Clustering of students of engineering mathematics based on their attitudes, orientations, motivations and intentions

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Abstract: - Insufficient skills in basic mathematics cause problems for those majoring in engineering at university level. The aim of this paper is to recognize the factors affecting on learning mathematics. Learning entails the learner's own initiative in the achievement of learning objectives. Factors with bearing on what students do include attitudes: orientations, intentions and motivations. The present study ascertains how new undergraduates of the Tampere University of Technology (TUT) differ in their attitudes to the study of mathematics. The data for the study comprise the responses to a questionnaire eliciting attitudes to the study of mathematics. On the basis of their attitudes students are classified into different learner groups. The learner groups that were found were Surface Oriented Learners, Peer Learners, Students Needing Support, Independent Learners and Skilful Students.

Key-Words: - attitudes, orientations, motivation, intention, groups, clustering, engineering, mathematics

1 Introduction and background

There has recently been public discussion about the learning of mathematics in Finnish schools and about the level of proficiency. In the Programme for International Student Assessment test (PISA) of 2003 the achievements of Finnish schoolchildren were good, yet at the same time over 200 teachers of mathematics at university level publicly expressed their concern regarding the decline in the level of proficiency in school mathematics – notably in basic algebraic routines. (see the special edition of the publication [in finnish] Matematiikkalehti Solmu at http://solmu.math.helsinki.fi.). A command of basic algebraic skills (calculating functions, cancelling, taking square roots. elementary functions. differentiations and integrations) is indispensable for basic studies in mathematics at university level.

In the PISA test of 2003 Finnish schoolchildren were ranked well, but at the same time more than 200 teachers of mathematics at university level publicly expressed concern at the decline in the level of school mathematics – notably the basic routines of algebra. Poor skills in basic mathematics causes problems for those majoring in engineering in which mathematics plays an important role. Given that traditionally mathematics has been a major consideration in the entrance requirements to universities of technology, increased student intake has resulted in a wider variation in mathematics skills.

In the teaching of mathematics for engineering due note should be taken of the needs of engineering Mathematics is a logical-deductive education. science, in which established axiomatic logical deduction leads to new findings whose practical significance is frequently secondary. In the application of mathematics the problems must be dressed in the guise of the mathematical model: in order to solve the equations of the model suitable methods must be found, the goodness of the model and the precision of the solution need to be assessed. Solutions must generally be arrived at by computer, thus knowledge of mathematical software and programming skills are essential.

Learning outcomes in mathematics are not dependent solely on good teaching, sufficient resources and other external considerations with bearing on learning. Learning implies activity on the part of the learner in order to achieve learning objectives. Factors with bearing on what the student does include attitudes: orientations, intentions and motivations. Orientation describes the student's conscious and unconscious study habits, intention the student's own conscious objective-setting and motivation the power to achieve the objectives set. The point of departure for the present study was the further development of teaching and learning in mathematics for engineering at the Tampere University of Technology (TUT). In autumn 2004 students beginning their studies completed a questionnaire. It contained questions intended to measure the attitudes of students to their studies.

Aim of the study was to ascertain how the students beginning their studies differ from each other with regard to their attitudes to the study of mathematics. Thus the study explores whether it is possible to classify students meaningfully on the basis of their attitudes into groups of different types of learners. What kinds of didactic adjustment should be made in order to cater for this diversity and develop measures supporting learning taking the different groups into consideration?

2 Concepts pertaining to learning

According to the constructivist view of learning, knowledge builds up in the learner's mind either by merging with existing knowledge structures or by adapting existing knowledge structures to fit new knowledge (Rauste-von Wright 1994). What was learned earlier is needed in order to assimilate what is still to come. The student's personal study habits, motivations, orientation and other individual characteristics have bearing on achievement in studies.

2.1 Orientations, intentions and motivations

2.1.1 Intention and motivation

Intention refers to the purpose, aim or choice generated by an actor on initiating some action. (Yrjönsuuri, 2002). The intention may, for instance, be to obtain a good mark, getting through a course or perhaps surviving on a minimum amount of effort. Intention describes the goal towards which the individual is moving. In studying mathematics the intention may, for example, be the application of the mathematics required in engineering sciences.

