Energy – Time Dichotomy in the Optimal Control of the Electrical Drives

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Abstract: A comparison between time optimal control and minimum energy control for electrical drives is performed. It is considered especially the drives with DC and induction motors. Some simulation and experimental results are presented.

Key-Words: electrical drives, optimal control, minimum time, minimum energy

1 Introduction

There are different requirements for the transient period and especially for the start of the electrical drives. The main demands refer to the productivity and to the energy consumption. If one of these aspects has a main importance for the technological process, the optimal control from the corresponding point of view can be adapted. If the interest is for a great productivity, the minimum time control of the electrical drive can be introduced, tacking also into account that this control can be easy achieved.

The optimal control from the energetic point of view is important because in the transient period the energy losses (especially the Joule losses) are very high. Although the optimal control [1], [2] of the electrical drives represents an important way for energy saving and there are numerous study dedicated to optimal control of the electrical drive systems, for different types of motors, criteria, or used methods (we mention, for instance [2], [3], [4], [5], [6], but many other papers can be indicated), the number of applications is nowadays very small. We appreciate that a cause of the reluctance in this direction is the complexity of the algorithm. However, the method proposed by authors [7], [8] for DC or brushless motors allow to carry out an easy implementation and therefore, it is useful to apply the optimal control, since the diminution of the energy losses in the motor windings is significant in the electromechanical transient process, by comparison with classical cascade control.

The mentioned papers put in evidence a know fact [2] that in many cases the demands of energy and time conditions are contradictory. Unfortunately, the design of electrical drives rarely tacke into account these aspects and does not offer users the possibility of choice of an adequate operating mode, in concordance with needs of the technological process and economical advantages.

The paper presents certain considerations referring to the optimal control of the electrical drives from the energetic and time response point of view, respectively. These considerations are presented in a simple mode, because the currents (and not voltages) are adopted as control variables. This procedure represents not only a computing simplification, but has a practical reason. Indeed, although the electrical drives are usually supplied from voltage sources, the motors are controlled by means of currents, e.g. the internal current loop in cascade control of the DC motors, or the frequent utilization of the current source inverters for induction motors. In this respect we mention that our previous results [8], [10] have indicated a high similitude between the two cases (the control by mean of currents and voltages). Tacking into account that the use of current as control variables leads to an easier implementation (based on cascade control with small transformations) this way was adapted in the paper.

2 The optimal control using one control variable

For the sake of a concrete presentation, the equations for a D.C. motor will be considered and some remarks referring to the induction motor will be indicated in the Section 3.

The mechanical equilibrium equation is

$$\dot{\Omega} = \frac{1}{J}(cI - M), \tag{1}$$

where Ω is the rotor speed, J is the inertia, I – the rotor current, M – the load torque and c – a machine constant. In the sequel we shall use the normalized (relative) values of the variables. In this respect, we introduce the normalized variables

$$i = I / I_N,\ m = M / M_N,\ \omega = \Omega / \Omega_N,\ \tau = t / T_N, \tag{2}$$

where the index N refers to the rated corresponding values, and

$$T_N = J\Omega_N / M_N \tag{3}$$

is a nominal time of the system.

In the steady state for rated values,

$$cI_N = M_N. \tag{4}$$

Tacking into account (2), (3) and (4), the equation (1) becomes

$$\dot{\omega} = i - m \tag{5}$$

The solution to this equation is

$$\omega(\tau) - \omega(0) = \int_0^\tau i(\theta)d\theta - \int_0^\tau m(\theta)d\theta. \tag{6}$$

If the final desired value ω_d of the speed is imposed, and if we consider $\omega(0) = 0$, equation (6) leads to

$$\omega_d = \omega(T) = \int_0^T i(\theta)d\theta - \int_0^T m(\theta)d\theta. \tag{7}$$

Remark 1: In many control problem, the adopted value for current depends on the load torque $m(t)$. The equation (7) indicates that in certain cases it is necessary to know especially the mean value

$$m_M = \frac{1}{T}\int_0^T m(\theta)d\theta \tag{8}$$

on the interval [0, T]. This fact implies to beforehand know at least the shape of $m(\tau), \tau \in [0, T]$ and to measure or estimate the magnitude of the load torque at the beginning of the control process. This aspect occurs in the optimal energy control problem, but not in the minimum time problem.

