Optimal Allocation of Heat Exchangers in an Aircraft Open Loop Bootstrap System

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Abstract: - Open loop bootstrap systems are being used in aircraft engines to cool the cabin air by saving power. Air is bled from then engine compressor and is used in an air refrigeration cycle to cool cabin air. The aim of the paper is to optimize from the thermodynamic standpoint the two heat exchangers (the main one and the regenerative one). The optimization criterion is minimum power necessary from the engine. The optimization constraint is a limited heat transfer capacity of the heat exchangers. We have developed a mathematical model of the system and we have analyzed two cases: ideal (isentropic efficiencies equal 1) and actual (isentropic efficiencies less than 1).

Key-Words: - open loop bootstrap systems, thermodynamic optimization, cross-flow heat exchangers, counterflow heat exchangers, optimal allocation

1. Introduction

The Take-Off Gross Weight (TOGW) penalty of an aircraft is a very important parameter that must be minimized in order to improve its payload/range capability. By minimizing TOGW, the performance of the aircraft is improved. A great deal of theoretical research work has been dedicated to finding optimal configurations for the systems of the aircraft in order to minimize the TOGW penalty, among which the environmental control system [1-4], [6]. The functions of the environmental control system of the aircraft are to control the cabin temperature, pressure, and humidity, to cool avionics, to defog the windshield, and to supply conditioned air to various systems and subsystems. When using ram air as a heat sink to reject the generated heat within the aircraft, the corresponding TOGW penalty can be defined as the sum of the ram air circuit equipment weight and the additional fuel. If the heat sink (ambient) is warmer than the load, cooling can be accomplished by means of a refrigeration cycle. The most common one is the so-called "air cycle" that acts as a reverse Brayton cycle, using compressed air as working fluid in an open loop bootstrap configuration. Open loop bootstrap systems use bleed air from the aircraft's engine compressor. This stream is compressed by a compressor and then is cooled by the ram air and passes through the turbine, which drives the compressor. As a consequence, the temperature of this stream after the turbine is lower than the temperature of the load. Cooling is performed in this way and heat is rejected overboard together with the air. The objective of our analysis is to find the optimal allocation of the heat transfer areas of the two heat exchangers that are parts of the aircraft's regenerative open loop bootstrap system, in order to minimize the power extracted from the engine compressor.

2. The Mathematical Model

Figure 1 shows the schematic of the regenerative open loop bootstrap system. There are two air streams (see also Figure 2): the engine air stream ($\dot{m}_e$) – bleed air extracted from the lower stages of the engine compressor C_1, and the ram air stream ($\dot{m}_a$)

1. The engine air circuit: After being compressed in the lower stages of the aircraft engine compressor C_1 from the ambient pressure p_a, respectively temperature T_a to p_1 and T_1, air flows through the compressor C of the open loop bootstrap system (OLBS), where its pressure and temperature are raised to p_2 and T_2 respectively. Next, the engine air stream passes through the cross-flow heat exchanger HE1 where its temperature is decreased to T_3 and further it is additionally cooled to the temperature T_4 in HE2, a cross-flow or a counterflow heat exchanger. In the turbine T, air is expanded to $p_5 = p_c$ and $T_5 < T_c$ and then it flows through the cabin, where it "absorbs" the cooling load $\dot{Q}_c$.