"Motivation refers to the power driving, directing and sustaining the actions of an individual (Tynjälä 1999.) Internal motivation is motivation which is not dependent on external rewards. External motivation refers to motivation which is influenced by external stimuli and rewards. Motivation may vary very considerably within a short space of time.

2.1.2 Orientations

Orientation refers to personal objectives, intentions, motives and expectations which govern how students study and learn. (Tynjälä, 1999.)

Orientations steer actions in study and learning. They constitute a holistic description of individual differences in the orientation towards learning and studying. (Tynjälä, 1999). Orientations can be divided, for example, into those of personal meaning, reproducing and achieving (Entwistle, 1986 adapting Biggs) and also non-academic orientation (Ramsden 1984). Personal meaning orientation is characterised by a search for the connections between matters, a critical and evaluative orientation to things and internal motivation (Ramsden 1984, Tynjälä, 1999). Personal meaning orientation frequently is considered a desirable orientation in studies. Reproducing orientation manifests itself in learning by rote and in a fear of failure. A student with a reproducing orientation does not seek correspondences between things (Ramsden 1984.) An achieving orientation for its part is characterised by a strategic approach, negative attitudes and achieving motivation. Achieving motivation drives the student to work for a course evaluation, credits or other comparable external accomplishments. (Ramsden, 1984; Tynjälä, 1999.) A non-academic orientation manifests in a negative attitude towards studies, an absence of systematicity in studies and a failure to be effective in studies (Ramsden, 1984).

Situational orientation is used to refer to an approach to studies which varies according to the situation. This may affect both intellectual and social activities. Actors may make use of several different situational orientations depending on the situation, varying these for the situation. Permanency in situational orientation is evidenced by the fact that situational orientation is primarily used in recurring situations (Yrjönsuuri, 2004)

2.2 On the orientation of university students

2.2.1 Situational orientations of undergraduates in technology

In autumn 2003 the situational orientation of undergraduates in technology of the Tampere University of Technology (TUT) was scrutinised. The study by Raija Yrjönsuuri (2004) was concerned with the situational orientations of students TUT information technology students of engineering mathematics. The study by Yrjönsuuri (2004) was based on four situational orientations in the learning of mathematics.

The situational orientations that were found were task orientation, dependency orientation, self orientation and defeatist orientation. Task orientation has characteristics features of

> -taking responsibility and initiative in the task -intention of learning mathematics

-tolerance of uncertainty

-evaluation of own learning

-contemplation of structures

-connections of content to be learned

-a precondition for deep processing of knowledge.

Dependency orientation has characteristics features of

- non-independent in task completion

- oriented towards remembering and learning by rote

- repetition of what has been learnt,

- adherence to external instructions and

- pursuit of social acceptance.

Self orientation has characteristics features of

- fear of failure,

-explanation of failure through external considerations,

- seeking means of survival,

- denying the value of study.

Defeatist orientation has characteristics features of - consequence of failure situations,

- actor perceives task to lack significance,

- lack of dedication to studies and

- denial of value of studies.

(Yrjönsuuri, 2004 and 2002).

2.3 Research on learning approaches and learning patterns

2.3.1 Surface and deep learning approaches

Marton and Säljö (1984) researched how university students learn from text. It was observed in this study that there were two separate ways of processing information, referred to as surface and deep learning approaches. It was seen that those students who did not form a conception of the main notion in the text were unsuccessful because they did not even look for one. Such surface approach students paid attention to individual, unrelated matters which they deemed important and endeavoured to memorise them as such (Entwistle, 1986). Their study motivation appeared to be external and to emanate from the expectations and demands of others.

Deep approach students for their part endeavoured to understand the new matter and to connect it to their existing knowledge. They called the author's claims into question and monitored critically what conclusions the author drew on the basis of the research data. (Entwistle, 1986.) These students were characterised by adhering to the core issue and they perceived themselves to be some kind of generators of knowledge. For the deep approach students learning was meaningful and they were motivated internally, thus they were interested in the matters to be studied for their own sake. (Marton & Säljö, 1984.)

