Multi-Objective Pareto Optimization of Centrifugal Pump Using Genetic Algorithms

N. NARIMAN-ZADEH, N. AMANIFARD, A. HAJILOO, P.GHALANDARI, B.HOSEINPOOR
Department of Mechanical Engineering
University of Guilan
Po. Box-3756, Rasht
IRAN

Abstract: Multi-objective genetic algorithm (GAs) is used for pump design pareto optimization, competing objectives for centrifugal pump design are total head (H), input power (Ps), hydraulic efficiency (η_H), and input parameter are capacity (Q), and the outer radius of the impeller (r_2). Multi-objective presents a set of compromised solution, and provides non-dominated optimal choices for designer.

Key-Words: Centrifugal Pump, Pareto Optimization, Head, Hydraulic Efficiency, Input Power, Multi-objective Optimization, GAs.

1 Introduction

Optimization in engineering design has always been of great importance and interest particularly in solving complex real-world design problems. Basically, the optimization process is defined as to find a set of values for a vector of design variables so that it leads to an optimum value of an objective or cost function. There are many calculus-based methods including gradient approaches to single objective optimization and are well documented in [1-2]. However, some basic difficulties in the gradient methods, such as their strong dependence on the initial guess, cause them to find local optima rather than global ones. Consequently, some other heuristic optimization methods, more importantly Genetic Algorithms (GAs) have been used extensively during the last decade. Such nature-inspired evolutionary algorithms [3-4] differ from other traditional calculus based techniques. The main difference is that GAs work with a population of candidate solutions not a single point in search space. This helps significantly to avoid being trapped in local optima [5] as long as the diversity of the population is well preserved. Such an advantage of evolutionary algorithms is very fruitful to solve many real-world optimal design or decision making problems which are indeed multi-objective. In these problems, there are several objective or cost functions (a vector of objectives) to be optimized (minimized or maximized) simultaneously. These objectives often conflict with each other so that improving one of them will deteriorate another. Therefore, there is no single optimal solution as the best with respect to all the objective functions. Instead, there is a set of optimal solutions, known as Pareto optimal solutions or Pareto front [6-9] for multi-objective optimization problems. The concept of Pareto front or set of optimal solutions in the space of objective functions in multi-objective optimization problems (MOPs) stands for a set of solutions that are non-dominated to each other but are superior to the rest of solutions in the search space. This means that it is not possible to find a single solution to be superior to all other solutions with respect to all objectives so that changing the vector of design variables in such a Pareto front consisting of these non-dominated solutions could not lead to the improvement of all objectives simultaneously. Consequently, such a change will lead to deteriorating of at least one objective. Thus, each solution of the Pareto set includes at least one objective inferior to that of another solution in that Pareto set, although both are superior to others in the rest of search space.

Centrifugal pumps are a group of turbomachines which are used in wide range of industrial systems as well as in home and office applications, such as process cooling water, chilled and hot water and industrial waste water systems, etc. Applications of this type of pumps are widespread when large head and low capacity is needed (in low specific speed).

By increasing the usage of pumping systems, designing of such systems which are optimum in operation is of great importance. Large head with high efficiency is expected from a centrifugal pump while they are in conflict with each other. If we have more head, more power should be use to prevail upon losses that decrease the efficiency. Therefore the design is important which can optimize both of them simultaneously.

In this point of view a multi-objective optimization has been performed by Oyama et al. [10] in order to redesign a single stage centrifugal pump for rocket engine. The Objectives were to maximize the total head and minimize the input power at a particular design point. Moreover, an on-line fuzzy optimization is also utilized by K. Benlarbi et al. [11] to maximize the global efficiency as well as maximize the driven speed and the water discharge rate of coupled centrifugal pump in a water pumping system.

In this paper, multi objective optimization is used for a water single stage centrifugal pump. This method has already been used by Amanifard et al. [12] for aerodynamic optimization of an axial compressor and valuable results have been obtained. The optimization objectives of the centrifugal pump are head (H), hydraulic efficiency (η_H), input power (Ps) which are competing parameters and the solution of this optimization is the pareto-optimal solution .The design parameters are pump capacity (Q) and the outer radius of the impeller (r_2).

