

Forecasting municipal solid waste generation based on grey fuzzy dynamic modeling

ZHU XIANG, LI DAOLIANG

College of Information and Electrical Engineering, China Agricultural University
and Key Laboratory of Modern Agriculture System Integration
No.17 Tsinghua East Road, Haidian District, Beijing, 100083, P.R.CHINA

Abstract: - There has been a significant increase in municipal solid waste generation in China during the last few decades. Both planning and design of municipal solid waste management systems require accurate prediction of solid waste generation. The lack of complete historical records of solid waste quantity and quality due to insufficient budget and unavailable management capacity has resulted in a situation that makes the long-term system planning and/or short-term expansion programs intangible. To effectively handle these problems based on limited data samples, a new analytical approach capable of addressing socioeconomic and environmental situations must be developed and applied for fulfilling the prediction analysis of solid waste generation with reasonable accuracy. This study presents a new approach—grey fuzzy dynamic modeling—for the prediction of solid waste generation in a fast-growing urban area based on a set of limited samples. The practical implementation has been accessed by a case study in the city of Beijing in China. It shows that such a new forecasting technique may achieve better prediction accuracy than those of the conventional grey dynamic model, least-squares regression method, and the fuzzy goal regression technique.

Key-Words: - Forecasting; solid waste generation; grey fuzzy dynamic modeling

1 Introduction

The twentieth century saw a dramatic increase in the production of urban solid waste, reflecting unprecedented global levels of economic activity [1]. The rapid urban growth has exerted heavy pressures on land and resources contained within the area surrounding cities, and resulted in serious environmental and social problems.

The prediction of municipal solid waste generation plays an important role in a solid waste management. Most of the traditional statistical forecasting models, such as the geometry average method, saturation curve method, least-squares regression method, and the curve extension method, are designed based on the configuration of semi-empirical mathematical models [2]. The structure of these models is simply an expression of cause-effect or an illustration of trend extension in order to verify the inherent

systematic features that are recognized as related to the observed database. In light of the evolution of structured or semi-structured forecasting techniques that have been developed in the scientific community, the fuzzy forecasting and grey dynamic modeling are viewed as two promising approaches for handling forecasting issues under uncertain environments [3].

This study presents a new approach—grey fuzzy dynamic modeling—for the prediction of municipal solid waste generation in an urban area based on a set of limited samples. This analysis replaces the least-squares regression method used in the GM model by the fuzzy goal regression method in order to present a better mathematical function and, as a result, to improve the overall prediction accuracy in a grey environment. A case study of solid waste generation in the city of Beijing in China demonstrates the significance and applicability of such an analytical approach.

2 Methodology

2.1 The basic concepts of grey system and grey dynamic model

Grey systems theory, describes random variables as a changeable interval number that varies with time factors and uses ‘color’ to represent the degree of uncertainty in a dynamic system [4]. It is believed that the uncertainties existing in the whitening process mainly come from the insufficiency of understandable information. The grey dynamic model (GM model) always describes the real world physical system as an energy system in a continuous domain. Hence, all the systematic responses in the real world physical system should be consistent with some sort of exponential pattern. With the solution from the proposed pseudo-differential equation, the next output from a grey environment can be predicted based on a few observed samples or data [5]. Such an approach does not need to pay more effort to directly formulating the underlying complicated physiochemical property or socioeconomic behavior in an unknown system [6].

Since the GM model can characterize such an unknown system and be able to forecast effectively based on a few data, it has showed its practicality in utilizing insufficient database [7]. Therefore, the GM model is proven as an effective method especially for many environmental or socioeconomic issues in China where the situation of lacking long-term monitoring database is vast. Such an exponential function may be thought of as an illustration of being consistent with the energy dissipation or accumulation phenomenon inherent in a specific grey system.

