Yaw Phase Mode Attitude Control Using Z Wheel Modeling for LEO Microsatellite

A.M. SI MOHAMMED[a], M. BENYETTOU[b], A. BOUDJEMAI[a]
Y. HASHIDA[c], M.N. SWEETING[c]

[a]Division de Mécanique Spatiale
Centre National des Techniques Spatiales
1, Avenue de la Palestine, Arzew
Algeria

[b]Université des Sciences et de la Technologie «Mohamed Boudiaf »
Laboratoire de Modélisation et Simulation, Oran
Algeria

[c]Surrey Satellite Technology Limited
Guilford, Surrey GU2 7XH
United Kingdom

Abstract: A control system is proposed for a low Earth orbit gravity gradient stabilised microsatellite using Z wheel. The microsatellite is 3-axis stabilized using a yaw reaction wheel, with dual redundant 3-axis magnetorquers. Two vector magnetometers and four dual sun sensors are carried in order to determine the full attitude.
The attitude was estimated using an Euler angles (small libration version) on based extended Kalman filter (EKF). After the satellite has been detumbled and deploy the gravity gradient boom, in order to have the accurate Nadir pointing we will use the Z zero-bias mode controller. The Z momentum wheel will be damped by the magnetorquers. This paper describes the attitude determination and control system design of LEO microsatellite using Z reaction wheel for yaw phase mode control and Z disturbance cancellation during X thruster firings for orbit maneuvers.

Keywords: Modelling, Simulation, Microsatellite, LEO, Attitude, Yaw, Control, Z Wheel, Magnetorquer.

1 Introduction

Small low cost satellites are becoming more important in the last few years when the possibility of piggyback launch opportunities. The aim of this control system is to achieve a stable Earth pointing attitude, maximizing the pointing accuracy and minimizing the control energy, within the limitation of the existing low cost technology.
A possible resource to be explored for improved performance of future low cost satellites is the processing capability of on board microprocessors. Innovative attitude control theory, more explicitly discrete time estimators, and control laws can be used to obtain this goal. As an example, a small satellite controller, making use of a gravity gradient (GG) boom, and coils (magnetorquers) to maintain an Earth pointing attitude [17].

The motion of a spacecraft presents two dynamic aspects of interest. The most obvious one is the trajectory traced by its center of mass which is governed by the classical Keplerian relations. The other is rotational motion about its center of mass, commonly referred to as libration, which is our attention. Due to the influence of internal and external torque, the undesirable orientation must be controlled for successful completion of a given mission [2].
A wide range of attitude control concepts has been proposed over the years and several have practical application. In general, they might be classified as active, passive, and semi passive procedures. The active approach use energy available on board the satellite. The passive and semi-passive systems, on the other hand, exploit the environmental forces for stabilization and control [15].

Passive stabilization techniques using gravity gradient torques have been in use for a long time, specifically for damping the libration motion of a spacecraft. This technique does not use any additional sensors or actuators, if the spacecraft can be designed in such a way that it is a gravity gradient stabilized. Even though this technique works well; it generally requires a long time to accomplish the libration damping (on the order of a few days). Moreover the attitude control errors are fairly loose (5° to 10°), which may be adequate to meet some mission requirements [12].

To improve the libration damping time and the attitude control errors, an active magnetic control technique using three torqurods has been suggested for a class of small satellites ranging in total mass from 40 to 200kg. This active control can reduce the libration damping time from days to a few orbits, and can achieve attitude control errors of less than 3° for roll, 2° for pitch, and 5° for yaw [14].

The proposal satellite attitude determination and control system uses a Z reaction wheels, gravity gradient boom (6 meter + 3 kg tip mass) and 3-axis magnetorquer rods. The magnetorquer rods do momentum maintenance and nutation damping for Z wheel, libration damping and yaw phase control.

