Computation of transmission system usage for power wheeling burden evaluation

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Abstract: - In this paper, the circuit theory is applied to develop a methodology able to evaluate and to charge electricity market participants for the transmission system usage. The developed formulation gives rise to generators/loads line flows responsibilities, after agreements between Purchasing and Selling Entities (PSEs) are settled in the market. The procedure developed adopts circuit theory to derive a relationship between complex power flowing in system lines and power injected by generators and loads. From the relationships derived dominant and opposite flows can be highlighted. This result enables the method to be used in clearing houses for transmission system usage compensation, and adopted for Bilateral Transaction as well as Power Pool-based markets, simply combining in different ways nodal contributions. Results are provided for the IEEE-14 bus test system confirming the effectiveness of the proposed method to selectively assign line flow burden to generators and loads.

Key-Words: - Power wheeling, Line flow, Bilateral Transactions, Power Pools, Simultaneous contracts.

1 Introduction

Vertically integrated utilities are being broken up worldwide, allowing end users to buy power from more distant generators. In this context new problems appeared, such as transmission pricing and congestion management. Related costs need to be covered by suppliers and consumers thus, in order to implement a nondiscriminatory market, it is necessary to know whether or not, and to what extent, each market participant contributes to transmission system usage. Several efforts focus on this task. The well known MW-Mile method evaluates the transmission capacity usage as a function of magnitude, path and distance traveled by the transacted power [1]. Another class of studies, based on the proportionality principle stated in [2] propose a topological analysis of network power flows [3-6]. They are based on the assumption that for any bus there are lines that inject power and others that evacuate power. A differential method in which power flows with and without transaction are compared to evaluate the impact of transaction on the network have been developed in [7]. In particular, distribution factors have been derived in terms of the ratio of active power flow due to injection and active net power flowing in lines. The method totally disregard reactive power flows which heavily contributes to system usage. Contributions of generators in supplying a set of buses have been determined adopting concepts like generator domain, commons, links and state graph in [8-10]. In systems without losses and loop flows it is possible to trace system power flows applying the graph theory [11]. Alternatively, as reported in [12], a tracing of real and imaginary currents from sources to sinks can be preventively evaluated and then converted into power contributions. Predefined node-to-line participation factors are based on sensitivity analysis which evaluates power flow variations following unitary changes of power injected/extracted by generators and loads [13-15].

This paper develop a methodology able to decompose power flows into a sum of contributions that depends on location as well as amount of injected and sunk powers made in transaction agreements. With this formulation, both active and reactive power contributions can be derived from each injection, and thus from each transactions. Moreover, useful information can be obtained about the direction of power contributions in relation to the net power flow lines to highlight dominant and opposite power flow contributions.

The concepts which form the basis of the proposed method and the algorithm required to put it into practice are described in the following sections with the help of the standard IEEE 14-bus test system.

2 Line flow unbundling

An explicit relationship between power flows and injected powers for a given operating point can be derived if one assumes that the actual system operating point derives from coexisting transactions settled in the market. In fact, they define the voltage profile (in terms of voltage magnitudes and phase angles) of the overall power system. Given such nodal voltages, line flows can be obtained and a relationship between them and the sum of transactions can be derived.

In a power system with N buses and L lines, $\mathcal{N} = \{1,2,\ldots,N\}$ denotes the set of buses, including the slack bus, and $\mathcal{L} = \{\ell_1, \ell_2, \ldots, \ell_L\}$ the set of transmission lines and transformers that connect the buses in the set N. $\boldsymbol{V} = [V_1, V_2, \ldots, V_N]^T$ denotes the N-dimensional vector of nodal voltages.