2.4 Learning patterns

Success in studies can be scrutinised through orientations and other educational concepts. Vermunt (1996) researched the study achievements of university students and Open University students and came to the conclusion that the differences in achievement between individuals was due to four components: cognitive processing strategy. metacognitive regulation strategies, conceptions of learning and study orientations. He calls this combination the learning pattern. (Vermunt, 2005.) the main characteristics of learning patterns are presented in Table 1 which is presented on page 5.

Vermunt's (1996, 2005) idea is that the approach to studies is characteristic of each student at a certain time. He does not perceive this to be an immutable personal trait, but rather that the approach takes shape as a result of the interaction of the individual's personal characteristics and situation-related influences.

2.5 Interactivity in learning

There is in learning a strong element of interactivity between teacher and learner, and also of interaction between students in the group. It has been possible to divide interactive group learning into three different forms: peer tutoring, cooperative learning, and peer collaboration (Damon & Phelps, 1989).

In peer tutoring one or more of the learners in the group direct the other members. Within the group it is possible to separate those students with weaker skills and knowledge, novices and those who are teaching them, the dominant students with a better command of the matter, i.e. the experts. Cooperative learning as a term covers various approaches to learning supporting group work. In peer collaboration students having the same level of skills and knowledge work together in order to resolve some challenging task which they could not cope with using only their individual knowledge. (Damon & Phelps, 1989)

3 Implementation of the research

This study aimed to explore students' attitudes to the study of mathematics through orientation, intention and motivation, and also through learning patterns. The study was accomplished in autumn 2004 at the beginning of studies as an overall survey of all those students beginning their studies in engineering mathematics at that time.

A questionnaire eliciting students' attitudes (called attitude questionnaire) to mathematics which constitutes the data used in this research was implemented together with the test of basic skills (See more in Pohjolainen et. al., 2006 [in finnish]). A total of 860 students took the test and the same students responded to the attitude questionnaire. The attitude questionnaire contains 55 statements and was carried out computer aided. Except for the first two statements, students had five possible response options, the extremities of which were totally disagree and totally agree The attitude questionnaire is presented as a whole in the study by Pohjolainen et al. (2006).

3.1 Research questions and methods

The following research questions were formulated for the present study:

1. How do students differ in their attitudes (orientation, motivation, intention)?

2. How should students with different attitudes and modes/means of studying be catered for in the teaching of mathematics?

Research methods included cross tabulation, comparison of averages, principal component analysis and cluster analysis. The data obtained from the attitude questionnaire were condensed using principal component analysis (Johnson & Wichern, 1998), after which observations were grouped using K-means clustering (Johnson & Wichern, 1998). The values for different sum variables corresponding principal components and for some single variables were interpreted. According to interpretations the clusters were meaningfully named.

3.2 Construction of questionnaire measures

The basis of the attitude questionnaire form was the orientation theory presented in the theory section (Ramsden 1984, Entwistle, 1986, Yrjönsuuri, 2002. The attitude questionnaire form was based on these orientation theories (personal meaning, reproducing, achieving, non-academic, task, dependent, self and defeatist orientation). In addition, questions designed to measure specifically intentions and motivations were added to the questionnaire.

More information on the construction of questionnaire measures can be found in research report of Pohjolainen et. al. (2006).

4 Analysis of the attitude questionnaire

4.1 Principal component analysis and sum variables formed on the basis thereof

First a principal component analysis of the attitude questionnaire data was performed using orthogonal Varimax rotation. The rotated component matrix resulting from the principal component analysis is presented as an appendix to Pohjolainen et al. (2006) The first 15 principal components obtained as a result of the principal component analysis explain 55.8% of the variation in the original variables.

The first 11 principal components of the strongly loaded variables formed sum variables whose reliability was tested with reliability analysis. The value of the coefficient of reliability should be greater than 0.6, but due to their meaningfulness sum variables 5, 7 and 11 were included in the further analysis.