The mathematical expression for various levels of GM model (i.e. the grey differential equation) is GM(n, h), as shown as Eq. (1), in which n corresponds to the dimension and h corresponds to the number of variables

$$\sum_{i=0}^n a_i \frac{d^{n-i} X_1^{(1)}}{dt^{n-i}} = \sum_{i=1}^{h-1} b_i X_{i+1}^{(1)} \quad (1)$$

where the variables $X_i^{(k)}(k)$ are defined

for those discrete types of observations after some sort of grey pre-treatment. The method of grey pre-treatment will be described later. The boundary conditions of Eq. (1) include:

$$X_1^{(0)}(0) = X_1^{(1)}(1) \quad (2)$$

$$X_i^{(1)}(k) = \sum_{j=1}^k X_i^{(0)}(j) \quad (3)$$

The most commonly used GM model is the GM(1, 1) model which represents the simplest form of the grey differential equation, as defined by Eq. (4).

$$\frac{dX^{(1)}}{dt} + a\xi^{(1)} = b \quad (4)$$

2.2 The basic concept of fuzzy sets theory and fuzzy goal regression

The focus of fuzzy sets theory is thus placed upon its non-statistical characteristics in nature. While the random variable is used for the description of uncertain statistical implication, the fuzzy membership function, which refers to the similarity of an element that belongs to a subjectively described set, is defined for illustrating the imprecision existing in real world system. The more an element or object can be said to belong to a fuzzy set A , the closer to 1 is its grade of membership.

Fuzzy sets theory is frequently applied in recent research for various types of systems analysis, covering forecasting, optimization, reasoning, and control issues [8–10].

A decision space in a fuzzy environment is thus defined as the intersection of those membership functions corresponding to those fuzzy objective(s) and constraint(s).

Therefore, if $\{\mu G_1, \mu G_2, \dots, \mu G_{2m}\}$ and $\{\mu C_1, \mu C_2, \dots, \mu C_p\}$ are denoted as the fuzzy membership functions for those fuzzy objectives $\{G_1, G_2, \dots, G_m\}$ and fuzzy constraints $\{C_1, C_2, \dots, C_p\}$ respectively in a decision space X , all the fuzzy membership functions of G_m and C_p can be combined together to form a new decision space D , which stands for a resultant fuzzy set generated from the intersection of all related G_m and C_n as shown below.

$$D = G_1 \cap G_2 \cap \dots \cap G_m \cap C_1 \cap C_2 \cap \dots \cap C_p \tag{5}$$

Since the decision D is defined as a fuzzy set, the optimal decision is any alternative $s \in S$ that can maximize the minimum attainable aspiration levels in decision making. The aspiration level of each fuzzy membership function is actually a common membership value achieved in the decision set, $\mu_D(s)$. Thus, the max min convolution requires maximizing the minimum membership values of those elements, as below [11]:

$$\max_s \mu_D = \max_s \min \{ \mu_{G_1}, \mu_{G_2}, \dots, \mu_{C_1}, \mu_{C_2}, \dots, \mu_{C_p} \} \tag{6}$$

Such an operation is actually an analogy of the non-fuzzy environment as the selection of activities simultaneously satisfies all the objective(s) and constraint(s). Hence, a goal programming problem with multiple fuzzy goals can be simplified as

$$CX_i \leq \tilde{f}_1 \tag{7}$$

$$CX_i \geq \tilde{f}_2 \tag{8}$$

$$A_k X \leq B_k \tag{9}$$

$$A_l X \geq B_l \tag{10}$$

$$X \geq 0 \tag{11}$$

where ' $\tilde{\geq}$ ' and ' $\tilde{\leq}$ ' denote the notion of fuzzified version of ' \geq ' and ' \leq ', which have the linguistic interpretation 'approximately greater than or equal to' and 'approximately less than or equal to', respectively. Eqs. (7) and (8) represent the fuzzy goal constraint. Eqs. (9) and (10) express the functional or definitional constraints with ' \geq ' and ' \leq ' relationships in the goal programming model.

In general, the non-increasing and non-decreasing linear membership functions are frequently used for the inequality constraints with 'approximately less than or equal to' and 'approximately greater than or equal to' relationships, respectively. It can be assumed that the membership values are linearly decreasing in Eq. (7) over the 'tolerance interval' δ_i and linearly increasing in Eq. (8) over the 'tolerance

interval' δ_j . Hence the most sensitive part of aspiration levels for decision makers is the range of tolerance intervals, δ_i and δ_j corresponding to these fuzzy goals. If a fuzzy goal programming model is designed to maximize several fuzzy goals with the fuzzy implication of 'the higher, the better', subject to a set of deterministic constraints, the problem can be further solved by introducing an intermediate control variable α [12].