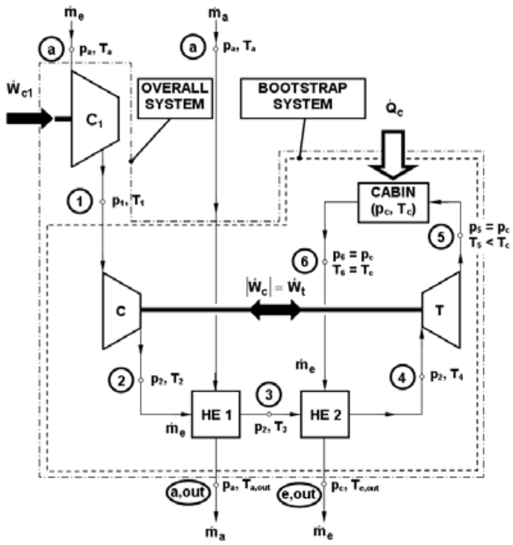

Fig. 1

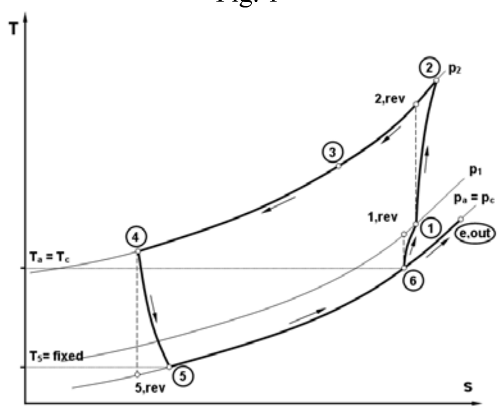

Fig. 2

The power supplied by the turbine T ($\dot{W}_t$) equals the shaft work rate absorbed by the compressor C ($\dot{W}_c$): $\left|\dot{W}_c\right| = \dot{W}_t$. The air parameters after the cabin are $p_6 = p_c$ and $T_6 = T_c$. At the end of the circuit, air flows through the cold side of the regenerative heat exchanger HE2, cooling the stream that flows to the turbine and is finally thrown overboard at p_c and $T_{e,\,out}$.

2. The ram air stream: Ram air enters the cross-flow heat exchanger HE1 at ambient parameters p_a and T_a, cools the engine air and exits at p_a and $T_{a,\,out}$. The simplified model from Figure 1 is based on the assumptions that there are no pressure losses in the two heat exchangers and that the specific heat of air is constant. The objective of the analysis is to find the optimal allocation of the two heat exchangers that requires the minimum power extracted from the engine compressor ($\dot{W}_{c1}$). The constraint is defined by the condition:

$$(UA) = (UA)_1 + (UA)_2 = \text{fixed} \tag{1}$$

where subscripts 1 and 2 correspond to the heat exchangers HE1 and HE2 respectively.

2.1. Equations

For each component of the overall system we write the equations that describe its behavior.

– the cabin:

$$T_5 = T_c - \frac{\dot{Q}_c}{\dot{m}_e c_p} \tag{2}$$

– the overall system:

$$\dot{W}_{c1} = \dot{Q}_c - \dot{m}_e c_p \left(T_{e,out} - T_a\right) - \dot{m}_a c_p \left(T_{a,out} - T_a\right) \tag{3}$$

– the engine compressor:

$$T_1 - T_a = T_a \frac{1}{\eta_{c1}} \left[\left(\frac{p_1}{p_a} \right)^b - 1 \right] \tag{4}$$

$$\dot{W}_{c1} = \dot{m}_e c_p T_a \frac{1}{\eta_{c1}} \left[1 - \left(\frac{p_1}{p_a} \right)^b \right] \tag{5}$$

– the bootstrap system compressor:

$$T_2 - T_1 = T_1 \frac{1}{\eta_c} \left[\left(\frac{p_2}{p_1} \right)^b - 1 \right] \tag{6}$$

$$\dot{W}_c = \dot{m}_e c_p T_1 \frac{1}{\eta_c} \left[1 - \left(\frac{p_2}{p_1} \right)^b \right] \tag{7}$$

– the ram air heat exchanger HE1:

Assume: $\dot{m}_a c_p = C_a = C_{max}$; $\dot{m}_e c_p = C_e = C_{min}$

Consequently:

$$\mu = \frac{C_{max}}{C_{min}} = \frac{\dot{m}_a}{\dot{m}_e} > 1 \tag{8}$$

The number of heat transfer units N_1:

$$N_1 = \frac{(UA)_1}{C_{min}} = \frac{(UA)_1}{\dot{m}_e c_p} \tag{9}$$

The effectiveness of the cross-flow heat exchanger HE1 [5]:

$$\varepsilon_1 = 1 - exp\left\{\mu N_1^{0.22}\left[exp\left(-\mu^{-1} N_1^{0.78}\right) - 1\right]\right\} \tag{10}$$