2 Centrifugal pump modeling

In this design a single-stage centrifugal pump have been used. According to fluid velocity triangles (velocity triangles are given in appendix A) the meridional velocity of fluid at the impeller discharge c_{m3} (*m/s*) is defined by equation (1).

$$c_{m3} = {Q \times 10^6}\big/{2\pi r_2 b_3} \tag{1}$$

Where Q, r_2 and b_3 are capacity of fluid [m^3/h], radius at impeller discharge [m/s], width at impeller discharge [*m*].

The theoretical head of the pump without prerotation calculated by Euler's formula, equation (2).

$$H_{theoritical} = \mu \frac{u_2^2}{g}\left(1 - \frac{c_{m3}}{u_2}\cot\beta_2\right) \tag{2}$$

That u_2, β_2 and μ are peripheral velocity at the impeller discharge [*m/s*], relative angle from tangential at the impeller discharge [*degree*] and slip factor.

The slip factor is difference between the theoretical and the absolute fluid tangential velocities and is defied by equation (3).

$$\mu = 1 - \frac{\pi \sin\beta_2}{z} \tag{3}$$

That z is number of the vane. The hydraulic efficiency is defined by

$$\eta_H = {H_{real}}\big/{H_{theoritical}} \tag{4}$$

Real head of the pump that is obtained from curve of the pump. Input power is calculated by

$$P_s = u_2 \mu (u_2 - c_{m3}\cot\beta_2) Q \tag{5}$$

There are explanations about these formulas in reference number [10].

3 Multi-objective optimization

Multi-objective optimization which is also called multicriteria optimization or vector optimization has been defined as finding a vector of decision variables satisfying constraints to give acceptable values to all objective functions [8]. In general, it can be mathematically defined as:

find the vector $X^* = [x_1^*, x_2^*, ..., x_n^*]^T$ to optimize

$$F(X) = [f_1(X), f_2(X), ..., f_k(X)]^T \tag{6}$$

subject to *m* inequality constraints

$$g_i(X) \le 0, \quad i = 1 \ to \ m \tag{7}$$

and *p* equality constraints

$$h_j(X) = 0 \quad , \quad \mathrm{j = 1 \ to \ p} \tag{8}$$

Where $X^* \in \Re^n$ is the vector of decision or design variables, and $F(X) \in \Re^k$ is the vector of objective functions which each of them be either minimized or maximized. However, without loss of generality, it is assumed that all objective functions are to be minimized. Such multi-objective minimization based on Pareto approach can be conducted using some definitions:

3.1 Definition of Pareto dominance

A vector $U = [u_1, u_2, ..., u_k] \in \Re^k$ is dominance to vector $V = [v_1, v_2, ..., v_k] \in \Re^k$ (denoted by $U \prec V$) if and only if $\forall i \in \{1,2,...,k\}$, $u_i \le v_i \ \wedge \ \exists j \in \{1,2,...,k\}$: $u_j < v_j$. In other words, there is at least one u_j which is smaller than v_j whilst the rest u's are either smaller or equal to corresponding v's.

3.2 Definition of Pareto optimality

A point $X^* \in \Omega$ (Ω is a feasible region in $\Re^n$ satisfying equations (2) and (3)) is said to be Pareto optimal (minimal) with respect to the all $X \in \Omega$ if and only if $F(X^*) < F(X)$. Alternatively, it can be readily restated as $\forall i \in \{1,2,...,k\}, \forall X \in \Omega - \{X^*\}$ $f_i(X^*) \le f_i(X)$ $\wedge$ $\exists j \in \{1,2,...,k\}$: $f_j(X^*) < f_j(X)$. In other words, the solution X^* is said to be Pareto optimal (minimal) if no other solution can be found to dominate X^* using the definition of Pareto dominance.

