Signature Analysis of Mechanical Watch Movements by Reassigned Spectrogram

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Abstract: - This paper presents a study on the signature analysis of mechanical watch movements. Time-frequency analysis is proposed to analyze the time-frequency features of mechanical watch sounds so as to diagnose faults of mechanical watches. It is done by a combination of Reassigned Spectrogram (RSP) and Finite Element Analysis (FEA). The RSP possesses excellent time and frequency resolution and very few misleading interference fringes in mapping the signal into a 2D function of time and frequency. It reveals shocks at different time of the escapement mechanism. By comparing the frequency components at different shocks to the modal frequencies of components of the escapement, one can detect possible malfunctions of the movement. Time-frequency analysis works robustly as a complementary fault diagnosing method to time-domain analysis. Finally, a number of practical examples are included to reflect the applicability of the proposed method.

Keywords: - Signature analysis, Mechanical watch movement, Reassigned spectrogram (RSP), Finite element analysis (FEA), Fault diagnosis

1 Introduction

A mechanical watch is a mechanical device for timekeeping. Appeared in the middle of 16th century, the mechanical watch is one of most complex mechanical mechanisms ever invented. Expert watch makers often listen to its ticks to diagnose faults. Different watches make different sounds. The basic "tick tick" sound comes from the impacts inside the escapement which is a feedback regulator that determines the timekeeping accuracy [1]. Taking advantage of this informative acoustic signal, several commercial systems are developed to estimate the timekeeping accuracy and diagnose faults [2][9]. However, these systems are based merely on time-domain signals. This limits the performance of these systems in fault diagnosis.

The escapement is composed of the balance wheel, the pallet fork and the escape wheel, which meshes with the fourth wheel through the escape pinion (see Fig. 1). One complete tick of the escapement can be divided into three phases: unlocking, impulse and drop [1]. During the unlocking phase, the impulse pin on the balance wheel strikes against the pallet fork notch and at the same time, the escape wheel falls back a small angle along the entry pallet jewel. After unlocking, the escape wheel tooth gives an impulse to the entry pallet jewel and the other side of the pallet fork notch catches up with the impulse pin and pushes the balance wheel forward. During the drop phase, the escape wheel tooth drops onto the exit pallet jewel and the pallet fork drops clockwise until it hits the banking pin. As the intensity of the shocks in the drop phase is the largest, the vibration of the escape wheel may cause vibration on the fourth wheel. The instantaneous sound frequencies correspond to the modal frequencies of the vibrating parts, including the balance wheel, the pallet fork, the escape wheel and the fourth wheel.

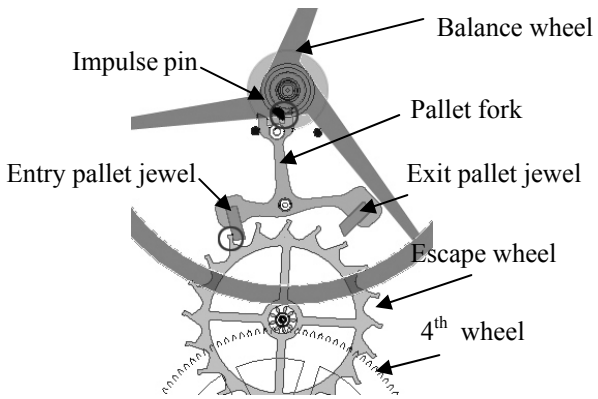


Fig. 1 The escapement

Fig. 2 shows a typical sound signal of one beat. A good watch should have 3 main audible sounds, corresponding to the three phases. Commercial systems make use of this feature and have developed some time-domain methods to estimate the accuracy

of the watch and detect faults. This method works well on some faults. However, for other faults, it is not robust. Fig. 3 shows some fault diagnosis methods that are used in Witschi® timing machine [9]: (a) not enough clearance between the horns and the impulse pin; (b) too much endshake in the pivot of the balance wheel; (c) fork horn touching the impulse pin (knocking). The differences between these three faults in the diagnosis interpretation are so tiny that one fault can easily be mistaken as another by using this diagnosis method. In real application, the waveform of the watch signal may not be as clear as those in Fig. 3, and this again makes the time-domain method unstable.