The Z wheels are used for the following control functions on satellite [1], [3], [7]

- Yaw control for push broom for Earth observation;
- Quick transfer between BBQ mode and yaw steering for thermal control;
- Z disturbance cancellation during X thruster firings for orbital control;

2 Attitude Dynamic Modelling

In common with boats and aircraft the orientation of a spacecraft can be defined by three angles (roll, pitch, and yaw). These angles are obtained from a sequence of right hand positive rotations from a reference X_R, Y_R, Z_R frame to a X_B, Y_B, Z_B set of spacecraft body axes. There are 12 possible sequences of rotations, which can be expressed using Euler angles. One example is a 2-1-3 sequence rotation. The first rotation is a pitch about the reference Y_R axis, this defines a pitch angle θ. The second rotation is a roll about the intermediate L axis, this define a roll angle φ. The last rotation is a yaw about the body Z_B axis, this define a yaw angle ψ. The attitude matrix, $\mathbf{A}$, which transforms an arbitrary vector from the reference X_R, Y_R, Z_R coordinates to the spacecraft body X_B, Y_B, Z_B coordinates can be expressed as [18]:

$$A = \begin{bmatrix} c\psi c\theta + s\psi s\phi s\theta & s\psi c\phi & -c\psi s\theta + c\psi s\phi c\theta \\ -s\psi c\theta + c\psi s\phi s\theta & c\psi c\phi & s\psi s\theta + c\psi s\phi c\theta \\ c\phi s\theta & -s\phi & c\phi c\theta \end{bmatrix} \quad (1)$$

Where

φ : Roll angle;
θ : Pitch angle;
ψ : Yaw angle;
c : cosine function;
s : sine function.

According to Euler's theorem any finite rotation of a rigid body can be expressed as a rotation through one angle (Φ) about a fixed axis ($\mathbf{e}$). Therefore, the transformation attitude matrix $\mathbf{A}$ can be obtained by the rotating angle Φ about the fixed axis $\mathbf{e}$. The Euler symmetric parameters q_1, q_2, q_3, q_4 in terms of angle Φ and rotation axis $\mathbf{e}$ are given by:

$$q_1 \equiv e_{ox} \sin\left(\frac{1}{2}\Phi\right) \quad (2.a)$$

$$q_2 \equiv e_{oy} \sin\left(\frac{1}{2}\Phi\right) \quad (2.b)$$

$$q_3 \equiv e_{oz} \sin\left(\frac{1}{2}\Phi\right) \quad (2.c)$$

$$q_4 \equiv \cos\left(\frac{1}{2}\Phi\right) \quad (2.d)$$

Where

$\mathbf{q} = [q_1 \quad q_2 \quad q_3 \quad q_4]^T$: attitude quaternion vector with respect to orbital coordinates;

$\mathbf{e} = [e_{ox} \quad e_{oy} \quad e_{oz}]^T$: Euler vector in orbital referenced coordinates;

Φ: rotation angle around the Euler vector.

The four Euler symmetric parameters are not independent, but satisfy the constraint,

$$q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1 \quad (3)$$

The attitude matrix is expressed in term of Euler symmetric parameters as [18]:

$$T_{BE} = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1 q_2 + q_3 q_4) & 2(q_1 q_3 - q_2 q_4) \\ 2(q_1 q_2 - q_3 q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2 q_3 + q_1 q_4) \\ 2(q_1 q_3 + q_2 q_4) & 2(q_2 q_3 - q_1 q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix} \quad (4)$$

This expression contains no trigonometric functions, which require time-consuming computation, and it can easily be referenced to the orbit coordinate system.
If the quaternion representation is used, the respective pitch, roll and yaw angles can be calculated as [17]:

$$\theta = \arctan\left\{\frac{A_{31}}{A_{33}}\right\} \quad (5.a)$$

$$\phi = \arcsin\{-A_{23}\} \quad (5.b)$$

$$\psi = \arctan\left\{\frac{A_{12}}{A_{22}}\right\} \quad (5.c)$$

The dynamics of the spacecraft in inertial space is governed by Euler's equations of motion can be expressed as follows in vector form [15], [18]

$$\mathbf{I}\dot{\boldsymbol{\omega}}_{\mathbf{B}}^{\mathbf{I}} = \mathbf{N}_{\mathbf{GG}} + \mathbf{N}_{\mathbf{D}} + \mathbf{N}_{\mathbf{M}} + \mathbf{N}_{\mathbf{T}} - \boldsymbol{\omega}_{\mathbf{B}}^{\mathbf{I}} \times (\mathbf{I}\boldsymbol{\omega}_{\mathbf{B}}^{\mathbf{I}} + \mathbf{h}) - \dot{\mathbf{h}} \quad (6)$$