By introducing the intermediate control variable, α , the fuzzy goal programming model, defined in Eqs. (9)–(10), becomes:

$$\text{Max } \alpha \tag{11}$$

Subject to

$$\mu(CX_i) \geq \alpha \tag{12}$$

$$A_k X \leq B_k \quad \forall k \tag{13}$$

$$A_l X \geq B_l \quad \forall l \tag{14}$$

$$X \geq 0 \tag{15}$$

Hence, the fuzzy structure represented in Eqs. (11) and (12) may be applied in the following fuzzy goal regression analysis. In the fuzzy goal regression analysis, the proposed linear regression model ($\sum a_i x_i$) can be viewed as a linear function that is designed to approach a target value of dependent variable Y . The regression effort for each set of observations, corresponding to dependent variables and independent variables, is to achieve the goal where the target value is approximated by a set of fuzzy parameters, a_i , multiplied by the observed input data, x_i . For n sets of observations, we have n fuzzy goals to be handled in the fuzzy programming model. The problem can therefore be stated as a maximization process of the minimum degree of fitting for these goal constraints. However, the achievement of each goal constraint, expressed by a triangular membership function, can be regarded as a superimposition of two membership functions, as shown in Fig. 1. The highest degree of fitting may be attained when the observed value of dependent variable Y is equal to the predicted value of $\sum a_i x_i$. Thus, each goal

constraint has to be separately expressed by two inequality constraints in the following method

$$Max \sum_{i=1}^n (h_{i1} + h_{i2}) \tag{16}$$

subject to:

$$\sum_{i=1}^n a_i x_i \leq Y_i + (1 - h_{i1})(k_2)(y_i)$$

$$\forall i = 1, \dots, n \tag{17}$$

$$\sum_{i=1}^n a_i x_i \geq Y_i - (1 - h_{i1})(k_1)(y_i)$$

$$\forall i = 1, \dots, n \tag{18}$$

$$0 \leq h_{i1}, h_{i2} \leq 1 \tag{19}$$

$$X \geq 0 \tag{20}$$

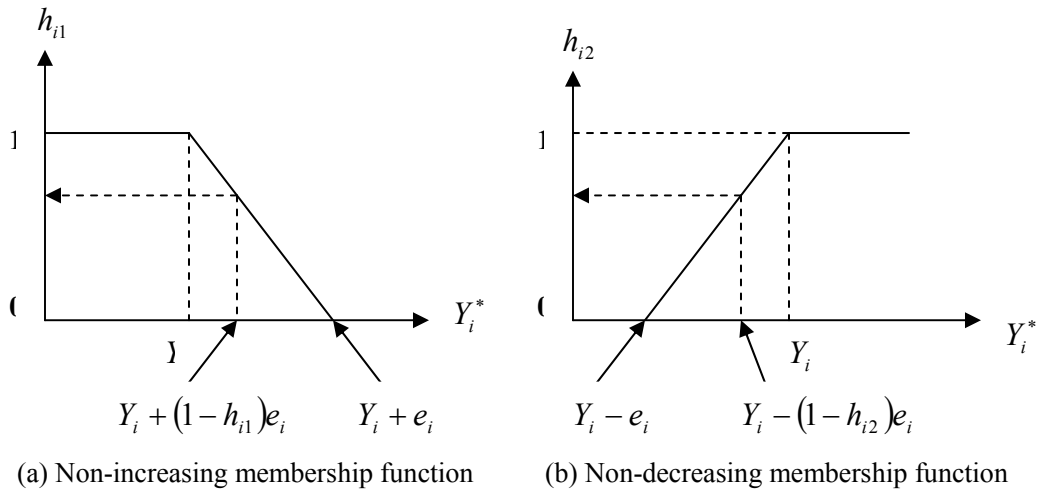


Fig.1. The membership functions used in fuzzy goal regression model

If considering the relative importance between various objectives and the effect resulting from the slopes of membership functions, Eq. (16) can be revised in a form of Eq. (21). After carrying out a series of analytical procedures in the above, multiple linear regression analysis can be integrated into a linear programming practice. It is believed that such a new forecasting method with the aid of the principles of fuzzy goal programming and fuzzy linear regression techniques may avoid the errors from the incomplete database and present better prediction accuracy.