The following two equations result:

$$T_2 - T_3 = \varepsilon_1 \left(T_2 - T_a\right) \tag{11}$$

$$T_{a,out} - T_a = \frac{\varepsilon_1}{\mu}\left(T_2 - T_a\right) \tag{12}$$

– the regenerative heat exchanger HE2:

$$\mu = \frac{\dot{m}_e}{\dot{m}_e} = 1 \tag{13}$$

$$N_2 = \frac{(UA)_2}{C_{min}} = \frac{(UA)_2}{\dot{m}_e c_p} \tag{14}$$

The effectiveness:

– cross-flow heat exchanger:

$$\varepsilon_2^a = 1 - exp\left\{N_2^{\,0.22}\left[exp\left(-N_2^{\,0.78}\right) - 1\right]\right\} \tag{15}$$

– counterflow heat exchanger:

$$\varepsilon_2^b = \frac{N_2}{N_2 + 1} \tag{16}$$

Consequently:

$$T_3 - T_4 = \varepsilon_2 (T_3 - T_c) \tag{17}$$

$$T_{e,out} - T_c = \varepsilon_2 (T_3 - T_c) \tag{18}$$

– the bootstrap system turbine T:

$$T_4 - T_c = T_4 \eta_t \left[1 - \left(\frac{p_2}{p_c}\right)^{-b}\right] - \frac{\dot{Q}_c}{\dot{m}_e c_p} \tag{19}$$

The shaft work rate of the turbine:

$$\dot{W}_t = \dot{m}_e c_p (T_4 - T_5) = \dot{m}_e c_p T_4 \eta_t \left[1 - \left(\frac{p_2}{p_c}\right)^{-b}\right] \tag{20}$$

– the energy conservation equation:

It follows that:

$$T_4 - T_c + \frac{\dot{Q}_c}{\dot{m}_e c_p} = T_2 - T_1 \tag{21}$$

– the heat exchangers inventory constraint:

The constraint:

$$N = N_1 + N_2 \tag{22}$$

We write Eqs.(1) through (22) in the dimensionless form.

2.2. The Function to be Minimized

In the previous section we have obtained a set equations that describe the model of the considered system. The objective of our analysis is to find the optimal allocation of the quantities *overall heat transfer coefficient × heat transfer area* $(UA)_{i,\ i=1,2}$ for the two heat exchangers, corresponding to the minimum shaft work rate necessary to rise the engine air's pressure in the engine compressor C_1 from p_a to p_1. This means that we shall have to determine the optimal value x_{opt} of the fraction x representing the ratio *(number of heat transfer units allocated to the first (main) heat exchanger) / (total number of heat transfer units)*. Since the function is very complicated, we consider two stages of our analysis: the ideal case, when all the isentropic efficiencies equal 1 and the actual case, when the isentropic efficiencies are less than 1.

In the ideal case the expression of the function to be minimized is:

$$\tau_1 = \frac{(K-1)(f_1 - a) - af_2}{1 - f_2 - K} \tag{23}$$

In the actual case the function to be minimized is:

$$b_1 \tau_1^3 + b_2 \tau_1^2 + b_3 \tau_1 + b_4 = 0 \tag{24}$$

3. Results and Discussion

We present the results that we have obtained by applying the above described model to the two cases. In the ideal case we have used the Newton-Raphson technique whereas in actual one we have used the Cardan formulas. In order to obtain the optimal heat exchangers allocation and the corresponding minimum values of the shaft work rate w_{c1}, we have selected the following fixed values: $\tau_a = \tau_c = 1$, $\pi_a = \pi_c = 1$, $q_c = 0.017$, $\eta_{c1} = 0.9$, $\eta_c = 0.8$, $\eta_t = 0.9$ (actual case). The selected parameters are the number of heat transfer units (N) and the thermal capacities ratio μ. The ranges of these two parameters are: $N = 1 \ldots 20$, $\mu = 1 \ldots 4$