Where $\boldsymbol{\omega}_B^I$, I, N_{GG}, N_D, N_M and N_T are respectively the inertially referenced body angular velocity vector, moment of inertia of spacecraft, gravity-gradient torque vector, applied magnetorquer control firing, unmodelled external disturbance torque vector such as aerodynamic or solar radiation pressure.
The rate of change of the quaternion is given by

$$\dot{\mathbf{q}} = \frac{1}{2}\boldsymbol{\Omega}\mathbf{q} = \frac{1}{2}\boldsymbol{\Lambda}(\mathbf{q})\boldsymbol{\omega}_B^O \quad (7)$$

Where

$$\boldsymbol{\Omega} = \begin{bmatrix} 0 & \omega_{oz} & -\omega_{oy} & \omega_{ox} \\ -\omega_{oz} & 0 & \omega_{ox} & \omega_{oy} \\ \omega_{oy} & -\omega_{ox} & 0 & \omega_{oz} \\ -\omega_{ox} & -\omega_{oy} & -\omega_{oz} & 0 \end{bmatrix} \quad (8)$$

$$\boldsymbol{\Lambda}(\mathbf{q}) = \begin{bmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix} \quad (9)$$

Where

$\boldsymbol{\omega}_B^O = [\omega_{ox} \quad \omega_{oy} \quad \omega_{oz}]^T$ = body angular velocity vector referenced to orbital coordinates.

The angular body rates referenced to the orbit coordinates can be obtained from the inertially referenced body rates by using the transformation matrix **A**:

$$\boldsymbol{\omega}_B^O = \boldsymbol{\omega}_B^I - \mathbf{A}\boldsymbol{\omega}_0 \quad (10)$$

If we assume the satellite in a near circular orbit with average orbital angular rate ω_o, then

$\boldsymbol{\omega}_0^B = [0 \quad -\omega_o \quad 0]^T$ is a constant rate vector.

The kinematic equations can derived by using a spacecraft referenced angular velocity vector $\boldsymbol{\omega}_B^R$ as follows [17], [18]:

$$\dot{\phi} = \omega_{Rx}\cos\psi - \omega_{Ry}\sin\psi \quad (11.a)$$

$$\dot{\theta} = (\omega_{Rx}\sin\psi + \omega_{Ry}\cos\psi)\sec\phi \quad (11.b)$$

$$\dot{\psi} = \omega_{Rz} + (\omega_{Rx}\sin\psi + \omega_{Ry}\cos\psi)\tan\phi \quad (11.c)$$

Where

$\boldsymbol{\omega}_B^R = [\omega_{Rx} \quad \omega_{Ry} \quad \omega_{Rz}]^T$ body relative angular velocity in any reference coordinate frame.

3 Attitude Determination Modelling

A Kalman filter is an optimal, recursive, data processing algorithm [1], [11] and [16] all address Kalman filtering for spacecraft attitude estimation.
The attitude was estimated using a Euler angles (small libration version) based extended Kalman filter (EKF) [2], [6]. This filter uses measurement vectors (in the body frame) from all the attitude sensors and by combining them with corresponding modeled vectors (in a reference frame) [10], [13] it estimates the attitude of the satellite.
The attitude sensors (magnetometer, sun sensor) will be used to determine the attitude of the satellite relative to the orbital frame. When using magnetic field data: a GPS receiver or an orbital propagator is used to obtain the position of the satellite. Using this position data, a model of the geomagnetic field, the International Geomagnetic Reference Field (IGRF) model, computes the geomagnetic **B**-field in orbit coordinates. On the other hand, the magnetic **B**-field is also measured by the 3-axis magnetometer in body coordinates. The attitude can then be solved from these two vectors over time.
The EKF cycle is given as follows [9]:

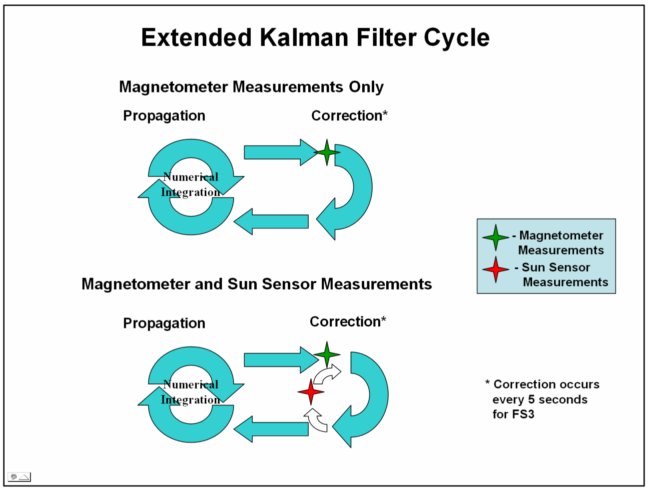

Fig. 1 Extended Kalman Filter Cycle

4 Magnetic Wheel Torquer Control Law

Reaction wheels are essentially torque motors with high-inertia rotors. They can spin in either direction. Roughly speaking one wheel provides for the control of one axis.