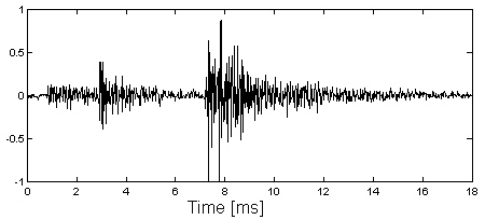


Fig. 2: A sound signal of a mechanical watch

(a) (b) (c)

Fig. 3 time-domain fault diagnosis

Since the sound is caused by the vibration of the escapement parts, we are motivated to analyze the sound from the time-frequency aspect. As each shock lasts very shortly, it can be approximated as impulse force. The damping ratio is very small and negligible, so the vibration frequencies are the modal frequencies of the escapement parts. Therefore, we propose to analyze the signal using the Time-Frequency Distribution (TFD) [3]. By comparing the results to the modal frequencies of the escapement parts that are found by means of Finite Element Analysis (FEA) [4], we can then find much more information about the mechanical movement and diagnose possible malfunctions robustly.

2 Modal Frequencies of Escapement

In general, the motion of a mechanical system can be expressed as a 2nd order differential equation [4]:

$$[M]\{\ddot{\delta}\}+[C]\{\dot{\delta}\}+[K]\{\delta\}=\{R(t)\} \quad (1)$$

where, M is the equivalent mass matrix, K is the stiffness matrix, C is the damping matrix and $R(t)$ is the external load matrix. When evaluating the modal frequency of the system, $\{R(t)\}$ is set to zero. Also, because the damping is usually rather small, equation (1) can be simplified as:

$$[M]\{\ddot{\delta}\}+[K]\{\delta\}=0 \quad (2)$$

Let $\omega_n^2 = \lambda$, where ω_n is the modal frequency of vibration, the eigen-function of equation (2) is found:

$$\det([K]+\lambda[M])=0 \quad (3)$$

Solving equation (3) will then give the modal frequencies. If we simplify the shocks in the escapement as impulse forces, then the response of the escapement parts can be given by:

$$x(t) = A\cos(\omega_n t - \phi) \quad (4)$$

where, A is the magnitude of the vibration determined by the initial conditions. It is seen that the frequencies of vibration correspond to the modal frequencies of the system.

Practically, the FEA is used for finding approximate solution of differential equations [15]. There are numerous commercial software for FEA such as ANSYS®, Algor® and etc. In our research, we use SolidWorks® to evaluate the modal frequencies of the escapement and the fourth wheel. The modal frequencies (0~24kHz) of the balance wheel, pallet fork, escape wheel and the fourth wheel are summarized in Table 1.

Watch Parts	Modal Frequencies (Hz)
Balance Wheel	481, 1752, 1753, 2488, 6417, 6419, 6766, 6768, 18331, 18892, 20223, 20228, 21537
Pallet Fork	16779, 21150, 22212
Escape Wheel	9117, 11262, 11271, 12667, 18728, 19425
4th Wheel	3752, 3755, 4361, 5646, 6645, 6658, 14177, 14178

Table 1: Modal frequencies of watch parts

3 Reassigned Spectrogram

Time-frequency distribution is a powerful tool for analyzing non-stationary signals [3]. It transfers a one-dimensional time-domain signal, $x(t)$, into a

two-dimensional function of time and frequency, $TFD(t, f)$. Hence, it can characterize signals over the time-frequency domain. There are a number of different TFDs, each one has its own advantages and drawbacks. Positive TFDs, such as spectrogram [3] and scalogram [5], do not produce cross-terms if the auto-terms do not overlap. However, the limitation of Heisenberg box [5] unavoidably makes the trade-off between time and frequency resolutions. The Wigner-Ville distribution (WVD) [6] provides the best time and frequency resolution but its practical application is restricted by the presence of spurious cross-terms, which appear in the middle of every two auto-terms in the TF plane. Smoothed versions of WVD, such as smoothed pseudo-Wigner-Ville distribution and Choi-Williams distribution [12] are developed to eliminate the cross-terms of WVD to a certain degree but again, such smoothing operation spreads out localized auto-terms.