3.1. The Ideal Case

<u>The cross-flow configuration</u>: The first remark to be made is that whatever the number of heat transfer units, x_{opt} has the same value for the same thermal capacities ratio. This fact can be explained by noticing that the effectivenesses vary in opposite directions in the same proportion. The variation of the dimensionless shaft work rate versus the number of heat transfer units for two values of μ (1 and 4) is plotted in Figures 3 and 4. The trend is identical: the optimal value of w_{c1} decreases as N increases, the only difference being that the lower values at the same number of heat transfer units correspond to higher values of μ. The slope of the curve is very steep at the beginning, but for higher values of N, the shaft work rate is practically constant. It is interesting to notice that the higher the thermal capacities ratio, the narrower the range of NTU corresponding to the steep slope of the curve. At $\mu = 4$, this range reduces to half its magnitude for $\mu = 1$. The variation of x_{opt} and of w_{c1} versus the ratio of thermal capacities μ for N = 5 and N = 20 is shown in Figures 5 and 6. As we have already emphasized, the only parameter that exerts an influence upon x_{opt} is μ and that is why its variation versus μ is the same on each of the four plots. The dimensionless shaft work rate w_{c1} exhibits a slight variation, decreasing as μ increases – see the scale of the plots. At low values of N, its variation is smooth along the entire range of μ, but the higher the number of heat transfer units, the more its evolution takes a stepwise shape (see Figure 6). However, one must bear in mind that the shaft work rate actually is almost constant, which means that the ratio of

thermal capacities exerts a very weak influence.

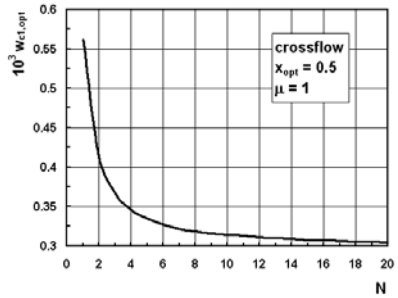

Fig. 3

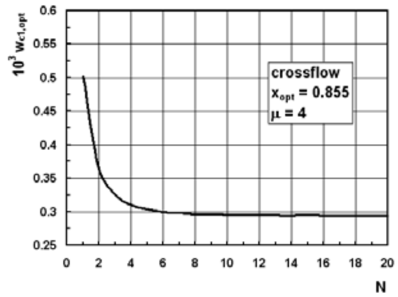

Fig. 4

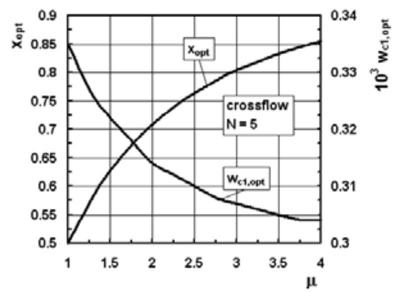

Fig. 5

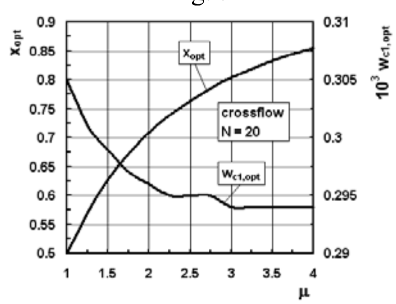

Fig. 6

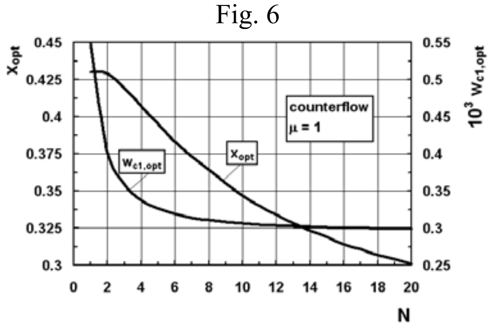

Fig. 7

The counterflow configuration: In this case, x_{opt} is obviously dependent on the thermal capacities ratio. Its variation, along with the evolution of w_{c1} are plotted versus N in Figures 7 and 8 for two values of μ (1 and 4). For μ > 1 the optimal heat exchangers allocation exhibits a maximum which "migrates" from N = 2 at μ = 1.5, to N = 5 at μ = 4, whereas its variation range narrows as μ increases. Likewise in the case of the cross-flow heat exchanger, the optimal (minimum) value of the dimensionless shaft work rate displays a very steep decrease at the beginning and then becomes constant. The steep slope interval varies from N = 11 for μ = 1 to N = 4 for μ = 4. This means that there is a large interval of N for which the power absorbed by the bootstrap system compressor is the lowest. In Figures 9 and 10 we have represented the variation of x_{opt} and of w_{c1} versus the ratio of thermal capacities μ for N = 5 and N = 20. The optimal value of the fraction x increases monotonically with μ, and the range is practically the same for every N. The dimensionless shaft work rate w_{c1} preserves the type of evolution present in the case of the cross-flow regenerative heat exchanger, with a smooth variation at the beginning, followed by a stepwise decrease, all in a very narrow range.