Each element $\ell \in \mathcal{L}$ is associated with the ordered pair $\ell = (i, j)$ using the convention whereby the direction of the power flow in the line ℓ is *from* node i *to* node j so that $\Re e(f_\ell) \geq 0$, where f_ℓ is the active and reactive power flowing in line ℓ. $\tilde{\boldsymbol{f}} = [f_1, f_2, \ldots, f_L]^T$ is defined as the L-dimensional vector of powers flowing in L lines. Following the same assumption, $\boldsymbol{I}_b = [I_1, I_2, \ldots, I_L]^T$ is given as the L-dimensional column vector of branch currents. With the series admittance of line ℓ given as $y_\ell = g_\ell - jb_\ell$, series line admittances are organized to form the (LxL)-dimensional primitive admittance matrix, $\boldsymbol{Y}_\ell$ [16].

The net active and reactive power injection are denoted at node $n \in \mathcal{N}$, by the complex power $S_n = P_n + jQ_n$.

Given that $\tilde{\boldsymbol{S}} = [S_1, S_2, \ldots, S_N]^T$ is the N-dimensional vector, representing the net power injected by the N buses of the system, the vector can be broken down in N terms of injections acting simultaneously in the power system as follows:

$$\tilde{\boldsymbol{S}} = \sum_{n \in \mathcal{N}} \tilde{\boldsymbol{S}}_n \tag{1}$$

where $\tilde{\boldsymbol{S}}_n = [0, 0, \ldots, S_n \ldots, 0]^T$ with $n \in \mathcal{N}$.

The (NxL)-dimensional node-to-branch reduced incidence matrix, $\boldsymbol{A}_{n\ell}$, can be obtained using graph theory.

The power flow in the generic ℓ–th branch, $\tilde{f}_\ell$, can be expressed as follows:

$$\tilde{f}_\ell = V_i I_\ell^* \tag{2}$$

where the apex * denotes the conjugate operator, and V_i is the voltage at node *from* i, from which the generic ℓ-th branch starts.

We can then obtain the (LxL)-dimensional matrix $\boldsymbol{V}_F = diag(V_i)$, where generic ℓ-th fill-in is the magnitude and phase angle voltage of the *from* node.

Equation (2) can be rewritten in compact form as follows:

$$\tilde{\boldsymbol{f}} = \boldsymbol{V}_F \boldsymbol{I}_b^* \tag{3}$$

The vector of branch currents can be obtained by the following relationship:

$$\boldsymbol{I}_b = \boldsymbol{Y}_\ell \boldsymbol{A}_\ell^T \boldsymbol{V} \tag{4}$$

Substituting (4) with (3), the following relationship between power flows and nodal voltages can be obtained:

$$\tilde{\boldsymbol{f}} = \boldsymbol{V}_F \boldsymbol{Y}_\ell^* \boldsymbol{A}_\ell^T \boldsymbol{V}^* \tag{5}$$

The linear and passive network gives rise to the following nodal current equation:

$$\boldsymbol{I} = \boldsymbol{Y}_n \boldsymbol{V} \tag{6}$$

where $\boldsymbol{I} = [I_1, I_2, \ldots, I_N]^T$ represents the vector of currents injected into nodes, and $\boldsymbol{Y}_n$ is the node-admittance matrix.

By inverting (6) the nodal voltage vector $\boldsymbol{V}$ can be obtained and then substituted with (5):

$$\tilde{\boldsymbol{f}} = \boldsymbol{V}_F \boldsymbol{Y}_\ell^* \boldsymbol{A}_\ell^T \left(\boldsymbol{Y}_n^{-1}\right)^* \boldsymbol{I}^* \tag{7}$$

After a load following solution has been obtained, the net power injected can be expressed in terms of magnitude and phase angle of node voltages and injected currents as follows:

$$\tilde{\boldsymbol{S}} = diag(\boldsymbol{V}) \boldsymbol{I}^* \tag{8}$$

From (8) the current injection vector can be obtained and substituted with (7) to obtain:

$$\tilde{\boldsymbol{f}} = \boldsymbol{V}_F \boldsymbol{Y}_\ell^* \boldsymbol{A}_\ell^T \left(\boldsymbol{Y}_n^{-1}\right)^* diag(\boldsymbol{V})^{-1} \tilde{\boldsymbol{S}} \tag{9}$$

Equation (8) shows how to evaluate power flows once the load flow calculation has been obtained. As can be noted, flows depend on the overall set of injections acting simultaneously on the system.