3.3 Definition of Pareto set

For a given MOP, a Pareto set $\mathcal{P}^*$ is a set in the decision variable space consisting of all the Pareto optimal vectors $\mathcal{P}^* = \{X \in \Omega | \nexists X' \in \Omega : F(X') < F(X)\}$. In other words, there is no other X' as a vector of decision variables in Ω that dominates any $X \in \mathcal{P}^*$.

3.4 Definition of Pareto front

For a given MOP, the Pareto front $\mathcal{PF}^*$ is a set of vector of objective functions which are obtained using the vectors of decision variables in the Pareto set $\mathcal{P}^*$, that is $\mathcal{PF}^* = \{F(X) = (f_1(X), f_2(X),...., f_k(X)) : X \in \mathcal{P}^*\}$.

In other words, the Pareto front $\mathcal{PF}^*$ is a set of the vectors of objective functions mapped from $\mathcal{P}^*$.

Evolutionary algorithms have been widely used for multi-objective optimization because of their natural properties suited for these types of problems. This is mostly because of their parallel or population-based search approach. However, it is very important that the genetic diversity within the population be preserved sufficiently [11]. This main issue in MOPs has been addressed by many related research works. Consequently, the premature convergence of MOEAs is prevented and the solutions are directed and distributed along the true Pareto front if such genetic diversity is well provided. The Pareto-based approach of NSGA-II [13] has been recently used in a wide area of engineering MOPs because of its simple yet efficient non-dominance ranking procedure in yielding different level of Pareto frontiers. However, the crowding approach in such state-of-the-art MOEA is not efficient as a diversity-preserving operator, particularly in problems with more than two objective functions. In fact, the crowding distance computed by routine in NSGA-II [13] may return an ambiguous value in such problems. The reason for such drawback is that sorting procedure of individuals based on each objective in this algorithm will cause different enclosing hyper-box. Thus, the overall crowding distance of an individual computed in this way may not exactly reflect the true measure of diversity or crowding property.

3.5 ϵ-elimination diversity algorithm

In the ϵ-elimination diversity approach that is used to replace the crowding distance assignment approach in NSGA-II [12], all the clones and ϵ-similar individuals are recognized and simply eliminated from the current population. Therefore, based on a pre-defined value of ϵ as the elimination threshold (ϵ=0.001 has been used in this paper), all the individuals in a front within this limit of a particular individual are eliminated. It should be noted that such ϵ-similarity must exist both in the space of objectives and in the space of the associated design variables. This will ensure that very different individuals in the space of design variables having ϵ-similarity in the space of objectives will not be eliminated from the population. The pseudo-code of the ϵ-elimination approach is depicted in Fig. 1. Evidently, the clones and ϵ-similar individuals are replaced from the population by the same number of new randomly generated individuals. Meanwhile, this will additionally help to explore the search space of the given MOP more effectively.

4 Multi-Objective Optimization of Centrifugal Pump

Many important parameters should be considered for centrifugal pump design. In choosing a centrifugal pump, Head is the main parameter. The real head of the pumps is obtained from experimental tests and is less than the theoretical head since there are losses in the pumps and their fittings. Therefore, the hydraulic efficiency is defined by the ratio of the real head and the design head. To simplify the design procedure, we consider single stage pump without prerotation at entrance.

Design parameters are capacity (Q) and the outer radius of the impeller. The objectives are head (H), hydraulic efficiency (η_H) and input power (Ps). The range of variation for input parameters are given in table 1. However, in this multi-objective analysis, some constant input parameters are already known or assumed as, *z=7*, r_1 *=56 mm*, *b=25 mm*, β_2 *=15°*, where z, r_1, b , β_2 , which are number of vanes, the input radius of impeller, width of the vanes, angle of output flow from impeller respectively. Pump rotative speed is considered *1450 rpm.*

```
Pseudo-code of ϵ-elimination
ϵ-elim=ϵ-elimination(pop)                    //pop includes
design variables and
                                  objective functions//
define       ϵ                    //Define elimination threshold
k=1                               //Front No.
i=1
until i+1<pop_size
        j=i+1
        until j< pop_size

    IF { || F(X(i), F(X(j)) || < ϵ ∧ || X(i), X(j) || < ϵ}
                       F(X(i), F(X(j))∈ PT_k*    X(i), X(j) ∈ P_k*

    THEN  pop= pop\pop(j)        // Remove the ϵ-similar
individual

r_new_ind = make_new_random_individual

                                  //Generate new random
        individual

pop=pop ∪ r_new_ind               //Add new randomly generated
                                        individual
```