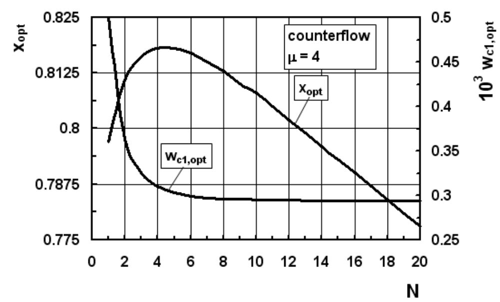

Fig. 8

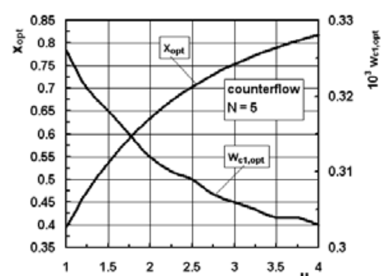

Fig. 9

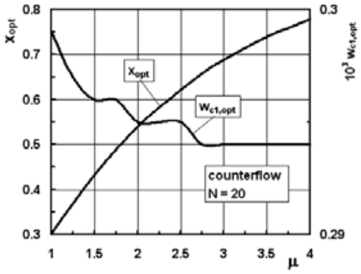

Fig. 10

3.2. The Actual Case

As we have emphasized, the mathematical relations that describe the model in the actual case are of extreme complexity, making it impossible to use the numerical techniques for the determination of the optimum of the functions to be studied. Therefore, the only way to arrive at a result (even if the approximation degree is high), is to compute the value of τ_1 (and implicitly w_{c1}) by assigning values to the variable x in the range 0 … 1 with small increments. The smaller the increments, the higher the precision. We have imposed an increment of 0.025, that we considered to be satisfactory for a sufficiently accurate analysis. Consequently, we have developed a computer software package modeling the Cardan approach for finding the roots of a cubic equation. By analyzing the behavior of the model, we have found that for the entire ranges of the variable and of the parameters (N and μ) the case that resulted was that corresponding to three real distinct roots. Two of them had values that did not meet the requirement $w_{c1} > 1$, only the third one did.

We have studied the variation of w_{c1} for the same ranges of the parameters N and μ as in the ideal case. The function $w_{c1} = F(x)_{\text{N and } \mu \text{ parameters}}$ exhibited the expected behavior only for the low values of N and μ (N = 1, 2, 3 and μ = 1 and 1.5), namely a shape somehow similar to the one that resulted in the ideal case. For higher values of the parameters, the variation became more and more irregular as N and μ increased, showing more maximums and minimums (some of the minimums having the same value for different values of x within the precision of the calculus). The optimal value of x was not following a predictable pattern, increasing and decreasing chaotically as the parameter varied. In this case it was impossible to plot the variation of x_{opt} and $w_{c1,opt}$.

At the end of each parameter range, the "suitable" root started to get values outside the acceptable interval as x was varying, and the model became more and more unstable making it impossible to select the optimal value. We noticed the same behavior by supposing a counterflow regenerative heat exchanger, this being the reason why we considered that data processing was sufficient only for the cross-flow configuration.

In such cases, it is possible to highlight a trend and to make predictions. In the other ones, when a trend is not evident, the only solution is to treat each situation separately and to select the optimal values.

Figure 11 displays the evolution of $w_{c1,opt}$ for μ = 1.5, resulting a very steep decrease in the range N = 1 to N = 4, followed by an almost linear diminution characterized by a slow slope. The difference from the ideal case is that the shaft work rate decreases continuously instead of remaining constant after the abrupt decrease at the beginning.