	T 1	T 1		
5	Learning pattern	Learning pattern		
Facto	Undirected	Reproduction directed		
	* virtually no	* processing by stages		
gies	processing	(operational)		
rate	* student has problems	* takes time to find		
ıg st	in almost all learning	important matters, but		
ognitive processing strategies	functions	difficulty in selecting		
Droc	* problems in isolating	them		
ive p	essential matters	* notes what must be		
gnit	* reads material again	learned by rote		
S	and again			
	* virtually no	* externally regulated		
JC	regulation of learning	* student observes signs		
ion	* student find	from teacher, e.g. test		
ulat	teacher's instructions	hints		
reg ing	unclear	*student needs a lot of		
itive	* student's way of	time for studies		
cogn	studying does not			
Metacognitive regulation o	change as studies			
Z	progress			
	*co-operation and	* reception of		
	•	-		
	stimuli	knowledge and		
	stimuli * wants more external	knowledge and learning by rote		
	* wants more external	learning by rote		
		learning by rote * main thing in studies is		
	* wants more external regulation	<pre>learning by rote * main thing in studies is to scrape through</pre>		
ng	* wants more external regulation* teacher needs to	learning by rote * main thing in studies is		
arning	 * wants more external regulation * teacher needs to explain carefully, 	<pre>learning by rote * main thing in studies is to scrape through * thinks studying is to</pre>		
of learning	 * wants more external regulation * teacher needs to explain carefully, provide summary, 	learning by rote * main thing in studies is to scrape through * thinks studying is to digest information		
ode of learning	 * wants more external regulation * teacher needs to explain carefully, provide summary, draw together what 	learning by rote * main thing in studies is to scrape through * thinks studying is to digest information through repetitive		
al mode of learning	 * wants more external regulation * teacher needs to explain carefully, provide summary, draw together what needs to be learnt and 	learning by rote * main thing in studies is to scrape through * thinks studying is to digest information through repetitive practice		
fental mode of learning	 * wants more external regulation * teacher needs to explain carefully, provide summary, draw together what needs to be learnt and what not, checks what 	learning by rote * main thing in studies is to scrape through * thinks studying is to digest information through repetitive practice * teacher must explain,		
Mental mode of learning	 * wants more external regulation * teacher needs to explain carefully, provide summary, draw together what needs to be learnt and what not, checks what students can do 	learning by rote * main thing in studies is to scrape through * thinks studying is to digest information through repetitive practice * teacher must explain, show connection		
Mental mode of learning	 * wants more external regulation * teacher needs to explain carefully, provide summary, draw together what needs to be learnt and what not, checks what students can do * students find co- 	learning by rote * main thing in studies is to scrape through * thinks studying is to digest information through repetitive practice * teacher must explain, show connection between things and		
Mental mode of learning	* wants more external regulation * teacher needs to explain carefully, provide summary, draw together what needs to be learnt and what not, checks what students can do * students find co- operation with peers	learning by rote * main thing in studies is to scrape through * thinks studying is to digest information through repetitive practice * teacher must explain, show connection between things and overall picture, ensure		
Mental mode of learning	* wants more external regulation * teacher needs to explain carefully, provide summary, draw together what needs to be learnt and what not, checks what students can do * students find co- operation with peers important	learning by rote * main thing in studies is to scrape through * thinks studying is to digest information through repetitive practice * teacher must explain, show connection between things and overall picture, ensure that student has		
Mental mode of learning	* wants more external regulation * teacher needs to explain carefully, provide summary, draw together what needs to be learnt and what not, checks what students can do * students find co- operation with peers important * student's task is to	learning by rote * main thing in studies is to scrape through * thinks studying is to digest information through repetitive practice * teacher must explain, show connection between things and overall picture, ensure that student has understood and say what		
Mental mode of learning	* wants more external regulation * teacher needs to explain carefully, provide summary, draw together what needs to be learnt and what not, checks what students can do * students find co- operation with peers important * student's task is to study regularly and get	learning by rote * main thing in studies is to scrape through * thinks studying is to digest information through repetitive practice * teacher must explain, show connection between things and overall picture, ensure that student has understood and say what is expected in test		
Mental mode of learning	* wants more external regulation * teacher needs to explain carefully, provide summary, draw together what needs to be learnt and what not, checks what students can do * students find co- operation with peers important * student's task is to study regularly and get	learning by rote * main thing in studies is to scrape through * thinks studying is to digest information through repetitive practice * teacher must explain, show connection between things and overall picture, ensure that student has understood and say what is expected in test * no need for student to		
Mental mode of learning	* wants more external regulation * teacher needs to explain carefully, provide summary, draw together what needs to be learnt and what not, checks what students can do * students find co- operation with peers important * student's task is to study regularly and get things into his head	<pre>learning by rote * main thing in studies is to scrape through * thinks studying is to digest information through repetitive practice * teacher must explain, show connection between things and overall picture, ensure that student has understood and say what is expected in test * no need for student to think critically</pre>		
ming Mental mode of learning	 * wants more external regulation * teacher needs to explain carefully, provide summary, draw together what needs to be learnt and what not, checks what students can do * students find co- operation with peers important * student's task is to study regularly and get things into his head * uncertain attitude 	<pre>learning by rote * main thing in studies is to scrape through * thinks studying is to digest information through repetitive practice * teacher must explain, show connection between things and overall picture, ensure that student has understood and say what is expected in test * no need for student to think critically </pre>		
Learning Mental mode of learning	 * wants more external regulation * teacher needs to explain carefully, provide summary, draw together what needs to be learnt and what not, checks what students can do * students find co- operation with peers important * student's task is to study regularly and get things into his head * uncertain attitude to studies 	learning by rote * main thing in studies is to scrape through * thinks studying is to digest information through repetitive practice * teacher must explain, show connection between things and overall picture, ensure that student has understood and say what is expected in test * no need for student to think critically * degree oriented * goal in studies is		
Learning Mental mode of learning	 * wants more external regulation * teacher needs to explain carefully, provide summary, draw together what needs to be learnt and what not, checks what students can do * students find co- operation with peers important * student's task is to study regularly and get things into his head * uncertain attitude to studies * wonders if the 	learning by rote * main thing in studies is to scrape through * thinks studying is to digest information through repetitive practice * teacher must explain, show connection between things and overall picture, ensure that student has understood and say what is expected in test * no need for student to think critically * degree oriented * goal in studies is getting credits and		

