Comparison of RF Inductor Performance Evaluation Methods

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Abstract: — Various model parameter calculation equations of RF CMOS inductors are compared. The calculation equations are using as variables either Y-, Z-parameters or S-parameters. Results are based on on-wafer RF S-parameter measurements of a 3.75 turn spiral inductor fabricated in AMIS 0.7 μm CMOS DM1P n-well technology. Results obtained with calculation equations for a differentially driven inductor topology show differences evaluating the same model parameter, depending on whether Z- and S-parameters are used. Thus, care must be taken by the designers when estimating an integrated inductor performance to comprehend the operating environment of the device and to choose the corresponding formulas accordingly.

Key-words: - RF CMOS inductors, model parameter evaluation, S-parameter measurements.

1 Introduction

Integrated inductors are basic components of RF integrated circuits. Accurate device models are required for RFIC circuit design. Model parameters that evaluate the performance of integrated inductors are the inductance, series resistance, and quality factor. Parameter extraction is based on a device model, such as the nine-element model [1]. On-wafer scattering parameter measurements are necessary for device model validation. S-parameter measurements are taken using a Vector Network Analyzer along with a set of Ground-Signal-Ground (GSG) or Ground-Signal (GS) probes. De-embedded S-parameters are converted to Y- and Z-parameters in order to be used for model parameter extraction.

Depending on the inductor configuration in the circuit different calculation equations are used to extract the model parameters. An equation using the Y_{11}-parameter as a variable, as well as an equation using the input reflection coefficient, Γ_{in}, defined from single-ended S-parameters, are used when the inductor is single-ended (grounded). An equation using the differential impedance Z_{dd} [2] as a variable, as well as an equation using a differential S-parameter [3], S_{dd}, are used when the inductor is differentially driven. Differential S-parameters have been defined in [4] to be used in differential networks.

In this work a comparison is made between the different calculation equations used in each inductor configuration in order to check their equivalency. Results are based on de-embedded Y-, Z-, and S-parameters calculated from on-wafer S-parameter measurements taken on a test chip. Comparison between results obtained for the differentially driven inductor show differences depending on whether the equation is based on Z or S parameters. Calculation equations using the Y or S parameters as variables produce the same results for the single-ended inductor. In the following a theoretical analysis of model parameter extraction methods is given in section 2. In section 3 results extracted from experimental data with the different evaluation methods are presented, followed by an analysis of the results. Conclusions are given in section 4.

2 Theoretical Analysis

2.1 Single-ended inductor

A general two-port network is shown in Fig. 1a. The nine-element model is taken as the inductor model. A π-type equivalent circuit shown in Fig. 1b is used to calculate the model parameters of the single-ended inductor, Since the second port is grounded in this case, the input admittance of the circuit is equal to Y_{11}.

Inductance, L_{Y11}, series resistance, R_{Y11}, and quality factor, Q_{Y11} are calculated in terms of the Y_{11} parameter as

$$L_{Y11} = \frac{1}{2\pi f}\,\mathrm{imag}\left(\frac{1}{Y_{11}}\right). \tag{1}$$

$$R_{Y11} = \mathrm{real}\left(\frac{1}{Y_{11}}\right). \tag{2}$$

$$Q_{Y11} = \frac{imag\left(\frac{1}{Y_{11}}\right)}{real\left(\frac{1}{Y_{11}}\right)}. \tag{3}$$

A different parameter extraction equation can be derived using the input reflection coefficient Γ_{in}

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = S_{11} - \frac{S_{12}S_{21}}{1 + S_{22}}. \tag{4}$$

$$Z_{in,se} = Zo\,\frac{1+\Gamma_{in}}{1-\Gamma_{in}} = \frac{(1+S_{11})(1+S_{22}) - S_{12}S_{21}}{(1-S_{11})(1+S_{22}) + S_{12}S_{21}} = \frac{1}{Y_{11}} \tag{5}$$

where Γ_L is the load reflection coefficient. Γ_L=-1 for the single-ended inductor. $Z_{in,se}$ is the input impedance. The right hand side of (5) is obtained by using the conversion relation between Y- and S-parameters [5]. Z_o is the characteristic port impedance (typically 50 Ω).