Finally, substituting (1) with (9), power flows can be obtained as follows:

$$\tilde{\boldsymbol{f}} = \boldsymbol{V}_F \boldsymbol{Y}_\ell^* \boldsymbol{A}_\ell^T \left(\boldsymbol{Y}_n^{-1}\right)^* diag(\boldsymbol{V})^{-1} \sum_{n \in \mathcal{N}} \tilde{\boldsymbol{S}}_n \tag{10}$$

The core of the proposed method relies on the assumption that all injections settled in the market define bus voltages that are common to all generators and loads. For the current operating point, defined by all injections in place, after a load flow calculation, eqn. (10) can be used to obtain the relationship between power flows and power injected by generators and loads.

With $\tilde{f}_\ell^n$ denoting the complex power flowing in generic ℓ-th line due to injection n-th, (10) can be rewritten as follows:

$$\tilde{\boldsymbol{f}} = \sum_{n \in \mathcal{N}} \left[\tilde{f}_1^n, \tilde{f}_2^n, \ldots, \tilde{f}_\ell^n, \ldots \tilde{f}_L^n\right]^T \tilde{\boldsymbol{S}}_n \tag{11}$$

3 Line flow factors

The contribution to complex power flow in lines from power injection can be expressed as a fraction of the net power flow of the line by adopting the following factor:

$$\Psi_\ell^n = \frac{\tilde{f}_\ell^n}{\left\|\tilde{f}_\ell\right\|} \quad \begin{array}{l}\ell \in \mathcal{L} \\ n \in \mathcal{N}\end{array} \tag{12}$$

Such contributions will constitute entries of the *line-to node contributions matrix*, whose dimension is (ℓxN). In (12) the complex power flowing in line ℓ-th due to injection n-th is normalized in relation to the apparent power of the line. Results in (12) are expressed in complex numbers which highlight active as well as reactive power contributions to the power flow line. With this formulation, information regarding direction of power flow can be obtained. While direction of power flow can be assumed for active power, nothing can be assumed for reactive power. In fact, the sign of reactive power can be interpreted either as direct or opposite flow as well as inductive or capacitive reactive power.

Node-to-line contributions, evaluated as in the above section, need to be opportunely combined on the basis on the adopted market rule. For this purpose we define the *aggregation matrix*, **M**, whose function is to combine contributions to line flows due to one generator and one load, in the case of bilateral market, or one or more generators to one or more loads in the case of Power Pool. In general, this matrix can combine both market structures, as is the case in the Italian market. We denote with nT the total number of bilateral transactions and with nP the total number of Pools, a generic element, $m_{i,j}$, of the (Nx(nT+nP+1))-dimensional matrix, **M**, will be equal to 1 if nodes i and j are involved in the transaction, otherwise it will be 0. Columns of the **M** matrix need to be expanded since they must incorporate the slack bus. In fact, the slack bus with its injection for loss compensation need to be considered separately from other buses exchanging power each other.

The adoption of the aggregation matrix permits to evaluate transaction or pool line contributions as follows:

$$\boldsymbol{L} = \Psi \boldsymbol{M} \tag{13}$$

where **L** is the (ℓx(nT+nP+1))-dimensional *line participation factors matrix* due to each transactions or pools.

The defined line-to-node contributions factors hold the following relationship:

$$L_\ell = \left\| \sum_{k=1}^{nT+nP+1} L_\ell^k \right\| = 1 \qquad \ell \in \mathcal{L} \tag{14}$$

4 Test Results

We tested the proposed methodology on the IEEE-14 Bus Test System, shown in figure 1 [17]. In all simulations generators were assumed as PV buses whereas load buses were considered as PQ buses. Reactive powers of loads were obtained considering load power factors equal to 0.9. Results are provided in p.u. on 100 MVA base.