Fig. 1 Pseudo-code of ϵ-elimination for preserving genetic diversity

capacity	outer radius of impeller
$0 \leq Q \leq 158$	$110 \leq r_2 \leq 130$

Table 1. Range of variation for input parameters

In the optimization process it is desired that both head and hydraulic efficiency to be maximized while minimization of input power is of interest. In this way, a population size of 45 has been chosen with crossover probability P_c and mutation probability P_m as 0.85 and 0.09 respectively using multi-objective genetic algorithm.

The Pareto front, of the three-objective optimization has been shown, in Fig. 2 in the plane of hydraulic efficiency and head and Fig. 3 in the plane of hydraulic efficiency and input power. As it is evident in the figures, improving one objective will cause another objective deteriorates accordingly.

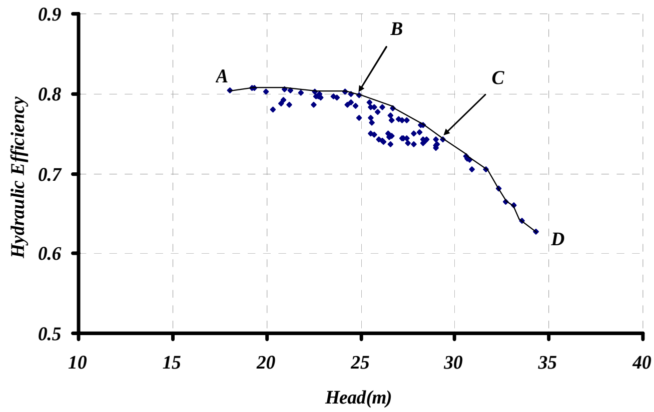

Fig.2 Pareto front of hydraulic efficiency and head

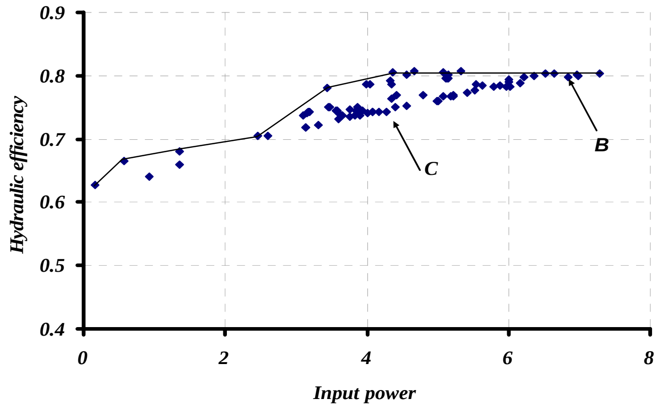

Fig.3 Pareto front of hydraulic efficiency and input power

The pareto front of head versus hydraulic efficiency can be divided into three distinct regions. In the first region (by moving from A to B), there are points with approximately constant hydraulic efficiency while the increase in head is noticeable. In this region, design point B has both high efficiency and large head and is thus superior to other points. On the other hand, in the second region (from B to C), high-intensity of optimum points is obvious. In this region by increasing the head, hydraulic efficiency decreases seven percent. Meanwhile a clear decrease in input power is evident (see Fig. 3). Nevertheless, it should be noticed that there is not any special advantages between the Pareto points, thus, each

point can be individually used by designers for special purpose.
Identification of some optimum points are also mentioned in table 2.