The inductance, series resistance, and quality factor are then defined in terms of S-parameters as

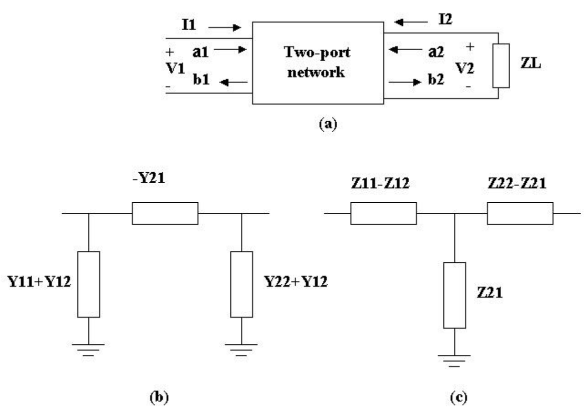

Fig. 1. (a) A general two-port network. (b) A π-type equivalent circuit. (c) A T-type equivalent circuit.

$$L_{s,se} = \frac{1}{2\pi f}\,\mathrm{imag}(Z_{in,se}). \tag{6}$$

$$R_{s,se} = \mathrm{real}(Z_{in,se}). \tag{7}$$

$$Q_{s,se} = \frac{\mathrm{imag}(Z_{in,se})}{\mathrm{real}(Z_{in,se})}. \tag{8}$$

Taking into account (5), L_{Y11}, R_{Y11}, and Q_{Y11} are equivalent to $L_{s,se}$, $R_{s,se}$, and $Q_{s,se}$ respectively.

2.2 Differentially Driven Inductor

A T-type equivalent circuit for the inductor is used in this case as shown in Fig. 1c. Differential and common mode voltages (V_d, V_c) and currents (I_d, I_c) are defined from single-ended voltages and currents (Fig. 1a), as

$$V_d = V_1 - V_2,\ I_d = \frac{I_1 - I_2}{2}. \qquad V_c = \frac{V_1 + V_2}{2},\ I_c = I_1 + I_2. \tag{9}$$

$$V_d = Z_{dc}I_c + Z_{dd}I_d. \qquad V_c = Z_{cc}I_c + Z_{cd}I_d. \tag{10}$$

A differential impedance $Z_{dd,z}$ is defined from (10) as [6]

$$Z_{dd,z} = Z_{dd} = \left.\frac{V_d}{I_d}\right|_{I_c=0} = Z_{11} + Z_{22} - Z_{12} - Z_{21}.$$

(11)

Inductance, series resistance, and quality factor are then defined in terms of the differential impedance $Z_{dd,z}$ as

$$L_{Zdd,z} = \frac{1}{2\pi f}\,\mathrm{imag}(Z_{dd,z}). \quad (12)$$

$$R_{Zdd,z} = \mathrm{real}(Z_{dd,z}). \quad (13)$$

$$Q_{Zdd,z} = \mathrm{imag}(Z_{dd,z}) \big/ \mathrm{real}(Z_{dd,z}). \quad (14)$$

Single-ended voltages and currents have been defined in terms of incident and reflected power waves as

$$V_1 = \sqrt{Z_o}(a_1 + b_1), V_2 = \sqrt{Z_o}(a_2 + b_2).$$
$$I_1 = \frac{a_1 - b_1}{\sqrt{Z_o}}, I_2 = \frac{a_2 - b_2}{\sqrt{Z_o}}. \quad (15)$$

where a_n, b_n are the incident and reflected power waves of port n, respectively.