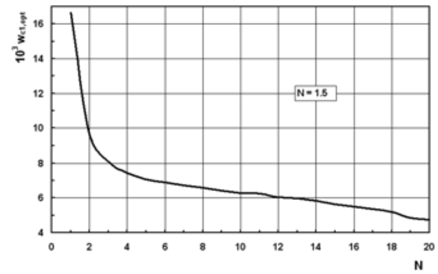

Fig. 11

4. Conclusions

Our work deals with the heat exchangers allocation optimization of an open loop bootstrap system that uses a regenerative heat exchanger as an additional equipment in order to improve the overall efficiency of the refrigeration unit. The optimization has been made by imposing a constraint upon the total number of heat transfer units allocated to the two heat exchangers (the main heat exchanger and the regenerative one). After building the mathematical model of the system's operation, we have obtained the solution of the equations set representing the function to be minimized. This function represents the dimensionless shaft work rate necessary to rise the pressure of the bleed engine air in the first stages of the engine compressor. This function results as the root of a cubic equation, if one suppose that the isentropic efficiencies of the engine compressor and of the bootstrap system compressor and turbine are less than 1 (the actual case). Due to the high complexity of the calculations necessary to apply a numerical approach in order to obtain the optimum of the function, we have adopted a simpler method: the calculus of the function for different values of the fraction x (the ratio between the NTU's of the main heat exchanger and the total number of heat transfer units allocated to the two heat exchangers). After tabulating these values, we have searched for the lowest one, which represents the minimum of the function. We have also considered the simple situation corresponding to the ideal processes characterized by isentropic efficiencies equal to 1. This hypothesis leads to a simple function, suitable for a numerical treatment by means of the Newton-Raphson method for determining the roots of transcendental equations. In this case, we have also considered two configurations for the regenerative heat exchanger, cross-flow and counterflow. The results in the actual case show that a coherent trend can be highlighted only for low values of the overall number of heat transfer units and of the thermal capacities ratio. At high values, the behavior of the function becomes erratic, making it impossible to evidence a trend, and even there are no roots of the cubic equation that meet the requirement $w_{c1} > 1$.

However, a fact is certain: the higher the overall number of heat transfer units or the thermal capacities ratio, the lower the shaft work rate w_{c1}. The ideal case shows that the optimal fraction x_{opt} does not depend on the overall NTU for the cross-flow scheme of HE2, the only parameter that exerts an influence being μ. If HE2 is a counterflow heat exchanger, x_{opt} depends on both N and μ and exhibits a maximum that migrates toward the higher values of N as the thermal capacities ratio increases. For both flow schemes x_{opt} increases as μ increases while N is kept constant. For μ = fixed, the shaft work rate shows in both cases a very steep decrease at low N's, followed by a quasi-constant value as N increases. If N is fixed, w_{c1} decreases too, but at an almost constant rate, displaying at the end a stepwise variation. It is important to notice in this case that this variation is however within a very narrow range: w_{c1} is practically constant as μ varies. As a final conclusion, the most favorable situations correspond to high values of the number of heat transfer units and of the thermal capacities ratio.

5. Nomenclature

A - heat transfer surface area
a - constant
b - ratio R/c_p
b_i - coefficients of the cubic equation
C - thermal capacity
c_p - specific heat at constant pressure
h - specific enthalpy
k - adiabatic exponent of air
$\dot{m}$ - mass flow rate
N - number of heat transfer units
p - pressure
$\dot{Q}_c$ - cooling load
q_p - dimensionless cooling load
R - air constant
T - temperature
U - overall heat transfer coefficient
$\dot{W}$ - shaft work rate
w - dimensionless shaft work rate
x - allocation of the heat exchanger HE1

Greek letter symbols

ε - heat exchanger effectiveness
η_{c1} - isentropic efficiency of the engine compressor
η_c - isentropic efficiency of the bootstrap system compressor
η_t - isentropic efficiency of the bootstrap system turbine
μ - thermal capacities ratio C_{max}/C_{min}
π - dimensionless pressure
τ - dimensionless temperature

Subscripts

a - ambient air
c - cooling load
e - engine air

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