Since watch signals are composed of synchronized frequency components at a certain time instance and that each frequency component lasts for only a short period of time; some frequency components are very close to each other. Therefore, it is necessary to find a TFD that possesses both a high time-frequency resolution (especially high frequency resolution) and a low level of cross-terms. Experiment results show that reassigned spectrogram is a proper choice for our application.

3.1 Spectrogram

The basic idea of STFT is to calculate the Fourier transform of the signal at time t within a small window h centered at time t. A spectrogram is the squared modulus of the STFT [3]:

$$STFT_x(t,\omega) = \int_{-\infty}^{+\infty} x(\tau)h^*(\tau - t)e^{i\omega\tau}d\tau \qquad (5)$$

$$SP_x(t,\omega) = \left|STFT_x(t,\omega)\right|^2 \qquad (6)$$

where, $g_{t,\omega}(\tau) = h(t-\tau)e^{i\omega\tau}$ is called time-frequency atoms. Each atom is obtained from the window h by a translation in time and a translation in frequency. It is seen that STFT correlates the signal with time-frequency atoms. The time-frequency resolution of g is represented by a Heisenberg box centered at (t, ω) with a time spread σ_t and a frequency spread σ_ω. In order to better measure a signal at a particular time and frequency (t, ω), it is natural to desire that σ_t and σ_ω be as narrow as possible. However, according to the Heisenberg uncertainty theorem, $\sigma_t\sigma_\omega \geq 1/2$. A shorter window length makes the time resolution becomes better but the frequency resolution becomes worse, and vice versa. Therefore, a trade-off between time and frequency resolution is unavoidable.

Instead of using its usual definition, the spectrogram can be equivalently expressed in the form of Cohen's class [6][8]:

$$SP_x(t,\omega) = \iint WV_x(s,\xi)WV_h(s-t,\xi-\omega)\frac{dsd\xi}{2\pi} \qquad (7)$$

where, WV_h is the WVD of the window h. It is seen that the spectrogram is a smooth version of the WVD using a smoothing kernel $\Phi_{TF} = WV_h$.

3.2 Reassigned Spectrogram (RSP)

Reassignment is a post-processing technique that can improve the readability of TFD with both good time and frequency concentration of the signal components and no misleading cross-terms [6][7][10].

From equation (7), we can see that the value that the spectrogram takes at a given point (t, ω) is not simply the value of this point. Instead, it is the weighted average of WVD contributions within the essential TF support of the kernel Φ_{TF}. In other words, an entire distribution of values is summarized to a number which is assigned to the geometric center (t, ω) of this TF domain. However, the energy distribution is not always geometrically symmetric. Moving this value to the center of gravity $(\hat{t},\hat{\omega})$ of the distribution is much more meaningful and this is the essence of reassignment. The reassigned spectrogram is defined as:

$$RSP_x(t',\omega';h) =$$

$$\iint SP_x(t,\omega;h)\delta(t'-\hat{t}_x(t,\omega))\delta(\omega'-\hat{\omega}_x(t,\omega))\frac{dtd\omega}{2\pi} \qquad (8)$$

The coordinates of center of gravity are given by:

$$\hat{t}_x(t,\omega) = t - \Re\left\{\frac{STFT_x(t,\omega;Th)\cdot STFT_x^*(t,\omega,h)}{\left|STFT_x(t,\omega,h)\right|^2}\right\} \qquad (9)$$

$$\hat{\omega}_x(t,\omega) = \omega + \Im\left\{\frac{STFT_x(t,\omega;Dh)\cdot STFT_x^*(t,\omega,h)}{\left|STFT_x(t,\omega,h)\right|^2}\right\} \tag{10}$$

where, $Th = t\cdot h(t)$, $Dh = dh(t) / dt$, $\Re$ and $\Im$ denote the real part and the imaginary part respectively.

3.3 Performance Evaluation of Spectrogram and RSP

3.3.1 Performance on synthetic watch signal

Based on the modal frequencies of the watch parts (see Table 1), a signal (sampling rate at 48kHz) is generated as shown in Fig. 4. This signal is very similar to a real watch signal and can be used to evaluate the performance of spectrogram and RSP on analyzing watch signals. Fig. 4 also shows the idealized TFD of this signal.