Using (9) and (15) the condition I_c=0 in (11) is equivalent to

$$I_c = 0 \Rightarrow I_1 = -I_2 \Rightarrow a_1 + a_2 = b_1 + b_2. \quad (16)$$

Single-ended S-parameters are defined in terms of incident and reflected power waves from

$$b_1 = S_{11} a_1 + S_{12} a_2.$$
$$b_2 = S_{21} a_1 + S_{21} a_2. \quad (17)$$

Using (15), (16), and (17), the differential impedance in (11) can be written in terms of incident and reflected power waves as

$$Z_{dd,z} = \frac{a_1 + b_1 - a_2 - b_2}{a_1 - b_1}. \quad (18)$$

Another definition of model parameters can be derived in terms of differential S-parameters. Differential and common mode S-parameters have been defined in [4] as

$$b_d = S_{dc} a_c + S_{dd} a_d. \quad (19)$$
$$b_c = S_{cc} a_c + S_{cd} a_d.$$

where differential and common mode incident (a_d, a_c) and reflected power waves (b_d, b_c) are defined from single-ended power waves as

$$a_c = \frac{a_1 + a_2}{\sqrt{2}}, a_d = \frac{a_1 - a_2}{\sqrt{2}}.$$
$$b_c = \frac{b_1 + b_2}{\sqrt{2}}, b_d = \frac{b_1 - b_2}{\sqrt{2}}. \quad (20)$$

A differential S-parameter, S_{dd}, is defined

$$S_{dd} = \left.\frac{b_d}{a_d}\right|_{a_c=0} = \left.\frac{b_2 - b_1}{a_2 - a_1}\right|_{a_c=0} = \left.\frac{b_1 - b_2}{2a_1}\right|_{a_c=0} \quad (21)$$
$$= \frac{S_{11} + S_{22} - S_{12} - S_{21}}{2}.$$

Using (20), the condition a_c=0 in (21) is equivalent to

$$a_c = 0 \Rightarrow a_1 = -a_2. \quad (22)$$

The right hand side in (21) has emerged taking into account (17) and (22).

A differential impedance, $Z_{dd,s}$ has been defined in terms of the differential S-parameters in [4] as

$$Z_{dd,s} = 2Z_o \frac{1 + S_{dd}}{1 - S_{dd}}. \quad (23)$$

Using (21), (23) can be written in terms of power waves and S-parameters as

$$Z_{dd,s} = 2Z_o \frac{2a_1 + b_1 - b_2}{2a_1 - b_1 + b_2} =$$
$$2Z_o \frac{2 + S_{11} + S_{22} - S_{12} - S_{12}}{2 - S_{11} - S_{22} + S_{12} + S_{21}}. \quad (24)$$

Inductance, series resistance, and quality factor are then defined in terms of the differential impedance $Z_{dd,s}$ from equations of the form of (12)-(14).

Comparing equations (18) and (24) in terms of power waves, it can be seen that they are not equivalent. They lead to different results for the differential impedance.

3 Results

The device under test (DUT) was a 1.75 turn square spiral inductor fabricated in AMIS 0.7 µm CMOS DM1P n-well technology. On wafer RF S-parameter measurements have been taken using a HP8510 Vector Network Analyzer and 150 µm pitch Picoprobe Ground-Signal-Ground probes in a frequency range from 1 to 15 GHz. A Line-Reflect-Match (LRM) calibration of the system performed on an impedance standard substrate calibrated the measurements up to the probe tips. An off-line calibration (de-embedding) was performed next to remove probe pad and metal interconnect line parasitic elements [7].

Figs. 2, 3, and 4 show the inductance, resistance and quality factor, respectively, calculated using the $Z_{dd,z}$, ((11)), $Z_{dd,s}$ ((24)) differential impedances and $Z_{in,se}$ ((5)) single-ended impedance.

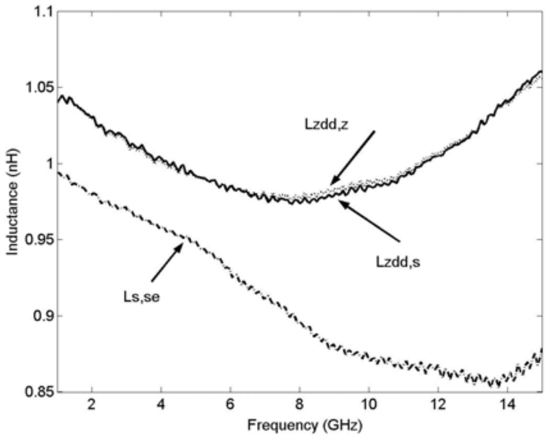

Fig. 2. Inductance of the 1.75 turn inductor calculated in terms of the $Z_{dd,z}$ (eq. (11)), $Z_{dd,s}$ (eq. (24)) differential impedances, and $Z_{in,se}$ (eq. (5)) single-ended impedance.