Ľ.	ac	Learning pattern	Learning pattern		
		Meaning directed	Application directed		
gnitive processing strategies		* deep processing	* concrete processing		
		* internal interest	* student pays attention		
		*distinguishes	to what can be applied		
		important matters and	in practice		
		tries to understand	* makes concrete and		
		* tries to make	applies what has been		
ve p		connections to prior	learned		
gniti		knowledge			
S					
		* mostly self-	* both self and		
f	learning	regulated	externally regulated		
on c		* if problems emerge,	* may sometimes find		
ılati		thinks why	studies too theoretical		
regu		* also uses processing	* feels he has		
itive		means to remedy	understood on		
letacognitive regulation o		situations	understanding		
			connection between		
Σ			theory and practice		
		* bo and day bodden a	* * 1 1 1		
		* knowledge building	* using knowledge		
		* perceives learning as	* teacher must ensure		
		a dialogue between	that matter does not		
		people	remain too theoretical.		
		* learning is	encourage student to		
ing		accommodating s.th.	think for himself,		
arn		new into prior	stimulate curiosity		
ode of le		knowledge	* student must apply		
		* teacher to explain	matter in practice,		
al m		things outside the	show interest by asking		
lent		book, give time for	teacher		
Z		thought			
		* good teaching puts			
		student's frame and			
		interest before tests			
		* responsibility for			
		learning with self			
u		* individual	* professional		
Itati		orientation	orientation		
rien		* studies out of	* desire to acquire		
ng o		interest, to develop as	skills and knowledge		
Learning orientatio		a person or from the	for work or hobbies		
Le		joy of studying			

Table 1. Learning patterns adapted from Vermunt (Vermunt, 1996; Vermunt 2005) The number of the sum variable indicates the number of the main component.

