for evaluation. It is clearly to note that the LCR is increased by the un-equal gain values, that is, as compares the results shown in Fig. 1, the most top one with $\beta=1.4$, with the results presented in Fig. 3, also the most top one with the case of $\psi=1$, the crossing ratio of the propagation intensity raises significantly. By the other view point to describe the effect of the un-equal branch gain for SC diversity with the results from comparison of Fig. 2 and Fig. 4. The divergence speed of the plot with equal branch gain $\psi=1$, and $\beta=1.4$ shown in Fig.4 is much faster than that of the plot with $\psi=1$ illustrated in Fig. 2. In words, the phenomena of the un-equal branch gain generated by the combining diversity hardware configuration definitely affect the system performance of wireless radio systems.

V. Conclusions

In this paper the average LCR and AFD normalized by the Doppler frequency versus normalized envelope level for a dual-branch SC receiver are discussed are investigated. Both of the equal and un-equal gain branches of the diversity paths are proposed and considered in the evaluation of system performance with the criterions of LCR and AFD, and the scenario of the suffered fading channel is considered characterized as correlated-Weibull fading model. The new formulas are derived and the numerical results are conduct for the purpose of accuracy validation. The different fading parameter of the Weibull distributed are compared each other in the numerical evaluation section. Besides, from the illustrated results it is worthwhile noting that the un-equal path gain, which is definitely affect the system performance of the SC diversity, should be taken into account for the analyzing or designing of the SC diversity system in wireless communication systems.

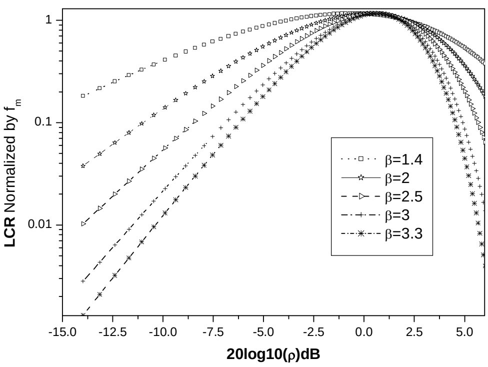

Fig. 1. Normalized average LCR of dual-branch SC receiver with equal branch gain versus normalized envelope level.

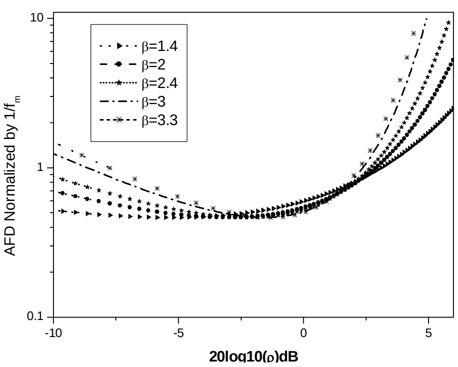

Fig. 2. Normalized average AFD of dual-branch SC receiver with equal branch gain versus normalized envelope level.

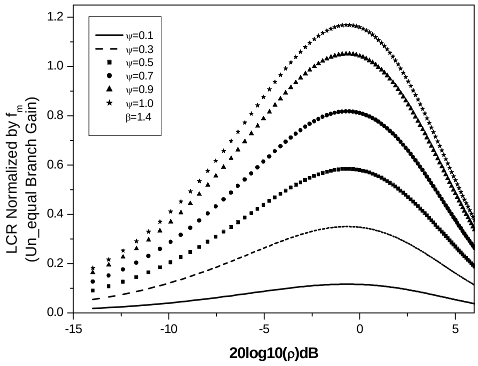

Fig. 3. Normalized LCR of SC diversity with un-equal branches gain versus normalized envelope level.

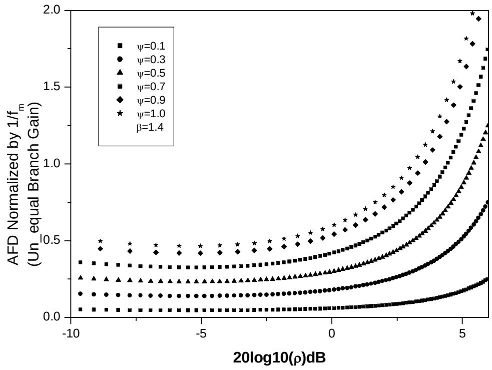

Fig. 4. Normalized AFD of a SC receiver with un-equal branches gain versus normalized envelope level

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