$$Max \sum_{i=1}^n \left(\frac{w_i}{k_1} h_{i1} + \frac{w_i}{k_2} h_{i2} \right) \tag{21}$$

2.3 Development of the grey fuzzy dynamic model

Due to the difficulty for previous method to obtain suitable *a* and *b* values with reasonable prediction accuracy in the circumstance of insufficient samples, the chance to use the

fuzzy goal regression model for finding the appropriate *a* and *b* values is interesting. The grey dynamic model (GFM) can then be developed by the integration of these two forecasting techniques. The parameter identification technique is therefore replaced by the fuzzy goal regression technique such that better prediction of *a* and *b* values are anticipated as the construction and operation of a GM model remains.

3 Case study

3.1 System environment of Beijing City

Beijing City, located in the northern part of China, is divided into eight administrative districts. The task of waste collection and shipping are handled by government in which eight independent collection teams are organized for cleaning up the waste streams in those administrative districts respectively. Previously studies focus on the determination

of optimal solid waste management strategy that illustrates possible interactions between recycling and incineration options [13] and its uncertainty analysis in decision-making [14]. This analysis serves as a companion study of Chang and Deng [13, 14] and tries to provide a systematic estimation if the recycling effect is not regarded as an influential factor of solid waste generation.

The major difficulty existing for the prediction analysis actually rests upon the inefficiency when encountering the incomplete database of solid waste generation. In developing countries, the minimum size of database is three for performing a basic GM or GFM modeling analysis [15]. Two out of three are prepared for calibration while the remaining one is used for verification of the proposed model. Using the GM(1,1) model to predict the future trend of solid waste generation with such a 10-year record seems reasonable. Extra measures for system planning of solid waste management are not required. This paper, however, first presents the application of the GFM(1,1) model for an advanced analysis. A strong improvement may be anticipated by performing a sensitivity analysis for the GFM method.

However, due to poor participation from the city residents, not enough income was generated to justify the continued operation of the recycling program. Ending the program, however, would shorten the lifespan of the landfills; therefore the City is considering building a material recovery facility (MRF) to continue the recycling program without requiring the support of the city residents.

3.2 Results and discussions

The implementation of the GFM model has to be accessed via three steps. The first step is to divide the ODS into two sub-sequences. One is used to calibrate the forecasting models and the other is reserved for the verification of the prediction accuracy. The second step is to establish the calibration and verification task and the final step is to fulfill the prediction analysis for practical application. The GM model serves as a base model for the purpose of comparison only. Both grey relational analysis and residual test indicate that the

GFM(1,1) method may exhibit a better prediction accuracy than the GM(1,1) method. Although the GM method can find out the general tendency of an unknown system, it still cannot fully avoid the imprecision resulting from the insufficient database.

The prediction analysis of solid waste generation characterized by the GFM method suggests that the planned incinerator with 900 tonnes/day capacity cannot handle the growing demand of solid waste generation in the near future. Once the proposed recycling program is unsuccessful, additional treatment and disposal alternatives should be taken into account in time.

4 Conclusion

Experience indicates the estimation of solid waste generation is crucial for the subsequent system planning of solid waste management in the urban region from both short- and long-term perspectives. However, a complete record of solid waste generation and composition is not always present. This analysis develops an effective tool for tackling those forecasting problems that are lacking a significant amount of data for determining regression models and that have vague relationships between the dependent variables and those socio-economic factors. The central idea is to utilize the fuzzy goal regression technique to improve the conventional grey dynamic model so as to minimize the discrepancy between the predicted values and the observed values. In developing countries, the minimum size of database is three in practicing a basic GM or GFM modeling analysis. Two are prepared for calibration, while the remaining one is used for verification. A case study of solid waste generation in the city of Beijing demonstrates the application potential of such an approach.

In fact, not only the selection of dimension and decision variable but also the choice of data sequence for the calibration and verification in the GFM modeling analysis may influence the prediction accuracy. This paper selected different dimensions in the GFM model to proceed a sensitivity analysis that may generate a set of predicted curves as

an allowable range in the prediction of solid waste generation. The sensitivity analysis also presents a comparative study between the conventional GM method and the newly derived GFM method. The general finding is that the number of dimension in the grey differential equation is an influential factor especially in models with higher dimension.

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