Fig. 5 shows the analysis results. Note that both the spectrogram and the RSP use a Hamming window of 120 points. Proper thresholds are chosen to give the best readability for each distribution. It is seen that the time and frequency resolution of RSP is much better than that of the spectrogram. For example, the frequencies 6,417Hz and 6,766Hz from 6ms to 10 ms are nearly mixed in the spectrogram, but are distinguished in the RSP. The reassignment provides a TF plot which resembles the idealized model to a greater extent. The RSP provides excellent time and frequency resolution although few interference fringes still persist (pointed with arrows in Fig. 5).

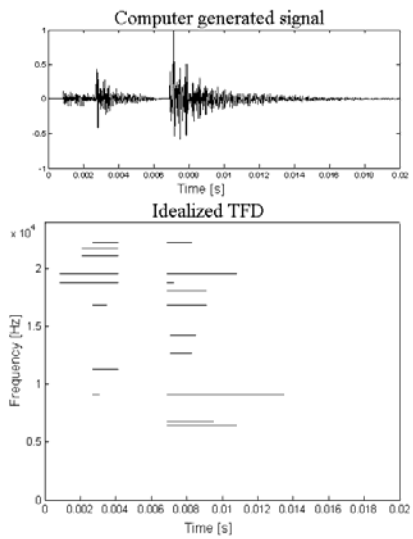
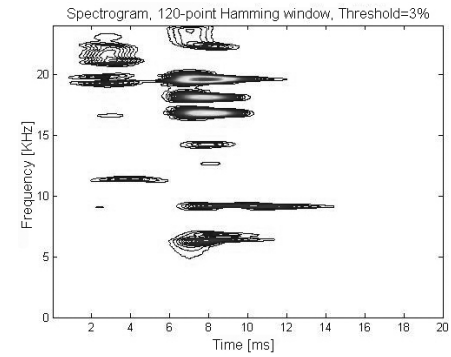
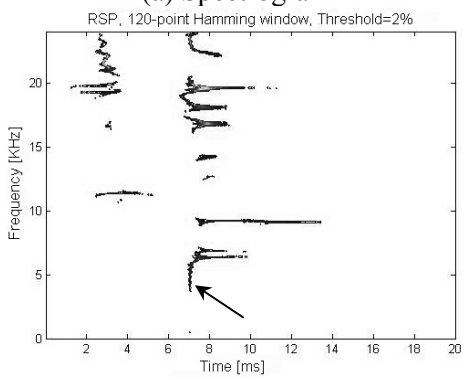


Fig. 4 Watch signal and idealized TFD



(a) Spectrogram

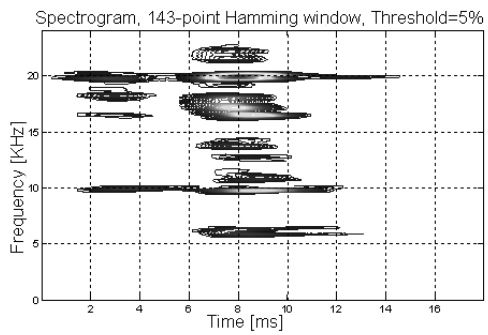


(b) RSP

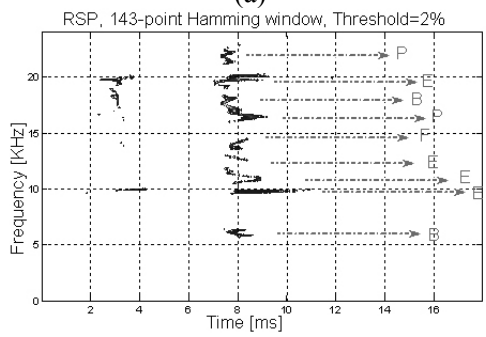
Fig. 5 Spectrogram and RSP of synthetic signal

3.3.2 Performance on real watch signal

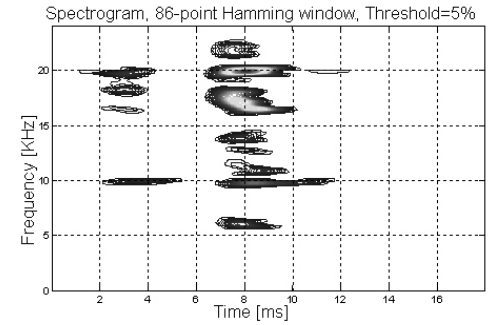
The performances of the spectrogram and the RSP are also evaluated using a real watch signal shown in Fig. 2. Fig. 6(a) and (b) are the spectrogram and the RSP using a 143-point Hamming window, while (c) and (d) are the spectrogram and the RSP using 86-point Hamming window.