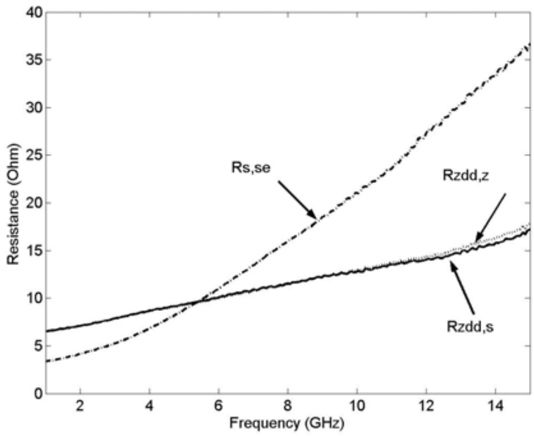

Fig. 3. Resistance of the 1.75 turn inductor calculated in terms of the $Z_{dd,z}$ (eq. (11)), $Z_{dd,s}$ (eq. (24)) differential impedances, and $Z_{in,se}$ (eq. (5)) single-ended impedance.

As it is shown in Figs. 2, 3, and 4, there are differences at high frequencies of the extracted model parameters depending on whether the $Z_{dd,z}$ or $Z_{dd,s}$ differential impedance is used in the calculations. Equation (16) (condition for the differential impedance $Z_{dd,z}$) implies that the same currents in magnitude (with 180^o phase difference) flow to ports 1 and 2, so that there is no signal leakage to ground. The low resistivity silicon substrate causes signal leakage, an effect which is more obvious at high frequencies, as seen from Figs. 2, 3, and 4.

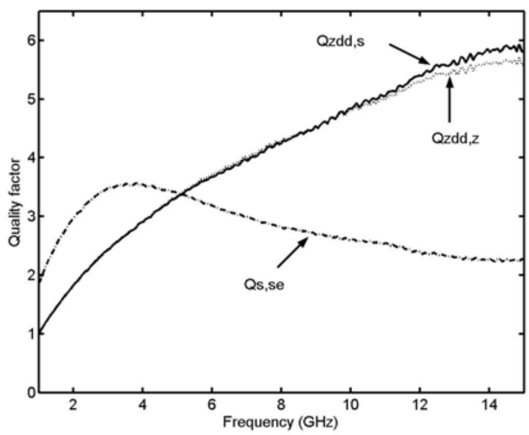

Fig. 4. Quality factor of the 1.75 turn inductor calculated in terms of the $Z_{dd,z}$ (eq. (11)), $Z_{dd,s}$ (eq. (24)) differential impedances, and $Z_{in,se}$ (eq. (5)) single-ended impedance.

4 Conclusions

This paper presents a comparative evaluation of various performance calculation equations of integrated RF CMOS inductors using different calculation formulas. In particular, the calculation of L, R and Q for a specific fabricated inductor based on the same measured S-parameters is assessed, utilizing single-ended as well as differential models. Y-, Z- and S-parameter based equations are used.

The calculation formulas utilized for single-ended inductor configuration (e.g. eq. (8) based on (5) and eq. (3)) give the same results. However, differences appear when differential configuration is considered and Eq. (11) or (24) is applied. The condition under which the differential $Z_{dd,z}$ is derived is mainly valid for SOI or GaAs technologies or at low frequencies for CMOS technology. Therefore, care must be taken by the designers when evaluating an integrated inductor performance to comprehend the operating environment of the device and to choose the corresponding formulas accordingly.

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