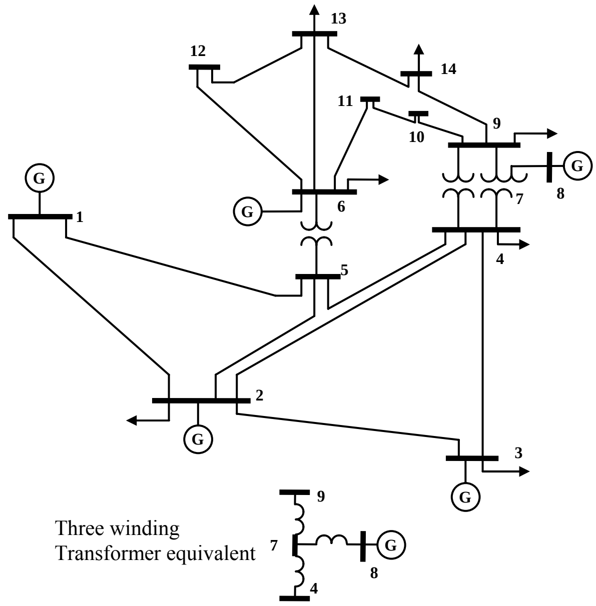

Fig. 1: The IEEE-14 bus test system.

We slightly modified the network changing compensators 3, 6 and 8 into generators. In particular, we assumed nodes # 2, 3, 6, and 8 as having net injection, whereas buses # 4, 9, 13 and 14 acted as pure loads. Remaining nodes were considered as pure transit nodes. Bus #1 was chosen to be the slack bus. In both cases we assumed powers injected into nodes as reported in Table 1.

TABLE 1 INJECTIONS DATA

Bus #	Generation Active Power [p.u.]	Generation Voltage Magnitude [p.u.]	Load Active Power [p.u.]	Load Reactive Power [p.u.]
1	--	1.060	--	--
2	0.60	1.045	--	--
3	0.80	1.010	--	--
4	--	--	0.40	0.19
6	0.40	1.070	--	--
8	0.20	1.090	--	--
9	--	--	0.60	0.29
13	--	--	0.20	0.10
14	--	--	0.80	0.39

Starting from these desired control variables, the load flow output gave the system operating point as reported in Tables 2 and 3.

TABLE 2 IEEE-14 BUS – LOAD FLOW SOLUTION

Bus	Voltage Magnitude	Generation		Load	
		Active Power	Reactive Power	Active Power	Reactive Power
#	[p.u.]	[p.u.]	[p.u.]	[p.u.]	[p.u.]
1	1.060	0.18	0.44	--	--
2	1.045	0.60	0.20	--	--
3	1.010	0.80	-0.33	--	--
4	0.991	--	--	0.40	0.19
5	1.005	--	--	--	--
6	1.070	0.40	0.57	--	--
7	1.012	--	--	--	--
8	1.090	0.20	0.49	--	--
9	0.967	--	--	0.60	0.29
10	0.984	--	--	--	--
11	1.025	--	--	--	--
12	1.037	--	--	--	--
13	0.995	--	--	0.20	0.10
14	0.831	--	--	0.80	0.39

TABLE 3 IEEE-14 NET LINE FLOWS [p.u.]

Line #	From bus #	To Bus #	Active Power flow [p.u.]	Reactive Power flow [p.u.]	Apparent Power flow [p.u.]
1	1	2	-0.07	0.29	0.30
2	1	5	0.25	0.21	0.33
3	2	3	-0.21	0.24	0.32
4	2	4	0.38	0.20	0.43
5	2	5	0.36	0.13	0.38
6	3	4	0.58	-0.08	0.59
7	4	5	-0.15	-0.28	0.31
8	4	7	0.41	-0.08	0.42
9	4	9	0.27	0.06	0.28
10	5	6	0.45	-0.26	0.52
11	6	11	0.19	0.15	0.24
12	6	12	0.15	0.07	0.16
13	6	13	0.51	0.36	0.63
14	7	8	-0.20	-0.45	0.49
15	7	9	0.61	0.44	0.75
16	9	10	-0.18	-0.13	0.22
17	9	14	0.46	0.29	0.54
18	10	11	-0.18	-0.13	0.22
19	12	13	0.14	0.06	0.16
20	13	14	0.43	0.28	0.51

Tables 4 and 5 report, respectively, shares expressed in p.u. of active and reactive power flowing in all the system lines.