In many applications, the load torque can be considered constant on the short interval of the start, and in this case, of course $m_M = m$ .For simplicity, in the sequel, we shall consider $m(\tau) = \text{const}$. If this condition is not satisfied, the above remark, referring to the mean value have to be considered and m will be replaced with m_M.

From the energetic point of view, the interest is to minimize the Joule losses, tacking into account that these ones are significantly greater than other losses in the transient period. For this reason, the criterion

$$I_E = \int_0^T i^2(\tau)d\tau \tag{9}$$

is adopted and we can formulate the optimal control problem

P1: Find $i(\tau)$ and $\omega(\tau)$ which minimizes the criterion (9) and satisfies the equation (5), and conditions $\omega(0) = 0,\ \omega(T) = \omega_d$.

From the productivity point of view, the interest is to minimize the transfer time T or the index

$$I_T = T = \int_0^T d\tau. \tag{10}$$

In all equations, the index T refers to a value obtained for minimum time problem and the index E refers to the optimal energy consumption problem.

It is obviously know that in minimum time problem, the restrictions have to be introduced, that is

$$|i(\tau)| \le i_M, \tag{11}$$

where i_M is a maximum acceptable value of the current. The minimum time problem is

P2: Find the control variable $i(t)$ and the state variable $\omega(t)$ so that the criterion (10) to be minimized, subject to (5) and (11).

The solution to this last problem can be easy find using the Pontriagyn minimum principle and it is

$$i_T^*(\tau) = i_M,\quad \tau \in [0, T]. \tag{12}$$

The minimum time is

$$T = \tau_T^* = \frac{\omega_d}{i_M - m} \tag{13}$$

and the normalized dissipated energy is

$$w_T^* = \frac{\omega_d}{i_M - m} i_M^2 \,. \tag{14}$$

In (14), $w^* = W / W_N$, where W is the Joule losses on the interval [0, T] and $W_N = rI_N^2 T_N$ is the normalized energy losses.

The control variable don't depend on the load torque (it is always i_M), but the transfer time τ_T^* depends on m.

The Hamiltonian in **P1** problem is $H = i^2 + \lambda(i - m)$ and from the Hamilton conditions, it results $\lambda(\tau) = \lambda = \text{const.}$ and

$$i(\tau) = i = \text{const.} \tag{15}$$

We obtain from the condition $\partial I_E / \partial i = 0$:

$$i_E^* = 2m \,, \tag{16}$$

the minimum Joule losses

$$w_E^* = 4m\omega_d \tag{16}$$

and the corresponding transfer time

$$\tau_E^* = \omega_d / m \,. \tag{17}$$

Remark 2: We conclude from (16) that the optimal current must ensure a electromagnetic torque having a double value of the load torque. This is a general feature of the electrical machine [11], valid for any motor type, using one or two control variable.

Remark 3: If the load torque has a small value, the transfer time (17) increases very much. In many applications this time must be limited to a maximum value τ_M and in this case $i_E = m + \frac{\omega_d}{\tau_M}$ and variation of the current is $\Delta i_E = i_E - i_E^* = \frac{\omega_d}{\tau_M - m} > 0$, since $\tau_M < \tau_E^*$.

Corresponding, the copper losses increase with

$$\begin{aligned} \Delta w_E = w_E - w_E^* = \tau_M i_E^2 - \tau_E^* i_e^{*2} = \\ = \tau_M \left(m - \frac{\omega_d}{\tau_M} \right) = \tau_M \Delta i_E^2 . \end{aligned} \tag{19}$$

The comparison between **P1** and **P2** problems can be performed referring the time transfer and the energy losses in the two cases.

Firstly, it is easy to remark that the solutions for both problems coincide if $i_M = 2$.

The differences between the two cases depend on the ratio $\mu = m / i_M$. One obtain from (13) and (18)

$$\frac{\tau_E^*}{\tau_T^*} = \frac{i_M - m}{m} = \frac{i}{\mu} - 1 \,, \tag{20}$$

with $\mu < 1$, since the motor cannot start if $i < m$.