The sum variables were named as follows [words in square brackets are abbreviations used in table 2] (the number in the parentheses is the coefficient of reliability):

1. [Uncert] Uncertainty of mathematical expertise (0.7545)

2. [Defeatist] Defeatist (0.7630)

3. [Posexp] Positive conception of own expertise (0.7545)

4. [Posmath] Positive attitude towards the study of mathematics (0.7038)

5. [Surface] Surface learning (0.5315)

6. [Rote] Learning by rote (0.6100)

7. [Deep] Deep learning (0.5216)

8. [Peer] Peer learning (0.6006)

9. [Weaklitrcy] Weakness in mathematical literacy (0.7673)

10. [Taskparall] Doing tasks in parallel (0.6149)

11. [Instrum] Instrumental learning (sufficient for the student for the task to look as if completed whether right or wrong) (0.5130)

The following statements were not strongly loaded on the former 11 principal components, thus they are processed in the following group analysis as they are (the number in front of the variable is the number on the questionnaire form/data):

4. [Effort] The fact that my efforts are appreciated inspired me to continue with my studies

5. [Byhand] When I am calculating I hope that someone will take me by the hand to advise me.

20. [Deduction] I learn best if I can use deduction in solving the task.

19. [Copying] I learn a lot by copying if I retain the thought with me

39. [Model] I succeed in solving the tasks when I take a model from the teacher

29. [Keepsols] I keep the solutions to the tasks strictly to myself.

42. [Fromdetails]I first learn the details then form a general conception of the matter.

3. [Depndsonme] Success in learning mathematics depends on me myself.

4.2 Grouping of students

The students were grouped into clusters according to the sum variables above (11) and the individual variables (Statements 3, 4, 5, 19, 20, 29, 39 and 42) using K-means cluster grouping. All variables used in the grouping into cluster centres were standardised prior to the analysis.

Next we present the solution of the five cluster centres identified by the researchers as informative. The solution of five cluster centres is presented in Table 2. The boxes in the table also show the average value of the standardised variable. If for some cluster centre the sum variable has a greater (smaller) value than in other cluster centres, the characteristic in more (less) common in the cluster centre concerned than in other cluster centres.

Final Cluster					
Centers					
	Cluster				
	1	2	3	4	5
Uncert	0.343	0.398	1.069	-0.187	-0.907
Defeatist	0.633	0.106	1.197	-0.101	-0.937
Posexp	-0.461	-0.247	-1.074	0.097	0.915
Posmath	-0.383	0.228	-1.236	-0.202	0.772
Surface	0.679	0.225	0.729	-0.239	-0.729
Rote	0.265	0.266	1.088	-0.251	-0.695
Deep	-0.637	0.296	-0.958	-0.275	0.784
Peer	-0.051	0.584	-0.024	-0.326	-0.213
Weaklitrcy	0.074	0.334	1.061	-0.129	-0.743
Taskparall	-0.140	0.469	0.696	-0.148	-0.556
Instrum	0.108	0.310	0.337	-0.230	-0.306
Effort	-0.323	0.529	-0.498	-0.383	0.266
Byhand	-0.158	0.669	0.805	-0.440	-0.528
Deduction	-0.336	0.248	-0.384	-0.422	0.512
Copying	0.463	0.261	0.191	-0.528	-0.131
Model	0.443	0.344	0.065	-0.640	-0.040
Keepsols	-0.376	0.010	0.449	0.249	-0.230
Fromdetails	-0.152	0.239	-0.026	-0.210	0.060
Depndsonme	0.390	0.108	-0.933	-0.326	0.411

Table 2. grouping of students on the basis of sum variables and individual variables into five cluster centres. Yellow indicates the greatest or greater variable for each cluster and blue the smallest or smaller values.

The clusters identified through K-means clustering are named were named as follows on the basis of the variables in Table 2: (1) Surface Oriented Learners (14.7%), (2) Peer Learners (24.0%), (3) Students Needing Support (12.5%), (4) Independent Learners (22.7%) and (5) Skilful Students (26.1%).

5 Conclusions

5.1 Orientation groups

The research questions were presented in Section 3.2: Research Question 1 concerned how students differ in their attitudes (orientation, motivation, intention).