Magnetorquers generate magnetic dipole moments whose interactions with the Earth's magnetic field produce the torques necessary to remove the excess momentum. The magnetic torque vector can be expressed as the cross product of the magnetic dipole moment M of the magnetic coils with the geomagnetic field strength B in the body frame [4], [14]:

$$\mathbf{N}_M = \mathbf{M} \times \mathbf{B} \tag{12}$$

Where

M : magnetic dipole control moment vector;

The following cross-product control law is used

$$\mathbf{M} = \frac{\mathbf{e} \times \mathbf{B}}{\|\mathbf{B}\|} \tag{13}$$

Where

B : Magnetometer measured magnetic field vector;

The error vector for a magnetorquer cross-product controller including Z wheel is given by [2]

$$\mathbf{e} = \begin{bmatrix} Kd_x \omega_{ox} / \omega_0 \\ Kd_y \omega_{oy} / \omega_0 \\ K_z (h_z - h_{z-ref}) \end{bmatrix} \tag{14}$$

Where

M : Magnetorquer switch-on time;
Kd : Derivative gain;
ω_0 : Orbit angular rate in rad/s;
ω_{0x}, ω_{0y}: X and Y orbit referenced angular rate of the satellite in rad/s;
K : Momentum maintenance gain constant;
$h_{z\text{-ref}}$: Reference yaw wheel momentum (nominally 0.052 Nms);
h_z : Yaw wheel momentum measurement in Nms;
ϕ,θ,ψ : Roll, Pitch and Yaw angle in rad;

The feedback control law for the Z wheel is given by

$$N_{zwheel} = Kd_z \omega_z^0 + Kp_z(\psi - \psi_{ref}) \tag{15}$$

$$h_{z\text{-wheel-cmd}} = \int N_{z-wheel} dt / I_{z-wheel} \tag{16}$$

Where Kp_z, Kd_z is the controller gain constant, ω_{oz} is the orbit reference angular rate, and Nh_{wheel} is the commanded wheel torque vector.

5 Simulation Results

The magnetic moment in the orthogonal X, Y and Z-axes was assumed to be equal to 10 Am^2 each. The Z reaction wheel has a MOI of 8.10^{-4} kgm^2 and the maximum speed is ± 5000 rpm. The maximum wheel torque is 5 milli-Nm.

We assume that we have gravity gradient torque and aerodynamic torque as external torque.

An International Geomagnetic reference Field (IGRF) model was used to obtain the geomagnetic field values.

A sampling period of TS = 10 seconds was utilised for the discrete filter algorithm.

To initialize the full state filter we use the yaw filter [5].

5.1 Optimal Gain Choice for the Magnetorquer Cross-Product Controller plus Z Wheel

The main goal of this section is how to choose the gain of the error vector Eq.(14) against the average magnetorquer power drain, the total accumulated on time of magnetorquer, Euler angles RMS and Euler angles RMS error.

For this simulation we are going to compute the Euler angles RMS and the Euler angles RMS error when the yaw angle is commanded to 0° [170°, respectively] and we are using estimator (magnetometers plus sun sensors).

Simulations done, the optimal gain is given as follows

Table 1. Optimal gain for the magnetorquer cross-product controller plus Z wheel

Kd_x	Kd_y	K_z
10	10	25

5.2 Yaw Phase Mode Accuracy State

Figure 2 to 5 presents the results of magnetorquer plus Z wheel yaw phase mode. The satellite is left to librate freely for the two orbits starting from an initial attitude of 3° roll, 0° pitch, 0° yaw, 0°/sec roll rate, 0°/sec pitch rate and 0.6°/sec yaw rate. At the start of the third orbit the magnetorquer plus Z reaction wheels activated and within one orbits the pitch and roll librations are damped to nadir pointing error of less than 2°, the yaw angle is controlled to 0°. At the start of the eighth orbit the yaw angle is commanded to 170° for six orbits.
The total accumulated on time of magnetorquer is approximately 9092 seconds during a active control window of 12 orbits (72000 seconds). This gives an average magnetorquer power drain of 0.10 Watt from the start until the attitude is achieved.