This variation is presented in Fig. 1.

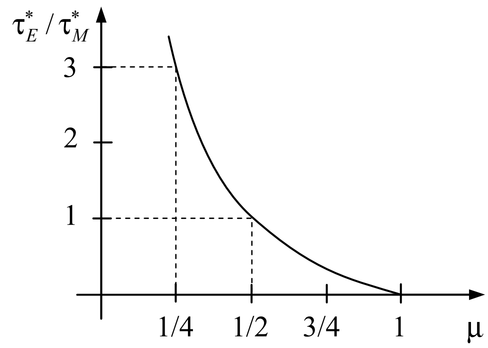

Fig. 1

Similarly, from (14) and (17), it results

$$\frac{w_E^*}{w_T^*} = \frac{4m(i_n - m)}{i_M^2} = 4\mu(1 - \mu) \tag{21}$$

and this variation is indicated in Fig. 2

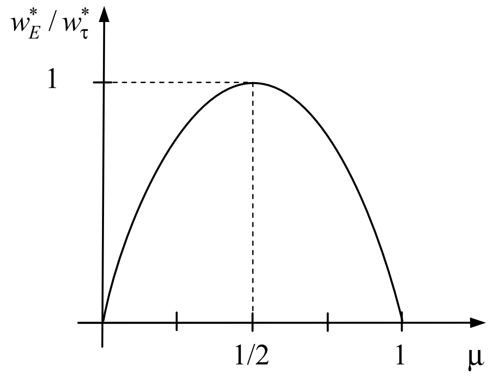

Fig. 2

The Fig. 1 and 2 show again the coincidence of the solutions for **P1** and **P2** problems if $i_M = 2m$. From energetic point of view, any other choice of i_M is unfavorable. The choice $1/2 < \mu < 1$ (or $m < i_M < 2m$) is not recommended from both point of view. If the productivity considerations have priority, one can adopt $i_M > 2m$ $(\mu < 1/2)$, but the energy losses increase in this case.

3 Optimal control with combined criterion

Tacking into account the contradictions between time and energy demands, it is not lack of interest to consider an optimal control with combined criterion:
P3: Find $i(\tau)$ and $\omega(t)$ which minimize

$$I_c = \int_0^T \left[\alpha + i^2(\tau)\right] d\tau , \tag{22}$$

subject to (5) and $\omega(0) = 0,\ \omega(T) = \omega_d$.

In (22) α is the weight coefficient for minimum time control problem: for $\alpha = 0$, one obtain the **P1** problem and for α very great, the problem becomes the minimum time one (a restriction for i have to be introduced in this case).

The Hamiltonian problem is $H = \alpha + i^2(\tau) + {}$ $+\lambda\left[i(\tau) - m(\tau)\right]$. Using a similar way as in Section 2, it results an optimal constant current and from the condition $\partial I / \partial i = 0$, one obtain the optimal control current

$$i_c^* = m + \sqrt{m^2 + \alpha} \tag{23}$$

and then

$$\tau_c^* = \omega_d / \sqrt{m^2 + \alpha} \tag{24}$$

and

$$w_c^* = i_c^{*2} \tau_c^* = \omega_d \left(m + \sqrt{m^2 + \alpha} \right) . \tag{25}$$

These energy losses are greater then in the **P1** case (17):

$$w_c^* - w_E^* = \omega_d \left(m - \sqrt{m^2 + \alpha} \right)^2 \Big/ \sqrt{m^2 + \alpha}$$

and $w_c^* = w_E^*$ if $\alpha = 0$.

Also, from (18) and (24), it results

$$\tau_E^* / \tau_c^* = \sqrt{m^2 + \alpha} \Big/ m > 1 .$$

Of course, for **P3** problem, the time transfer is improved, but the energy losses increase by comparison with **P2** case.