As can be noted, nodes 5, 7, 10, 11, and 12 exhibit contributions equal to zero since they do not have any net injection, i.e, they are pure transit nodes. Moreover, some power flow contributions exhibit an opposite sign in relation to that of the net line flow, which can be read as opposite flow contributions. Their influence on dominant flows, i.e. on contributions with the same sign as the net line flow, is in opposition.

For test purposes, in the last column of both tables we reported the sum of the node-to-line contributions. It can be observed that the total contributions exactly match the net active and reactive line flows, as reported in table 3.

Because the lack of space the line-to node contributions matrix was omitted, deriving only the final line participation factors matrix.

We supposed that four transactions, defined by the following aggregation matrix, took place in the market:

TABLE 6 AGGREGATION MATRIX - **M**

		Slack Bus	Transaction #			
			T_1	T_2	T_3	T_4
Bus #	1	**1**	0	0	0	0
	2	0	**1**	0	0	0
	3	0	0	**1**	0	0
	4	0	0	0	**1**	0
	5	0	0	0	0	0
	6	0	0	0	**1**	0
	7	0	0	0	0	0
	8	0	0	0	0	**1**
	9	0	**1**	0	0	0
	10	0	0	0	0	0
	11	0	0	0	0	0
	12	0	0	0	0	0
	13	0	0	0	0	**1**
	14	0	0	**1**	0	0

The matrix **L** of the line participation factors, obtained applying eqn. (13) is as follows:

5 Conclusions

In this paper a method for determining the impact of generation and load values on transmission line flows has been developed. The allocation process is of much interest as competitive electricity markets necessitate equitable methods for allocating transmission usage in order to set transmission usage charges and congestion charges on an unbiased and an open-access basis. In this method, line flows are shared among generators and loads on a nodal basis enabling the scheme to be used in markets where Power Pool is superposed to Bilateral Market as in the case of the Italian electricity market.

Efficient factors have been derived with aim of implementing a practical tool to allocate transmission usage payments. By using these factors, allocation can be attributed to generators, loads, or transaction-related net power injections. The procedure developed adopts circuit theory to derive a relationship between complex power flowing in system lines and power injected by generators and loads at a given operating point. Relationships derived show power flow contributions from each injection with a proper sign to indicate if a given injection contributes to the net power flow in the same or opposite direction as the dominant flow.

TABLE 4 ACTIVE LINE FLOW SHARES [p.u.]