Surface Oriented Learners are uncertain about their own expertise. Their attitudes are not the most positive and their studying is characterised by copying or studying with the help of examples. However, they do take responsibility for their own learning and trust themselves, as it is their conception that their success in the study of mathematics depends on them. However, these students do not pursue deep approach. Compared to other groups they consider it less important to call what is taught into question. This is indeed understandable, since calling knowledge into question undermines the preconditions for learning by rote and surface approach. The intention in studies is to get through the course and take the and the significance of degree. studving mathematics is derived from the needs of their own respective degree programmes. Solving tasks is not kept strictly to themselves but may be shared with peers.

In this group Entwistle's (1986, adapted from Biggs) reproducing and achieving orientation and Yrjönsuuri's dependency orientation are emphasised. In Vermunt's learning patterns (Table 1) this is oriented towards reproduction. Students with an approach to reproduction perceive the teacher as a dispenser of knowledge and the student as its recipient. The student does not need to think critically.

Peer Learners are more social compared to the other groups and like to study together with their peers. Their attitude to the study of mathematics is positive. The teacher's support and attention and the example provided by the teacher are important. Copying, studying by means of examples and learning by rote are their methods of study, but there is also an attempt at deep learning.

In this group Entwistle's (1986, adapted from Biggs) reproducing orientation and Yrjönsuuri's dependency orientation are emphasised. Vermunt's approach to learning is directed towards reproduction, but also partly not directed and partly directed towards meaning. Peer Learners appear to make most use of use processing by stages (Table 1) since compared to other groups they study the details first and then build up entities from them. Students Needing Support are extremely uncertain of their mathematical expertise compared to other groups and easily abandon their studies. Their attitudes towards the study of mathematics are moreover weak. These students in need of support study mathematics by learning by rote and they find the language of mathematics difficult to understand. They hope that someone will come and take them by the hand to advise them; the examples provided by the teacher are not sufficient. They do not take responsibility for their own learning. It suffices for them to get the tasks looking as if they had been completed (instrumental learning).

In the actions of this group we see Ramsden's (1984) non-academic orientation and Yrjönsuuri's defeatist and self orientations. The interest in mathematics of Students Needing Support is influenced by their degree programmes and possible completion of a degree. According to Vermunt their attitude to their studies is uncertain. They wonder if they have made the right choice of field of study.

Independent Learners go more their own way than do students in other groups, at least in the study of mathematics. According to the variables in the cluster analysis the group appeared more passive than the other groups, but its good achievements told a different story. Table 2 shows that the averages of the group ran parallel in several variables with those of the group of Skilful Students. Those classified as Independent Learners have a positive conception of their own capabilities and do not resort to learning by rote, reproducing orientation or surface approach. Compared to the Skilful Students group, however, Independent Learners are not as positive about studying mathematics, do not pursue deep study and do not find recognition of their efforts particularly important and do not use creative reasoning when solving tasks. They moreover study in their own way and the significance of other students in studying is not as strongly emphasised as in other groups. They also keep the solutions to their task to themselves. Examination of the original statements showed that in the opinion of this group the statements "Learning mathematical structures is unnecessary" was given the second highest ranking.

It appears somewhat contradictory that in the opinion of this group the average for the statement "Success in learning mathematics depends on me myself", was low, being lower only in the Students Needing Support group. Such a response may reflect the group's indifference to learning mathematics.

Examination of Vermunt's approaches suggests that some of the Independent Learners might be application directed. This group is characterised by believing that they understand the matter if they understand the relation between theory and practice. They may consider the study of mathematics too theoretical if the connection to the real world cannot immediately be demonstrated. Those who are application directed render what they learn concrete and apply it; their studying is both self-directed and externally directed. Application directed study includes professional orientation. The teachers can see that some of the TUT students of engineering are those who apply knowledge, considering practicality more important than theory. Theory is appreciated only if it provides immediately practical solutions.

Skilful Students have a positive attitude to studying mathematics and a positive conception of their own skills. Skilful Students pursued deep learning and used learning by rote least in their studies. Copying and examples were not as important to them as they were to other groups. Skilful Students do not give up easily when doing their tasks.

In Entwistle's terms Skilful Students are personal meaning oriented and for Yrjönsuuri they are taskoriented. In Vermunt's approach clearly meaning directed. The motivation of these students is internal and the strategies regulating studies are selfdirected. Learning is the construction of knowledge and the teacher ought to recount matters not in the textbook and allow students time for their own thinking as they learn best if they can use creative reasoning and take responsibility for their own learning. The goal of studying is personal: interest, the joy of studying, developing as a person.