4 The optimal control using two control variables

4.1 The D.C. motor case

If the machine iron is nonsaturated the equation (1) becomes in this case (after normalization)

$$\dot{\omega}(t) = k i_1 i_2 - m , \tag{26}$$

where $i_1 = I_1 / I_{1N}$ and $i_2 = I_2 / I_{2N}$ correspond to the stator and the rotor currents, respectively.

For minimum time problem, the Hamiltonian is $H = 1 + \lambda(\tau)(i_1 i_2 - m)$. The canonical equation $\partial H / \partial\omega = -\dot{\lambda}$ leads to $\lambda(\tau) = \lambda = \text{const.}$ Since H linearly depends on i_1 and i_2 respectively, the minimum principle is satisfied if the currents have the maximum acceptable values i_{1M} and i_{2M}. Therefore, no difference appears by comparison with one control variable case.

For the minimum energy problem, the criterion is

$$I_E = \int_0^T \left[r_1 i_1^2(\tau) + r_2 i_2^2(\tau) \right] d\tau , \tag{28}$$

where r_1 and r_2 are the stator and rotor windings resistances.

The authors have proved [11] that the condition (16) also holds in this case and it is achieved if

$$r_1 i_1^2 = r_2 i_2^2 , \tag{29}$$

that is the rotor and stator losses have to be equal. Of course, such a condition can be carried out only for small m. In other cases, the stator currents reach its maximum value $i_1 = 1$ and the behaviour is similar as in the Section 2.

4.2 The induction motor case

In this case, the criterion for minimum energy problem is similar with (28), with the same significances of the variables. We mention only the result proved in [11]: the optimal control is obtained if (16) holds. The components i_{1d} and i_{1q} (in a *d-q* frame) of the stator current are established [11]. Again, the obtained values can be adopted only for small load torque. Otherwise, the maximum value for i_{1d} component (i.e. the maximum value of the flux) is achieved and the control is performed as in the Section 2.

For the minimum time problem, the conclusion is similar as in the D.C. motor case: the both components of the stator current must have the maximum acceptable values.

In conclusion, if the load torque is small, it is not lake of interest to consider two control variables (the expression are indicated in [11]) in the minimum energy problem. Otherwise, and also in the minimum time problem, it is sufficient to adopt only one control current, as in Section 2.

5 System implementation and experimental results

The above established control is an *ideal optimal control*. We say that the control is ideal because of the assumption referring to the currents: namely, we supposed these can have abrupt, non-inertial variations.

The fact that we know the ideal optimal values of the currents suggests an easy implementation: the controller must ensure the desired value of the currents with a great accuracy. A better accuracy ensures a better proximity to ideal optimal control. Of course, a task for load torque estimation and for computing of the desired current values has to be introduced in the minimum energy problem. This task is not necessary in the minimum time problem. Our attention will be focused in sequel on the proper optimal controller, described above. There are different ways in this direction and we shall present only a structure based on a cascade control with PI controllers: the output of the speed controller is transmitted to the input of the current controller via a saturation bloc. The level of the saturation is variable and corresponds to the desired value of the current. This specific part (the internal loop of the cascade) is presented in Fig.3, where S represents the saturation bloc, C is a PI controller, P is the controlled plant, and i^0 represents the output of the external (speed) controller. The level of saturation of the block S is i_M for minimum time problem or is established in dependence of the load torque *m*, in the minimum energy problem.

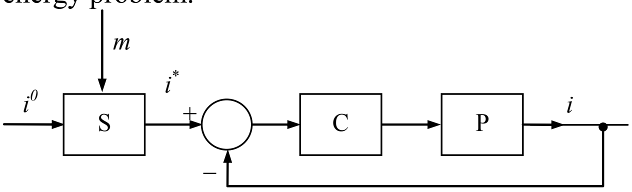

Fig. 3 Structure for optimal current control

The next figures present a selection of simulation and experimental tests, performed for the proposed structure, for DC or induction motors and for different operation conditions. We have supposed in all cases that the load torque is known.