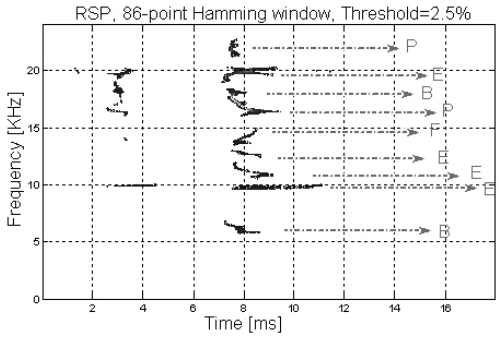
(a)



(b)



(c)



(d)

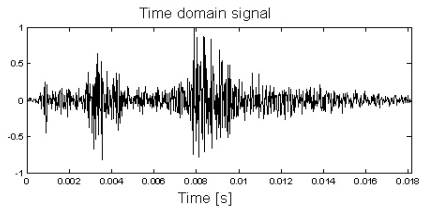
Fig. 6 Spectrogram and RSP of real watch sigal

It is seen that the result given by spectrogram heavily depends on the length of the short-window. It is obvious that the 143-point window gives better frequency resolution than that of the 86-point window. Comparing Fig. 6(b) with Fig. 6(d), we can see that the interdependency between signal and window length is dramatically reduced after reassignment. The RSPs using different window length give almost the same results and possess excellent resolutions. Furthermore, nearly all of the frequency components in the RSPs can match their counterparts among the modal frequencies given in Table 1 (B represents balance wheel, P represents pallet fork, E represents escape wheel and F represents fourth wheel). It can be seen that RSP reveals the insight of the escapement motion [11].

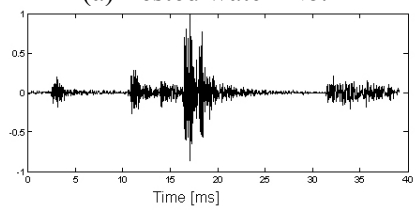
4 Fault Detection

As mentioned before, time-domain analysis does not work robustly in diagnosing certain faults of the mechanical watch. Since RSP reveals the shocks at different time of the movement, we therefore propose time-frequency analysis. We use the examples in Fig. 3 to show the advantages of time-frequency analysis.

Fig. 7 shows the signals of two tested watches and both of them have some faults. The clearance between the horns and the impulse pin is too small in tested watch No. 1. For tested watch No. 2, there are too much endshake in the pivot of the balance wheel. However, their time-domain signals are not like the diagnosis interpretations shown in Fig. 3(a) and (b). The signal of the first tested watch resembles the signal of a good watch very much. After comparing the RSP of the tested watch (Fig. 8(a)) and that of the good watch (Fig. 6(b) or (d)), we can tell their differences. It is seen from Fig. 8(a), during the third phase, the frequency of the balance wheel (about 6419Hz) appears twice, the first one at about 8.2ms and the other is at about 12ms. This implies that there is an additional shock at about 12ms and in reality. This is a shock between the impulse pin and the pallet fork notch. Subsequently, the shock causes vibration of the escape wheel as a result of the destitution of clearance between the horns and the impulse pin.