Line #	From bus #	To Bus #	Node # 1	2	3	4	5	6	7	8	9	10	11	12	13	14	**Total**
1	1	2	0.11	-0.15	-0.13	0.03	0.00	-0.02	0.00	-0.01	0.03	0.00	0.00	0.00	0.01	0.04	-0.07
2	1	5	0.03	0.01	-0.06	0.06	0.00	-0.06	0.00	-0.02	0.10	0.00	0.00	0.00	0.04	0.15	0.25
3	2	3	0.02	0.08	-0.36	0.02	0.00	-0.01	0.00	-0.01	0.02	0.00	0.00	0.00	0.00	0.02	-0.21
4	2	4	0.02	0.09	-0.03	0.09	0.00	-0.05	0.00	-0.03	0.11	0.00	0.00	0.00	0.03	0.15	0.38
5	2	5	0.00	0.06	-0.03	0.07	0.00	-0.08	0.00	-0.03	0.12	0.00	0.00	0.00	0.05	0.19	0.36
6	3	4	0.00	0.00	0.34	0.07	0.00	-0.04	0.00	-0.02	0.09	0.00	0.00	0.00	0.03	0.12	0.58
7	4	5	-0.07	-0.13	0.02	-0.08	0.00	-0.11	0.00	-0.02	0.04	0.00	0.00	0.00	0.06	0.15	-0.15
8	4	7	0.14	0.42	0.52	-0.29	0.00	0.17	0.00	0.04	-0.17	0.00	0.00	0.00	-0.09	-0.32	0.41
9	4	9	0.07	0.18	0.23	-0.13	0.00	0.06	0.00	0.04	-0.04	0.00	0.00	0.00	-0.03	-0.10	0.27
10	5	6	0.19	0.64	0.85	-0.43	0.00	0.15	0.00	0.11	-0.44	0.00	0.00	0.00	-0.09	-0.52	0.45
11	6	11	-0.05	-0.10	-0.11	0.08	0.00	0.02	0.00	-0.08	0.23	0.00	0.00	0.00	0.00	0.20	0.19
12	6	12	-0.01	-0.01	-0.01	0.01	0.00	0.00	0.00	-0.01	0.03	0.00	0.00	0.00	0.04	0.11	0.15
13	6	13	-0.03	-0.05	-0.06	0.04	0.00	0.01	0.00	-0.04	0.12	0.00	0.00	0.00	0.14	0.38	0.51
14	7	8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.20	0.00	0.00	0.00	0.00	0.00	0.00	-0.20
15	7	9	0.06	0.13	0.15	-0.10	0.00	0.00	0.00	0.14	0.12	0.00	0.00	0.00	0.00	0.10	0.61
16	9	10	0.05	0.09	0.10	-0.07	0.00	-0.02	0.00	0.08	-0.21	0.00	0.00	0.00	0.00	-0.19	-0.18
17	9	14	0.03	0.06	0.06	-0.05	0.00	-0.01	0.00	0.05	-0.14	0.00	0.00	0.00	0.03	0.42	0.46
18	10	11	0.05	0.10	0.10	-0.08	0.00	-0.02	0.00	0.08	-0.22	0.00	0.00	0.00	0.00	-0.19	-0.18
19	12	13	-0.01	-0.01	-0.01	0.01	0.00	0.00	0.00	-0.01	0.03	0.00	0.00	0.00	0.04	0.10	0.14
20	13	14	-0.03	-0.06	-0.06	0.05	0.00	0.01	0.00	-0.05	0.14	0.00	0.00	0.00	-0.03	0.46	0.43

TABLE 5 REACTIVE LINE FLOW SHARES [p.u.]

Line #	From bus #	To Bus #	Node # 1	2	3	4	5	6	7	8	9	10	11	12	13	14	**Total**
1	1	2	0.26	-0.05	0.06	0.01	0.00	-0.02	0.00	-0.03	0.02	0.00	0.00	0.00	0.00	0.03	0.29
2	1	5	0.08	0.01	0.03	0.04	0.00	-0.13	0.00	-0.09	0.08	0.00	0.00	0.00	0.03	0.15	0.21
3	2	3	0.05	0.03	0.15	0.01	0.00	-0.01	0.00	-0.02	0.02	0.00	0.00	0.00	0.00	0.02	0.24
4	2	4	0.04	0.03	0.02	0.05	0.00	-0.10	0.00	-0.10	0.09	0.00	0.00	0.00	0.02	0.15	0.20
5	2	5	0.01	0.02	0.03	0.04	0.00	-0.15	0.00	-0.09	0.08	0.00	0.00	0.00	0.03	0.16	0.13
6	3	4	-0.01	0.00	-0.17	0.04	0.00	-0.09	0.00	-0.08	0.08	0.00	0.00	0.00	0.02	0.12	-0.08
7	4	5	-0.11	-0.01	0.02	-0.06	0.00	-0.20	0.00	0.04	-0.05	0.00	0.00	0.00	0.04	0.05	-0.28
8	4	7	0.28	0.10	-0.32	-0.12	0.00	0.30	0.00	0.03	-0.07	0.00	0.00	0.00	-0.06	-0.22	-0.08
9	4	9	0.12	0.04	-0.14	-0.05	0.00	0.11	0.00	0.06	0.00	0.00	0.00	0.00	-0.02	-0.06	0.06
10	5	6	0.47	0.21	-0.41	-0.23	0.00	0.25	0.00	0.44	-0.38	0.00	0.00	0.00	-0.07	-0.54	-0.26
11	6	11	-0.06	0.00	0.11	0.02	0.00	0.02	0.00	-0.12	0.09	0.00	0.00	0.00	0.00	0.10	0.15
12	6	12	-0.01	0.00	0.02	0.00	0.00	0.00	0.00	-0.01	0.01	0.00	0.00	0.00	0.01	0.05	0.07
13	6	13	-0.03	0.00	0.06	0.01	0.00	0.01	0.00	-0.06	0.04	0.00	0.00	0.00	0.08	0.25	0.36
14	7	8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.45	0.00	0.00	0.00	0.00	0.00	0.00	-0.45
15	7	9	0.08	0.01	-0.13	-0.02	0.00	0.01	0.00	0.25	0.12	0.00	0.00	0.00	0.00	0.13	0.44
16	9	10	0.05	0.00	-0.10	-0.01	0.00	-0.02	0.00	0.11	-0.07	0.00	0.00	0.00	0.00	-0.08	-0.13
17	9	14	0.03	-0.01	-0.07	-0.01	0.00	-0.01	0.00	0.07	-0.04	0.00	0.00	0.00	0.01	0.30	0.29
18	10	11	0.05	0.00	-0.10	-0.01	0.00	-0.02	0.00	0.11	-0.07	0.00	0.00	0.00	0.00	-0.09	-0.13
19	12	13	-0.01	0.00	0.02	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.01	0.04	0.06
20	13	14	-0.03	0.00	0.07	0.01	0.00	0.01	0.00	-0.07	0.04	0.00	0.00	0.00	-0.02	0.26	0.28