For a drive system with DC motor (having rated data: 3kW, 220V, 17.5A, 1200rpm) with m=15Nm, ωd=125 rad/s, J=0.08Nms^2/rad, certain results obtained by simulation tests are presented in Fig.4 The first case corresponds to the optimal energy control (I=2M/c, and M= 0.75M_N), and the second figure is for time optimal control with limit value for current I_M= 3 I_N. Of course, the drive system is faster in the last case, but the energy consumption increase with 25%.

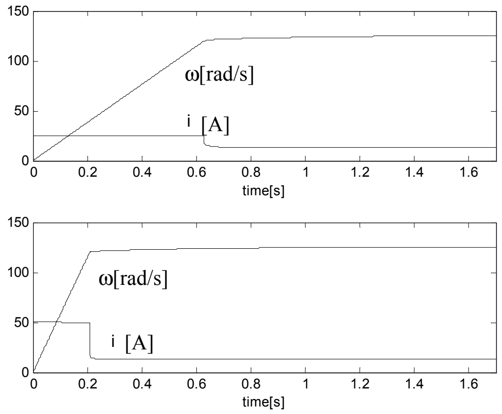

Fig.4. Behaviour of the drive system for minimum energy control and minimum time control

Fig. 5 shows two situations when the load torque has a variation at the moment t_l = T/2 (a step variation from 0.5M_N to M_N). A control variant is to adopt I=2M/c on each interval (first figure). Other variant (optimal) is to adopt I = 2M_{med}/c on all interval. In this case, the energy losses decrease with about 12%.

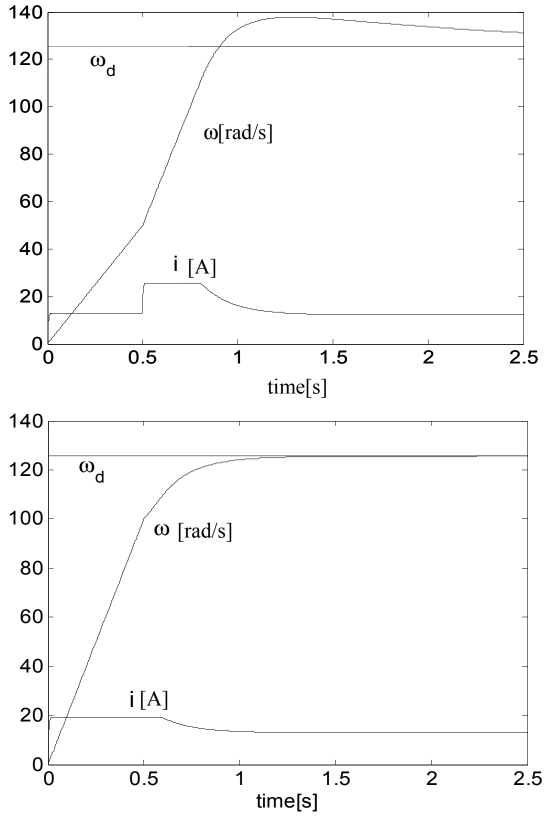

Fig. 5 Behaviour of the optimal system for the variable load torque

The Fig 6 presents the experimental results for an optimal energy control for an electrical drive system with a induction motor (with rated data 4kW, 380V, 8,64A, 1430 rpm).

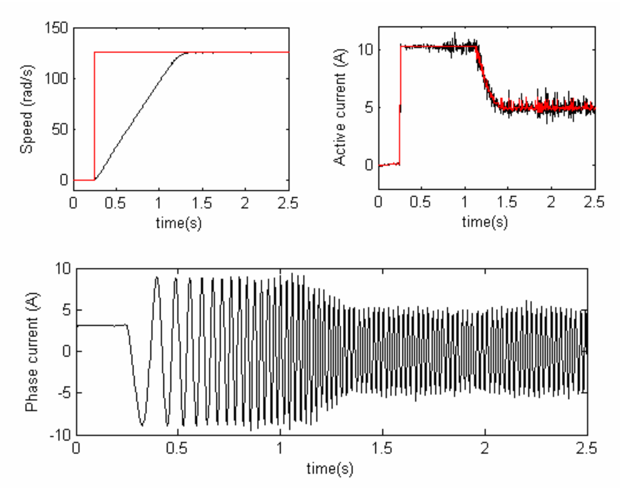

Fig. 6. Behaviour of the drive system with induction motor for optimal energy control (experimental results): speed, active current and stator current

One can remark a good behaviour of the drive systems in all cases. Of course, from energetic point of view, the better situation is the optimal control. Note that in certain situation, the cascade control can achieve similar performances, namely in the case when current is limited to the value $I_M= 2I_N$ and $M=2M_N$. The smallest energy losses are obtained not because of controllers design based on usual criteria, but because of limitation of the current at the same value as in the optimal control. But, for other load torque, the optimal control leads to a value up to 15% smaller as for the cascade control.