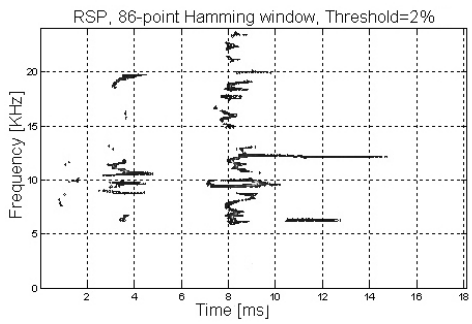


(a) Tested watch No.1

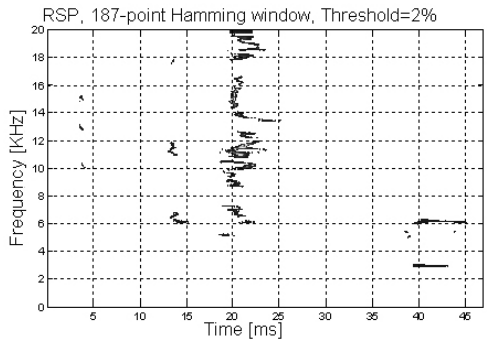


(b) Tested watch No.2

Fig. 7 Time-domain signals



(a) Tested watch No.1



(b) Tested watch No.2

Fig. 8 RSPs of two tested watches

From Fig 8(b), we can see that there is one additional peak appearing in the time-domain signal. With the help of Fig. 8(b), one can easily tell that this watch does not suffer from faults (a) and (c) in Fig. 3, because there are only modal frequencies of the balance wheel (about 2488Hz and 6419Hz). This additional peak is caused by excessive axial end-shaking in the pivot of the balance wheel [9].

Other types of faults can also be detected by analyzing RSPs in a similar manner.

5 Conclusions

This paper presents the signature analysis of mechanical watch using sound signals. The following conclusions can be drawn:

(a) The sound frequencies of a mechanical watch movement correspond to modal frequencies of the balance wheel, the pallet fork, escape wheel and the fourth wheel.

(b) Time-frequency analysis can be done by a combination of FEA which gives the modal frequencies and RTFD which gives the frequencies of the mechanical watch components.

(c) Reassigned spectrogram provides excellent time-frequency resolution with few misleading interference terms. Therefore, it is useful in detecting defects and errors of the mechanical watches.

(d) Time-frequency analysis reveals the insights of the escapement motion and has an obvious advantage over time-domain analysis in diagnosing possible faults of the escapement.

References:

[1] C. A. Reymondin, G. Monnier, D. Jeanneret and U. Pelaratti, *The Theory of Horology*, Swiss Federation of Technical Colleges, Switzerland, 1999.

[2] http://www.bmumford.com/microset.html, The official website of MicroSet.

[3] S. Qian, D. Chen, *Joint Time-frequency Analysis: Methods and Applications*, Prentice Hall, Upper Saddle River, N.J., USA, 1996.

[4] H. Benaroya, *Mechanical vibration: Analysis, Uncertainties, and Control*, Prentice Hall, Upper Saddle River, N.J., USA, 1998.

[5] S. Mallat, *A Wavelet Tour of Signal Processing*, Academic, London, 1999.

[6] A. Papandreou-Suppappola, *Applications in Time-Frequency Signal Processing. Part 5: Time-Frequency Reassignment: From Principles to Algorithms*, CRC Press, Boca Raton, Fla., USA, 2003.

[7] Z. Peng, F. Chu, Y. He, Vibration Signal Analysis and Feature Extraction Based On Reassigned Wavelet Scalogram, *Journal of Sound and Vibration,* Vol.253, No.5, 2002, pp. 1087-1100.

[8] L. Cohen, Time-frequency distributions - A Review, *Proceedings of The IEEE,* Vol.77, No.7, 1989, pp. 941-981.

[9]http://www.witschi.com/download/Training_EN.pdf, Basic Training, *Witschi Electronic Ltd.*, Switzerland, 2003.

[10] F. Auger, P. Flandrin, Improving the Readability of Time-Frequency and Time-Scale Representations by the Reassignment Method, *IEEE Transactions on Signal Processing,* Vol.43, 1995, pp. 1068-1089.

[11] S. Su, R. Du, Signature Analysis of Mechanical Watch Movements, *The Second Asian Int. Symposium on Mechantronics*, Hong Kong, December, 2006.

[12] H. Choi, W. Williams, Improved Time/Frequency Representation of Multicomponent Signals using Exponentials Kernels, *IEEE Trans Signal Processing*, Vol. 37, 1989, pp. 862-871.