Table 2. Euler angles RMS (Yaw angle is commanded to 0 deg)

Roll	Pitch	Yaw
0.06 deg	0.08 deg	0.20 deg

Table 3. Error Euler angles RMS (Yaw angle is commanded to 170 deg)

Roll	Pitch	Yaw
0.05 deg	0.20 deg	0.25 deg

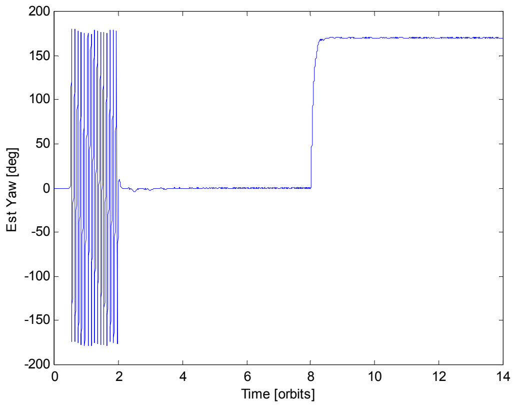

Fig. 2 Estimated yaw angle during yaw phase control

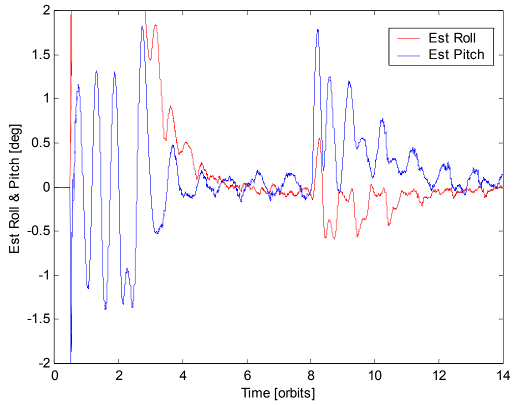

Fig. 3 Estimated roll/pitch angle during yaw phase control

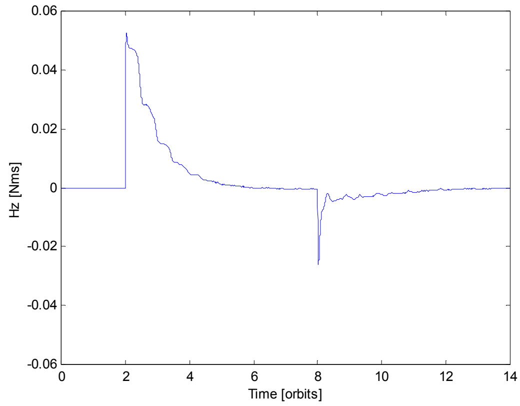

Fig. 4 Z Wheel momentum during yaw phase control

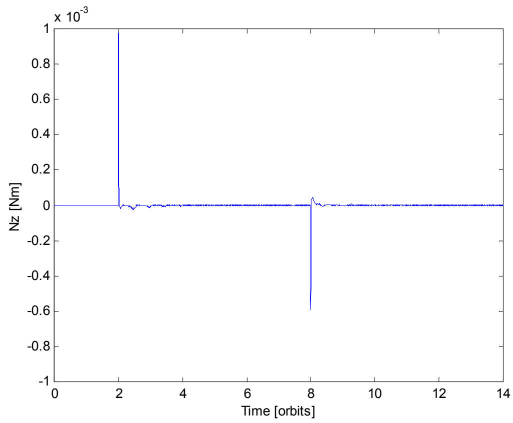

Fig. 5 Z Wheel torque during yaw phase control

5.3 Z Reaction Wheel Libration Damping Thruster Torque

We assume that the thruster is in the X-axis with the following torque Nx = 0.0 Nm, Ny = 0.001 Nm and Nz = 0.005 Nm. For this simulation we are going to simulate the same scenario above.

Figure 6 to 12 shows the orbital maneuver to damp the thruster disturbance torque; the firing time of the thruster is 10 seconds at 33000 seconds (5.5 orbits) and at 66000 seconds (11 orbits). Damping of the attitude disturbance is achieved within a fraction of an orbit. The total accumulated on time of magnetorquer is approximately 10360 seconds during an active control window of 12 orbits (72 000 seconds). This gives an average magnetorquer power drain of 0.12 Watt from the start until the attitude is achieved.