6 Conclusions

The minimum energy control and minimum time control of the electrical drives have contradictory demands. Generally, the endeavor of the diminish of the transfer time leads to the increase of the energy losses.

In all cases, the minimum time control can be achieved using only one variable control. For optimal energy control, the use of two variables present interest only for very small load torque.

The structure of the optimal control is not complicated and is based on a usual cascade one, if we adopt the currents as control variable. This assumption has a practical justification, based on the fact that motor currents are frequently controlled in the variable speed drives.

There is a great similitude in optimal control of different motor types.

The performed simulation and experimental tests show a very good behaviour of the optimal control systems and indicate an opportunity for energy consumption decrease in the transient period of an electrical drive.

References

[1] B.D.O. Anderson, J.B. Moore, *Optimal Control. Linear Quadratic Methods*, Prentice Hall, 1990.

[2] M. Athans and P.L. Falb, *Optimal Control*, Mc Graw Hill, New York, 1966.

[3] C. De Capua and C. Landi, Measurement Station Performance Optimization for Testing on High Efficiency Variable Speed Drives, *IEEE Trans. on Instrumentation and Measurement*, vol. 48, no. 6, 1999, pp. 1149-1154.

[4] I.T. Wallace, D.W. Novotny, R.D. Lorenz and D.M. Divan, Verification of Enhanced Dynamic Torque per Ampere Capability in Saturated Induction Machines, *IEEE Transactions on Industry Applications*, vol. 30, no. 5, Sept./Oct. 1994, pp. 1193-1201.

[5] M. Ghribi, Y. Dube and K. Al-Haddad, Linear Quadratic Control of a DC Motor, *Proceedings of the 3rd European Conference on Power Electronics and Applications, EPE'89*, Aachen 1989, pp. 261-265.

[6] C. Botan, F. Ostafi and A. Onea, Discrete Optimal Control with Fixed End-point for an Electrical Drive System, *Proceedings of 11th International Power Electronics and Motion Control Conference, EPE PEMC*, Riga, sept. 2004, vol. 4, pp.344-348.

[7] T. Egami, H. Morita and T. Tsuchiya, Efficiency Optimized Model Reference Adaptive Control System for a DC Motor, *IEEE Transactions on Industrial Electronics,* vol. 37, no. 1, Feb. 1990, pp. 28-33.

[8] C. Botan, A. Onea, F. Ostafi, A Solution to the LQ-Problem with Finite Final Time, *WSES Int. Conf. on Automation and Information (AITA 2001),* Skiatos, Greece, *in Advanced in Automation, Multimedia and Video Systems, and Modern Computer Science,* Ed. Kulev V.V., D'Attellis C.E., Mastorakis N.E.,WSES Press, 2001, pp.62-66.

[9] C. Boțan, F. Ostafi, A. Onea, A Solution to the Optimal Tracking Problem for Linear Systems, *WSEAS Transactions on Systems*, Vol. 1, ISSN 1109-2777, 2002, pp. 185-189.

[10] A. Onea and C. Botan, Real Time LQ-Optimal Control of a D.C. Drive System, *Proceedings of the 24th Annual Conference of the IEEE Industrial Electronics Society, IECON'98*, Aachen, 1998, pp.1417-1421.

[11] C. Botan, V. Horga, F. Ostafi, M. Albu, M Ratoi, General Aspects of the Electrical Drive Systems Optimal Control, submitted to *12th European Conference on Power Electronics and Applications* - EPE 2007, September 2007, Aalborg, Denmark.