5.2 Teaching alteration and further research

The second research question was also presented in section 3.2. It concerned how should students with different attitudes and modes/means of studying be catered for in the teaching of mathematics?

The first thing to recognize is to tune in to fact that there are several different learner types among students. Also it is crucial to know different learner types so that one can know how and in which direction the teaching should be developed. After that it is possible to cater different learners with different teaching arrangements. In fact from the autumn 2006 the exercises in Mathematics for Engineers were arranged in such a way that half of the exercise groups were directed to Skilful Students and Independent Learners and the other half of groups were directed to Students Needing Support, Peer Learners and Surface Oriented Learners. This enables for most of the students a possibility to learn more close to their own learning orientation than in traditional exercise group arrangement.

It should be noted that for best learning results different learners may need different kind of course contents to support their studies. For instance we may provide for Skilful Learners and Independent Learners such lectures or exercises that provide more theory and concentrate less on details. Such suggestions will be presented in the further paper.

The results mentioned above provide a lot of information to develop teaching but it also brings out further research questions: how do other factors e.g. the achievements in mathematics and in further engineering studies affect/predict each other, how could the orientation groups be taken into account more efficiently from learning point of view e.g. by producing material inspiring learners to learn mathematics.

In fact some changes considering new exercise groups, new application oriented material and other teaching arrangements have been done at TUT. These teaching arrangements will be reported in the further paper.

References:

[1] Damon, W. & Phelps, E, *Critical distinction among three approaches to peer education*, International Journal of Educational Research. Vol. 13, No. 1, pp. 9-19, 1989.

[2] Entwistle, N. Styles of Learning and Teaching. An Integrated Outline of Educational Psychology for students, Teachers, and Lecturers, Chichester, John Wiley & Sons. 293 pages, 1986.

[3] Johnson, R.A. & Wichern, D.W. *Applied Multivariate Statistical Analysis*, Upper Saddle River, Prentice-Hall, 816 pages, 1998.

[4] Marton, F. & Säljö, R. *Approaches to Learning.* in Marton, F., Hounsell, D. & Entwistle, N. (ed.) The Experience of Learning, Edinburgh, Scottish academic Press, pp. 36-55, 1984.

[5] Pohjolainen, S., Raassina, H., Silius, K., Huikkola, M. & Turunen, E, *TTY:n insinöörimatematiikan opiskelijoiden asenteet,* *taidot ja opetuksen kehittäminen*, Tampere University of Technology, Institute of Mathematics, 127 pages, 2006.

[6] Ramsden, P., *The Context of Learning*, In: Marton, F., Hounsell, D. & Entwistle, N. (ed.) The Experience of Learning. Edinburgh, Scottish Academic Press, pp. 144-164, 1984.

[7] Rauste-von Wright, M. & von Wright, J, *Oppiminen ja koulutus*, Juva, WSOY, pp. 114-134, 1994.

[8] Solmun erikoisnumerot 1 & 2, 2005-2006: //solmu.math.helsinki.fi/ (checked 29.5.2006).

[9] Tynjälä, P., Oppiminen tiedon rakentamisena. Konstruktivistisen oppimiskäsityksen perusteita, Helsinki, Kirjayhtymä, 214 pages, 1999.

[10] Vermunt, J.D., *Metacognitive, cognitive and affective aspects of learning styles and strategies: A phenomenographic analysis,* Higher Education, Vol. 31. pp. 25-50, 1996.

[11] Vermunt, J.D., *Relations between student learning patterns and personal and contextual factors and academic performance*, Higher Education, Vol. 49, pp. 205-234, 2005.

[12] Yrjönsuuri, R.,. *Opit kun haluat. Matematiikkaa ja yhteistyötä*, Hamina, Oy Kotkan kirjapaino Ab, 177 pages, 2002

[13] Yrjönsuuri, R., *Matematiikan opiskelun kokeminen ja tilanneorientaatiot*, Unpublished bookdraft, 57 pages, 2004.