TABLE 6 LINE PARTICIPATION FACTORS MATRIX - **L**

	Slack Bus		Transaction #							
			T_1		T_2		T_3		T_4	
Line #	Active	Reactive	Active	Reactive	Active	Reactive	Active	Reactive	Active	Reactive
1	0.37	0.86	-0.37	-0.09	-0.27	0.30	0.05	-0.01	-0.01	-0.09
2	0.08	0.25	0.35	0.26	0.28	0.57	0.02	-0.29	0.04	-0.17
3	0.06	0.14	0.30	0.14	-1.06	0.52	0.04	0.01	-0.01	-0.05
4	0.05	0.09	0.47	0.27	0.26	0.39	0.09	-0.11	0.01	-0.18
5	0.01	0.03	0.49	0.28	0.42	0.51	-0.01	-0.30	0.03	-0.16
6	-0.01	-0.02	0.15	0.13	0.79	-0.07	0.04	-0.08	0.01	-0.10
7	-0.23	-0.36	-0.29	-0.18	0.54	0.21	-0.60	-0.82	0.11	0.25
8	0.35	0.68	0.58	0.07	0.49	-1.28	-0.31	0.42	-0.13	-0.08
9	0.24	0.45	0.51	0.15	0.46	-0.72	-0.25	0.22	0.01	0.14
10	0.36	0.90	0.38	-0.34	0.62	-1.84	-0.54	0.06	0.04	0.72
11	-0.20	-0.24	0.53	0.35	0.36	0.86	0.41	0.16	-0.32	-0.50
12	-0.05	-0.04	0.12	0.05	0.59	0.39	0.09	0.02	0.16	0.00
13	-0.04	-0.05	0.11	0.07	0.51	0.49	0.08	0.03	0.16	0.03
14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.41	-0.91
15	0.08	0.10	0.34	0.17	0.33	0.00	-0.13	-0.02	0.19	0.34
16	0.22	0.23	-0.55	-0.32	-0.41	-0.84	-0.41	-0.14	0.34	0.48
17	0.06	0.06	-0.15	-0.08	0.90	0.44	-0.11	-0.03	0.15	0.15
18	0.21	0.23	-0.54	-0.33	-0.40	-0.84	-0.41	-0.14	0.34	0.48
19	-0.05	-0.04	0.12	0.05	0.60	0.38	0.09	0.01	0.16	0.00
20	-0.06	-0.06	0.16	0.09	0.78	0.65	0.12	0.04	-0.16	-0.17

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