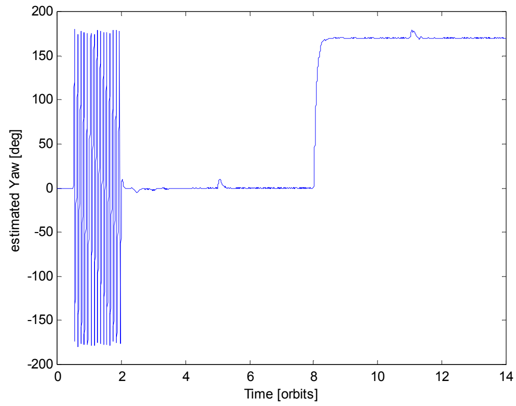

Fig. 6 Estimated yaw angle during yaw phase control damping of thruster disturbances

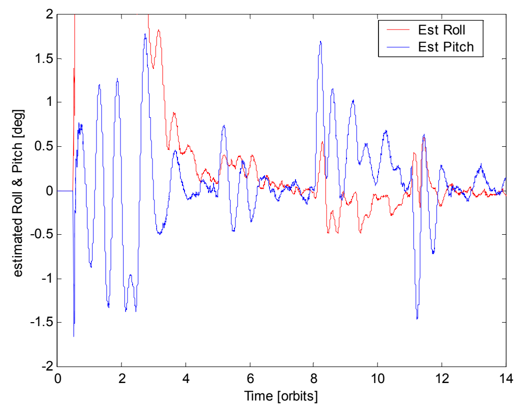

Fig. 7 Estimated roll/pitch angle during yaw phase control damping of thruster disturbances

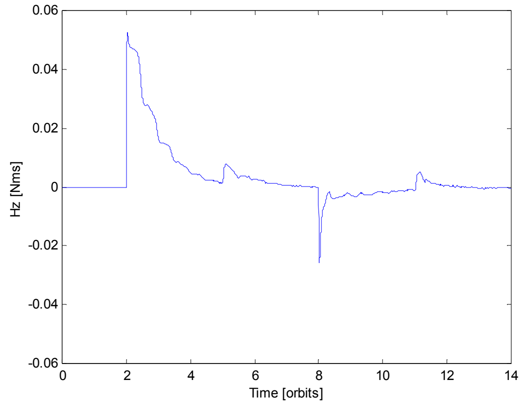

Fig. 8 Z Wheel momentum during yaw phase control damping of thruster disturbances

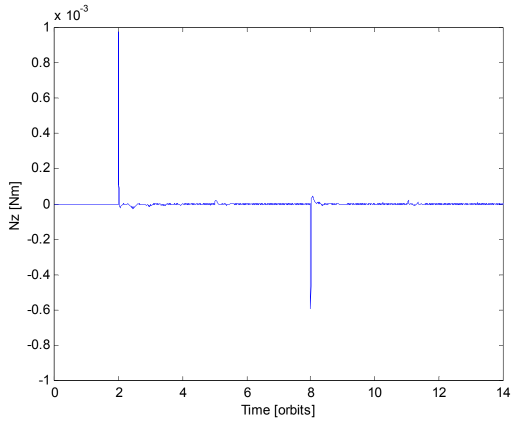

Fig. 9 Z Wheel torque during yaw phase control damping of thruster disturbances

6 Conclusion

The proposed attitude determination and control system was tested on 3-axis stabilized satellite, using yaw reaction wheel, with dual redundant 3-axis magnetorquers. We have demonstrated successful operation of Z wheel controller on a gravity gradient stabilised satellite.

In order to damp libration thruster torque is better to use Z wheel plus magnetorquer than magnetorquer because

- Less total accumulated on-time of magnetorquer;
- Less average magnetorquer power drain.

To conclude, a low cost and light weight attitude determination and control system was proposed to be used by three axis Nadir stabilised platform satellite.

References:

[1] A. Gelb, *Applied Optimal Estimation*, The M.I.T. Press, Cambridge - Massachusetts – London, 1988.

[2] A.C. Stickler and K.T. Alfriend, "An Elementary Magnetic Attitude Control System", *AIAA Mechanics and Control of Flight Conference*, Anaheim California, 5-9 Aug. 1974.