	$r_2(mm)$	$Q(m^3/h)$	$H(m)$	η_H	$P_s(kw)$
A	112	88.9	18	80	7.3
B	128	100	25	80	6.8
C	128.3	53.4	29.4	74	4.3
D	128.8	10	34.3	63	1.69

Table 2. Identification of some optimum points

5 Conclusion

Multi-objective Pareto based on optimization of centrifugal pump has been successfully used. Current pareto optimal solutions displays tradeoff information between maximization of head and hydraulic efficiency and minimization of the input power offs which would have not been achieved without such optimization process. Such tradeoff information is very helpful to a higher-level decision-maker in selecting a design with other considerations.

References:

[1] J.S. Arora, *Introduction to Optimum Design*. McGraw-Hill, 1989

[2] S.S. Rao, Engineering *Optimization: Theory and Practice*. John Wiley & Sons, 1996

[3] D.E. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning*. Addison-Wesley, New York, 1989

[4] T. Back, D.B. Fogel, Z. Michalewicz, *Handbook of Evolutionary Computation*. Institute of Physics Publishing and New York: Oxford University Press, 1997

[5] G. Renner, A. Ekart, Genetic algorithms in computer aided design. *Computer-Aided Design*, Vol.35, 2003, pp 709-726

[6] N. Srinivas, K. Deb, Multiobjective optimization Using Nondominated Sorting in Genetic Algorithms. *Evolutionary Computation*, Vol. 2, No. 3, 1994, pp 221-248

[7] C.M. Fonseca, P.J. Fleming, Genetic algorithms for multi-objective optimization: Formulation, discussion and generalization. *Proc. Of the Fifth Int. Conf. On genetic Algorithms*, Forrest S. (Ed.), San Mateo, CA, Morgan Kaufmann, 1993, pp 416-423

[8] C.A. Coello Coello, A comprehensive survey of evolutionary based multi-objective optimization techniques, *Knowledge and Information Systems: An Int. Journal*, Vol. 3, 1993, pp 269-308, 1999.

[9] V. Pareto, *Cours d'economic ploitique*. Lausanne, Switzerland, Rouge, 1896.

[10] A. Oyama, M.S. Liou, Multiobjective optimization of rocket engine pumps using evolutionary algoritm, *AIAA Paper 2001-2581, 15th AIAA Computational Fluid Dynamics Conference,* Anaheim, California, June, 2001.

[11] K. Benlarbi, L. Mokrani, M.S. Nait-said, "A fuzzy global efficiency optimization of a photovoltaic water pumping system", *Journal of Solar Energy,* Vol. 77, 2004, pp. 203-216.

[12] N. Amanifard, N. Nariman-Zadeh, A. Jamali, M. H. Farahani, R. Farzane-Kari, "Multi-Objective pareto optimization of axial compressor stages using genetic algorithms", WSEAS Transactions on Information Science and Application, Issue 10, Volume 3, 2006, pp. 1873-1878 .

[13] K. Atashkari, N. Nariman-Zadeh, A. Pilechi, A. Jamali and X. Yao "Thermodynamic Pareto optimization of turbojet engines using multi-objective genetic algorithms", *International Journal of Thermal Sciences,* Vol.44, No11, 2005, pp.1061-1071.

Appendix A

Velocity triangles of pump impeller can be viewed in figure 4.

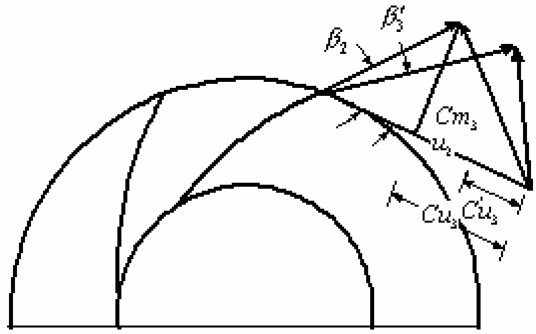

Fig.4 Velocity triangles