[3] A.M. Si Mohammed, M.N. Sweeting, J. R. Cooksley, “An Attitude Determination and Control System of the Alsat-1 First Algerian Microsatellite”, *Proceeding IEEE Recent Advances in Space Technologies, RAST 2003*, 20-22 November, 2003, Istumbul, Turkey.

[4] A.M. Si Mohammed, M.N. Sweeting, J. R. Cooksley. “Modelling and Simulation of Magnetic Control and its Application on Alsat-1 First Algerian Microsatellite”. *Proceeding 18th European Simulation Multiconference, Networked Simulations and Simulated Networks, SCS – The Society for Modeling and Simulation International*, 13-16 June, Magdeburg, Germany, 2004.

[5] A.M. Si Mohammed, M. Benyettou, M.N. Sweeting, J. R. Cooksley, “Initial Attitude Acquisition Result of the Alsat-1 First Algerian Microsatellite in Orbit”, *Proceeding IEEE International Conference on Networking, Sensing and Control*, March 19-22, 2005, Tucson, Arizona, USA.

[6] A.M. Si Mohammed, M. Benyettou, M.N. Sweeting, J. R. Cooksley, “Full Attitude Determination Specification - Small Libration Version - of the Alsat-1 First Algerian Microsatellite in Orbit”, *Proceeding IEEE Recent Advances in Space Technologies, RAST 2005*, 9-11 June, 2005, Istumbul, Turkey.

[7] A.M. Si Mohammed, M. Benyettou, M.N. Sweeting, J. R. Cooksley, “Imaging Mode Results of the Alsat-1 First Algerian Microsatellite in Orbit”, *Proceeding IEEE Recent Advances in Space Technologies, RAST 2005*, 9-11 June, 2005, Istumbul, Turkey.

[8] A.M. Si Mohammed, M. Benyettou, S. Chouraqui, A. Boudjemai, Y. Hashida, “*Magnetorquer Control for Orbital Manoeuvre of low earth orbit Macrosatellite*”, Journal of WSEAS Transactions on Communications, Vol. 5, Issue 5, pp. 944-947, May 2006.

[9] C.C. Matthew, J. Hale, P. Vergez, M. J. Meerman, Y. Hashida, “Kalman Filtering and the Attitude Determination and Control Task”, Department of Astronautics, USAF Academy, American Institute of Aeronautics and Astronautics, 2006.

[10] E.J. Lefferts, F.L. Markley, M.D. Shuster, “*Kalman Filtering for Spacecraft Attitude estimation*”, Journal of Guidance and control, Vol. 5, No. 5, pp. 417-429, Sept. 1982.

[11] F. Martel, P.K. Pal, M. Psiaki, “*Three Axis Attitude Determination via Kalman Filtering of Magnetometer Data*”, Paper No. 17 for the Flight Mechanics Estimation Symposium, NASA/Goddard Space Flight Center, Greebelt Maryland, May 10 & 11, 1988.

[12] F. Martel, P.K. Pal and M. Psiaki “*Active Magnetic Control System for Gravity Gradient Stabilized Spacecraft*”, Proceedings of the 2th AIAA/USU Conference on Small Satellites, Utah State University, Sept.1988.

[13] M.D. Shuster, S.D. Oh, “*Three Axis Attitude Determination from Vector Measurements*”, Journal of Guidance and Control, Vol. 4, No.1, pp. 70-77, Jan. 1981.

[14] M.S. Hodgart "Gravity Gradient and Magnetorquing Attitude Control for Low Cost Low Earth Orbit Satellites- the UoSAT Experience", Ph.D. Submission at University of Surrey, June 1989.

[15] J.R. Wertz, 1991. *Space Mission Analysis and Design*. Space Technology Library, Kluwer Academic Publishers, Dordrecht – Boston – London.

[16] R.G. Brown, P.Y.C. Hwang, *Introduction to Random Signals and Applied Kalman Filtering*, John Whiley and Sons, New York – Chichester – Brisbane – Toronto – Singapore, 1992.

[17] W.H. Steyn, “A Multi-Mode Attitude Determination and Control System for Small Satellite”, Ph.D (Engineering), University of Stellenbosch, December 1995.

[18] Y. Hashida, ADCS *Design for Future UoSAT Standard Platform*, Surrey Space Centre, Guilford, UK, August 2004.